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A New Continuous Wavelet Family and its Application to Analysis of Elementary Periodic Signals

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Abstract: ***The successive derivatives of both gaussian functions and Cauchy–Lorentz distribution functions are the family of one dimensional continuous wavelet and in this paper we demostrated that the product of these functions are also forms a family of one dimensional continuous wavelet and are successfully use in Analysis of elementary periodic signals.***

Keywords: One dimensional continuous wavelets, Wavelet Analysis, Wavelet Scalogram.

1. Introduction

In this paper we introduce a new continuous wavelet family and we perform both Fourier analysis and wavelet analysis (using this new wavelet) of elementary periodic signals. Then, we compare the spectral information found in each of them. This is done by two examples, the first one in Analysis of a Sum of Two Periodic Signals and the second one in Analysis of a Complicated Signal [8]. In section-2 we discuss the requirement for a function to be Mother wavelet [1], [2], [3] , [4] and [5], in section-3 we verified all the conditions for new family of function to be a wavelet [6] and in the last section-4 we demonstrate the use of the new wavelets in analyzing of elementary periodic signals. [7], [8] and [9].

2. Requirement for Continuous Wavelet

In order to be classified as a Mother wavelet, a function must satisfy certain mathematical criteria. These are:

2.1. The wavelet must have finite energy i.e.

$$\Psi(t) \in L^2(R) \text{ Or } E = \int_{-\infty}^{\infty} |\Psi(t)|^2 \, dt < \infty \tag{1}$$

2.2. If $\hat{\Psi}(\omega)$ is the Fourier transform of $\Psi(t)$ i.e.

$$\hat{\Psi}(\omega) = \int_{-\infty}^{\infty} \Psi(t) e^{-i\omega t} dt \tag{2}$$

Then the following condition must hold

$$C_\psi = \int_0^{\infty} \frac{\left|\hat{\Psi}(\omega)\right|^2}{\omega} df < \infty \tag{3}$$

(The above equation is known as admissibility condition and C_ψ is called the admissibility constant)

This implies that the wavelet has no zero frequency component i.e. $\hat{\Psi}(0) = 0$ or to put this in another way, the wavelet $\Psi(t)$ must have zero mean i.e.

$$\int_{-\infty}^{\infty} \Psi(t) \, dt = 0 \tag{4}$$

3. The New Wavelet Family

The present continuous wavelet family is built by taking the successive differentiation of the function $\psi(t) = e^{-t^2} \cdot \frac{1}{1+t^2}$ which is the product of the Gaussian distribution function $\left(e^{-t^2}\right)$ and Cauchy–Lorentz distribution function $\left(\frac{1}{1+t^2}\right)$.

The integer k is the parameter of this family and represents the order of the wavelet of the family i.e. $\psi_k(t)$ is a Continuous Wavelet for each k and

$$\psi_k(t) = C_k \frac{d^k}{dx^k}(\psi(t)) \tag{5}$$

In the previous formula, C_k is such that