International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2014): 5.611

A Realistic Study of the Rayleigh-Bénard Problem in the Newtonian Nanofluids with a Uniform Heat Source: Free-Free Case

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Abstract: The main purpose in this investigation is to use the Buongiorno's mathematical model for studying the effect of an internal heat source which produces a constant volumetric heat on the onset of convective instability in a confined medium , filled of a Newtonian nanofluid layer and heated from below, this layer is assumed to have a low concentration of nanoparticles. The linear study in the free - free case shows that the thermal stability depends of the volumetric heat delivered by the internal source, the Brownian motion, the thermophoresis of the nanoparticles and other thermo-physical properties of nanoparticles. The studied problem will be solved analytically by converting our boundary value problem to an initial value problem, after this step we will approach the searched solutions with polynomials of high degree.

Keywords: linear stability, nanofluid, Buongiorno's model, internal heat source, free-free case.

1. Introduction

For Increasing the effective thermal conductivity of the coolant fluids in a confined geometry, we prefer using fluids containing nano-sized metallic particles (about 1-100nm) in suspension to obtain a nanofluid characterized by a high effective thermal conductivity compared to the regulars fluids (water - oil - ethylene glycol). The experiment shows that the presence of the nanoparticles in a base fluid allows us to obtain a significant growth in the thermal conductivity of the mixture (base fluid + nanoparticles), for this purpose we find that the nanofluids are currently used in the cooling of advanced electronic or nuclear systems.

The nanofluid term was introduced by Choi [1] in 1995 and remains usually used to characterize this type of colloidal suspension. Buongiorno [2] was the first researcher who treated the convective transport problem in nanofluids, he was established the conservation equations of a non-homogeneous equilibrium model of nanofluids for mass, momentum and heat transport. The thermal problem of instability in nanofluids with rigid-free and free-free boundaries was studied by Tzou [3,4] using the eigenfunction expansions method. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [5], they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

In this paper, we will study the Rayleigh -Bénard problem for the Newtonian nanofluids with a uniform heat source in the free-free case using a new type of boundary conditions for the nanoparticles which assumes the nanoparticle flux must be zero on the impermeable boundaries. D.A. Nield and A.V. Kuznetsov [6] are considered as the first ones who were used this type of boundary conditions for the nanoparticles.

The new model of boundary conditions for the nanoparticles is physically more realistic since it combines the contribution of the Brownian motion and the thermophoresis of the nanoparticles, for this reason we find currently several authors [7-11] are using this type of model to study the natural convection in nanofluids.

To show the accuracy of our method in this study, we will check some results treated by D.Yadav et al. [12] concerning the study of the convective instability of regular fluids in presence of an internal heat source which produces a constant volumetric heat in the free-free case.

2. Mathematical Formulation

We consider an infinite horizontal layer of an incompressible Newtonian nanofluid characterized by a low concentration of nanoparticles , heated uniformly from below and confined between two identical horizontal surfaces where the temperature is constant and the nanoparticle flux is zero on the boundaries, this layer will be subjected to an internal heat source which will provide a constant volumetric heat Q_s and also to the gravity field $\vec{g}$ (see **Figure 1**).The thermo-physical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the analytical formulation except for the density variation in the momentum equation which is based on the Boussinesq approximations .

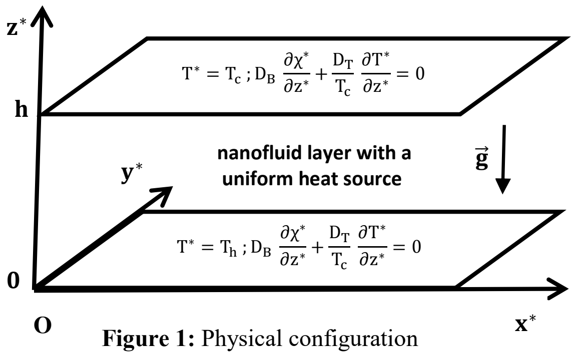

Figure 1: Physical configuration

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Within the framework of the assumptions which were made by Buongiorno [2] and Tzou [3,4] in their publications for the Newtonian nanofluids, we can write the basic equations of conservation which govern our problem in dimensionless form as follows:

$$\vec{\nabla}^*.\vec{V}^* = 0 \tag{1}$$

$$\rho_f\left[\frac{\partial\vec{V}^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\vec{V}^*\right] = -\vec{\nabla}^*P^* + \eta\vec{\nabla}^{*2}\vec{V}^* + \{\rho_0[1-\beta(T^*-T_c)](1-\chi^*) + \rho_p\chi^*\}\vec{g} \tag{2}$$

$$(\rho c)_f\left[\frac{\partial T^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)T^*\right] = \kappa\vec{\nabla}^{*2}T^* + Q_s + (\rho c)_p\left[D_B\vec{\nabla}^*\chi^*.\vec{\nabla}^*T^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^*T^*.\vec{\nabla}^*T^*\right] \tag{3}$$

$$\frac{\partial\chi^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\chi^* = D_B\vec{\nabla}^{*2}\chi^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^{*2}T^* \tag{4}$$

where ρ_f is the density of the base fluid , ρ_0 is the fluid density at reference temperature T_c , ρ_p is the nanoparticle density , β is the thermal expansion coefficient of the base fluid , $\vec{V}^*$ is the velocity vector , t^* is the time , P^* is the pressure , T^* is the temperature , χ^* is the volume fraction of nanoparticles , η is the viscosity of the nanofluid , κ is the thermal conductivity of the nanofluid , D_B is the Brownian diffusion coefficient , D_T is the thermophoretic diffusion coefficient , $(\rho c)_f$ is the heat capacity of the base fluid , $(\rho c)_p$ is the heat capacity of the nanoparticle , (x^*, y^*, z^*) are the cartesian coordinates , $\vec{\nabla}^*$ is the vector differential operator .

In this study the asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks).

If we consider the following dimensionless variables:

$$(x^*; y^*; z^*) = h(x; y; z) \; ; \; t^* = \frac{h^2}{\alpha}t \; ; \; \vec{V}^* = \frac{\alpha}{h}\vec{V}$$

$$P^* = \frac{\eta\alpha}{h^2}P \; ; \; T^* - T_c = (T_h - T_c)T \; ; \; \chi^* - \chi_0^* = \chi_0^*\chi$$

Then, we can get from equations (1)-(4) the following adimensional forms:

$$\vec{\nabla}.\vec{V} = 0 \tag{5}$$

$$P_r^{-1}\left[\frac{\partial\vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\vec{\nabla}(P + R_M z) + \vec{\nabla}^2\vec{V} + [(1-\chi_0^*)R_a T - R_N\chi - \chi_0^* R_a T\chi]\vec{e}_z \tag{6}$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T = \vec{\nabla}^2 T + H_s + N_B L_e^{-1}\vec{\nabla}\chi.\vec{\nabla}T + N_A N_B L_e^{-1}\vec{\nabla}T.\vec{\nabla}T \tag{7}$$

$$\frac{\partial\chi}{\partial t} + (\vec{V}.\vec{\nabla})\chi = L_e^{-1}\vec{\nabla}^2\chi + N_A L_e^{-1}\vec{\nabla}^2 T \tag{8}$$

Where $P_r = \eta/\rho_f\alpha$ is the Prandtl number , $L_e = \alpha/D_B$ is the Lewis number , $H_s = Q_s h^2/\kappa(T_h - T_c)$ is the dimensionless constant heat source strength , $R_a = \rho_0 g\beta h^3(T_h - T_c)/\eta\alpha$ is the thermal Rayleigh number , $R_M = [\rho_0(1-\chi_0^*) + \rho_p\chi_0^*]gh^3/\eta\alpha$ is the basic density Rayleigh number , $R_N = (\rho_p - \rho_0)\chi_0^* gh^3/\eta\alpha$ is the concentration Rayleigh number, $N_A = D_T(T_h - T_c)/D_B T_c\chi_0^*$ is the modified diffusivity ratio , $N_B = (\rho c)_p\chi_0^*/(\rho c)_f$ is the modified particle-density increment , $\alpha = \kappa/(\rho c)_f$ is the thermal diffusivity of the nanofluid , χ_0^* is the reference value for the nanoparticle volume fraction .

2.1 Basic Solution

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying in the z-direction only, therefore:

$$\vec{V}_b = \vec{0} \tag{9}$$

$$T_b = 1 \; ; \; \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \;\; \text{at} \;\; z = 0 \tag{10}$$

$$T_b = 0 \; ; \; \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \;\; \text{at} \;\; z = 1 \tag{11}$$

If we introduce the precedent results into equations (6)-(8), we obtain:

$$\vec{\nabla}(P_b + R_M z) = [(1-\chi_0^*)R_a T - R_N\chi - \chi_0^* R_a T\chi]\vec{e}_z \tag{12}$$

$$\frac{d^2T_b}{dz^2} + N_B L_e^{-1}\left(\frac{d\chi_b}{dz}\frac{dT_b}{dz}\right) + N_A N_B L_e^{-1}\left(\frac{dT_b}{dz}\right)^2 = -H_s \tag{13}$$

$$\frac{d^2\chi_b}{dz^2} + N_A\frac{d^2T_b}{dz^2} = 0 \tag{14}$$

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

$$\chi_b = N_A(1 - T_b) + \chi_0 \tag{15}$$

Where $\chi_0 = (\chi^* - \chi_0^*)/\chi_0^*$ is the relative nanoparticle volume fraction at $z = 0$.

If we take into account the expression (15), we can get after simplification of the equation (13):

$$\frac{d^2T_b}{dz^2} = -H_s \tag{16}$$

Finally, we obtain after an integrating of the equation (16) between 0 and 1:

$$T_b = -\frac{1}{2}H_s z^2 + \left(\frac{1}{2}H_s - 1\right)z + 1 \tag{17}$$

$$\chi_b = \frac{1}{2}N_A H_s z^2 - N_A\left(\frac{1}{2}H_s - 1\right)z + \chi_0 \tag{18}$$

2.2 Stability Analysis

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows

$$T = T_b + T' ; \vec{V} = \vec{V}_b + \vec{V}' \; ; P = P_b + P' \; ; \; \chi = \chi_b + \chi' \tag{19}$$

In the framework of the Boussinesq approximations, we can neglect the terms coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

$$\vec{\nabla}.\vec{V}' = 0 \tag{20}$$

$$P_r^{-1}\frac{\partial\vec{V}'}{\partial t} = -\vec{\nabla}P' + (R_a T' - R_N\chi')\vec{e}_z + \vec{\nabla}^2\vec{V}' \tag{21}$$

$$\frac{\partial T'}{\partial t} + f_1 w' = \vec{\nabla}^2 T' + f_2\frac{\partial T'}{\partial z} + f_3\frac{\partial\chi'}{\partial z} \tag{22}$$

$$\frac{\partial\chi'}{\partial t} + f_4 w' = N_A L_e^{-1}\vec{\nabla}^2 T' + L_e^{-1}\vec{\nabla}^2\chi' \tag{23}$$

Where $f_1 = DT_b$, $f_2 = N_B L_e^{-1}D(\chi_b + 2N_A T_b)$, $f_3 = N_B L_e^{-1}DT_b$
$f_4 = D\chi_b$ and $D = d/dz$.

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After application of the curl operator twice to equation (21) and using the equation (20), we obtain the following z-component of the momentum equation:

$$P_r^{-1}\frac{\partial}{\partial t}\vec{\nabla}^2 w' = \vec{\nabla}^4 w' + R_a \vec{\nabla}_2^2 T' - R_N \vec{\nabla}_2^2 \chi' \quad \textbf{(24)}$$

Where $\vec{\nabla}_2^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$ is the two-dimensional Laplacian operator on the horizontal plane.

Analyzing the disturbances into normal modes, we can simplify the equations (22) - (24) by assuming that the perturbation quantities are of the form:

$$(w', T', \chi') = (\mathcal{w}(z), \mathcal{T}(z), \mathcal{X}(z))e^{[i(k_x x + k_y y)+\sigma t]} \quad \textbf{(25)}$$

After introducing the expressions (25) into equations (22) - (24), we obtain:

$$P_r^{-1}\sigma(D^2 - k^2)\mathcal{w} = (D^2 - k^2)^2\mathcal{w} - k^2 R_a \mathcal{T} + k^2 R_N \mathcal{X} \quad \textbf{(26)}$$

$$\sigma\mathcal{T} + f_1\mathcal{w} = (D^2 - k^2)\mathcal{T} + f_2 D\mathcal{T} + f_3 D\mathcal{X} \quad \textbf{(27)}$$

$$\sigma\mathcal{X} + f_4\mathcal{w} = N_A L_e^{-1}(D^2 - k^2)\mathcal{T} + L_e^{-1}(D^2 - k^2)\mathcal{X} \quad \textbf{(28)}$$

Where σ is the dimensionless growth rate, k_x and k_y are respectively the dimensionless waves numbers along the x and y directions , $k = \sqrt{k_x^2 + k_y^2}$ is the resultant dimensionless wave number.

In the free-free case, the equations (26) - (28) will be solved subject to the following boundary conditions:

$$\mathcal{w} = D^2\,\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = 0 \quad \text{at} \quad z = 0; 1 \quad \textbf{(29)}$$

2.3 Method of Solution

In this study we assume that the principle of exchange of stability is valid. As we are interested in a stationary stability study characterized by $\sigma = 0$, then the equations (26)-(28) become:

$$(D^2 - k^2)^2\mathcal{w} - k^2 R_a\mathcal{T} + k^2 R_N\mathcal{X} = 0 \quad \textbf{(30)}$$

$$f_1\mathcal{w} - (D^2 - k^2)\mathcal{T} - f_2 D\mathcal{T} - f_3 D\mathcal{X} = 0 \quad \textbf{(31)}$$

$$f_4\mathcal{w} - N_A L_e^{-1}(D^2 - k^2)\mathcal{T} - L_e^{-1}(D^2 - k^2)\mathcal{X} = 0 \quad \textbf{(32)}$$

We can solve the equations (30) - (32) which are subjected to the conditions (29), by making a suitable change of variables that makes the number of variables equal to the number of boundary conditions, to obtain a set of eight first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz}u_i(z) = a_{ij}u_j(z)\,; \quad 1 \le i, j \le 8 \quad \textbf{(33)}$$

With:

$$a_{ij} = a_{ij}(z, k, R_a, H_s, N_B, L_e, R_N, N_A)$$

The solution of the system (33) in matrix notation can be written as follows:

$$U = BC \quad \textbf{(34)}$$

Where $B = \left(\left(b_{ij}(z)\right)_{\substack{1\le i\le 8\\ 1\le j\le 8}}\right)$ is a square matrix of order 8×8, $U = \left(\left(u_i(z)\right)_{1\le i\le 8}\right)^T$ is the unknown vector column of our problem , $C = \left(\left(c_j\right)_{1\le j\le 8}\right)^T$ is a constant vector column.

If we assume that the matrix B is written in the following form:

$$B = \left(\left(u_i^j(z)\right)_{\substack{1\le i\le 8\\ 1\le j\le 8}}\right) \quad \textbf{(35)}$$

Therefore, the use of four boundary conditions at $z = 0$, allows us to write each variable $u_i(z)$ as a linear combination only for four functions $u_i^j(z)$, such that:

$$b_{ij}(0) = u_i^j(0) = \delta_{ij} \quad \textbf{(36)}$$

After introducing the new expressions of the variables $u_i(z)$ in the system (33), we will obtain the following equations:

$$\frac{d}{dz}u_i^j(z) = a_{il}u_l^j(z)\,; \quad 1 \le i, l, j \le 8 \quad \textbf{(37)}$$

For each value of j , we must solve a set of eight first order ordinary differential equations which are subjected to the initial conditions (36) , by approaching these variables with real power series defined in the interval $[0,1]$ and truncated at the order N , such that:

$$u_i^j(z) = \sum_{p=0}^{p=N} d_p^{i,j} z^p \quad \textbf{(38)}$$

A linear combination of the solutions $u_i^j(z)$ satisfying the boundary conditions (29) at $z = 1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a, k, H_s, N_B, L_e, R_N, N_A) = 0 \quad \textbf{(39)}$$

If we give to each control parameter $(H_s, N_B, L_e, R_N, N_A)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number R_a which corresponds to a fixed wave number k and verifies the dispersion relation (39). After that, we will find a set of points (k, R_a) which help us to plot our curve and find the critical value (k_c, R_{ac}) which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

2.4 Validation of the Method

The truncation order N which correspond at the convergence of our method is determined, when the five digits after the comma of the critical thermal Rayleigh number R_{ac} remain unchanged (see **Table 1**).

To validate our method, we compared our results with those obtained by Dhananjay Yadav et al. **[12]** concerning the Rayleigh-Bénard problem for the regular fluids in the presence of an internal heat source. To make this careful comparison, we must take into consideration the restrictions: $L_e^{-1} = R_N = N_A = N_B = 0$ in the governing equations of our problem (see **Table 2**).

According to the below results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. Briefly, the convergence of the results depends greatly on the truncation order N of the power series and also of the heat source strength H_s. Finally, to ensure the accuracy of our obtained critical values for the studied Newtonian nanofluids, we will take as truncation order : $N = 32$

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Table 1: The exact stationary instability threshold of a Newtonian nanofluid for $H_s = 20$ and $H_s = 60$.

$N_B = 0.01, L_e = 100, R_N = 1, N_A = 0.1$				
N	$H_s = 20$		$H_s = 60$	
	k_c	R_{ac}	k_c	R_{ac}
25	2.47227	449.84172	2.72311	209.35582
26	2.47233	449.83882	2.72358	209.34532
27	2.47232	449.83940	2.72346	209.34765
28	2.47232	449.83935	2.72348	209.34743
29	2.47232	449.83936	2.72348	209.34749
30	2.47232	449.83935	2.72348	209.34743
31	2.47232	449.83936	2.72348	209.34746
32	2.47232	449.83936	2.72348	209.34745
33	2.47232	449.83936	2.72348	209.34745
34	2.47232	449.83936	2.72348	209.34745
35	2.47232	449.83936	2.72348	209.34745
Exact value	**2.47232**	**449.83936**	**2.72348**	**209.34745**

Table 2: The obtained stationary instability threshold by D. Yadav et al. **[12]** and us, for the regular fluids for various values of H_s.

H_s	D. Yadav et all		Present study		
	k_c	R_{ac}	k_c	R_{ac}	N
0	2.221	657.51128	2.22144	657.51138	18
1	2.223	656.69234	2.22287	656.69244	20
2	2.227	654.25598	2.22715	654.25608	23
10	2.340	589.42140	2.33980	589.42136	28
20	2.527	473.46423	2.52736	473.46404	29
30	2.657	381.12034	2.65717	381.12011	29
40	2.739	315.26816	2.73859	315.26794	31
60	2.829	232.11493	2.82940	232.11473	31

3. Results and Conclusion

To study the effect of a parameter $(H_s, N_B, L_e, R_N, N_A)$ on the onset of the convective instability for the newtonian nanofluids in the presence of an internal heat source which produces a constant volumetric heat, we must determine the variation of the critical thermal Rayleigh number R_{ac} as a function of the heat source strength H_s for different values of this parameter (see **Figure 2 - Figure 5**).

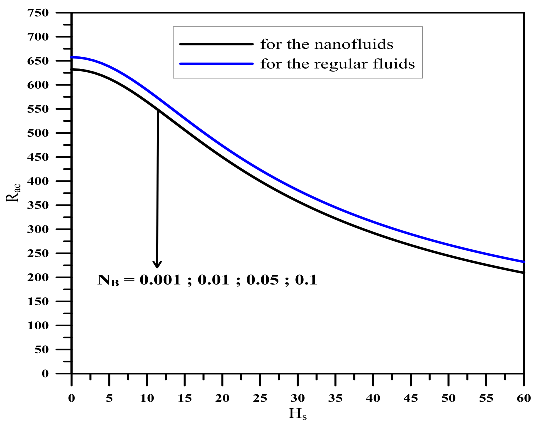

Figure 2: Plot of R_{ac} as a function of H_s and N_B for $L_e = 100, R_N = 1, N_A = 0.1$

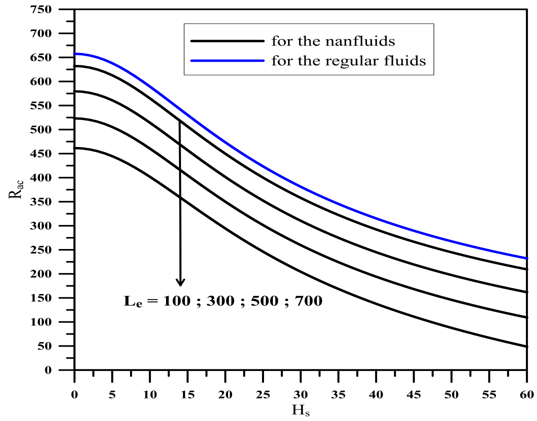

Figure 3: Plot of R_{ac} as a function of H_s and L_e for $N_B = 0.01, R_N = 1, N_A = 0.1$

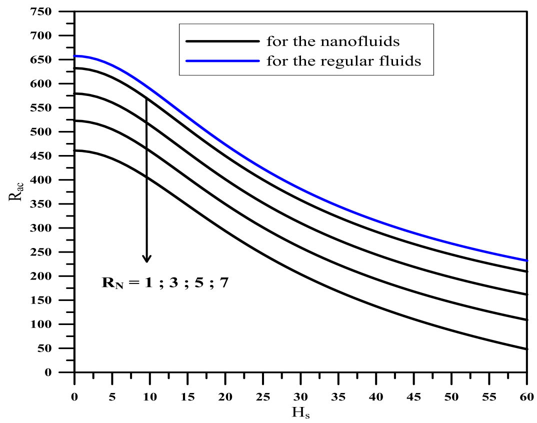

Figure 4: Plot of R_{ac} as a function of H_s and R_N for $N_B = 0.01, L_e = 100, N_A = 0.1$

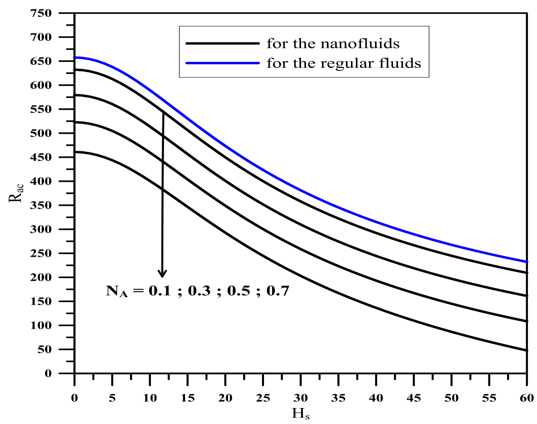

Figure 5: Plot of R_{ac} as a function of H_s and N_A for $N_B = 0.01, L_e = 100, R_N = 1$

In this paper, we have examined the effect of a uniform heat source on the onset of convection in a Newtonian nanofluid layer heated uniformly from below in the case where the nanoparticle flux is zero on the impermeable boundaries, this study shows that the modified particle-density increment N_B has a negligible effect on the onset of the convective instability in the Newtonian nanofluids , therefore the contribution of Brownian motion and thermophoresis in the thermal energy equation (7) can be neglected. Rather, the Brownian motion and thermophoresis directly enter in the equation (8) expressing the conservation of nanoparticles.

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The precedent Figures show also that an increase either in the heat source strength H_s , in the Lewis number L_e , in the concentration Rayleigh number R_N or in the modified diffusivity ratio N_A allows us to accelerate the onset of the convection, hence they have a destabilizing effect.

The obtained results may be summarized as follows;

- The heat source strength H_s increases the energy supply to the system, and hence increases the driving force which accelerates the onset of the convection.

- The modified particle-density increment N_B has a negligible effect, because this parameter always appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number L_e near the temperature gradient and the volume fraction gradient of nanoparticles , such that:

$$N_B \sim 10^{-3} - 10^{-1} \;;\; L_e \sim 10^2 - 10^3$$

- The presence of the nanoparticles in a base fluid allows us to destabilize it, this result can be interpreted as an increase in the volume fraction of nanoparticles, increases the Brownian motion and the thermophoresis of the nanoparticles, which cause the destabilizing effect.
- The regular fluids are more stable than the nanofluids.
- An increase in the temperature difference between the horizontal plates allows us to decrease the critical thermal Rayleigh number R_{ac} , this result can be explained by the increase in the buoyancy forces which destabilizes the system.

- To ensure the stability of the nanofluids, we can use the less dense nanoparticles or the ones which are having a small heat capacity.

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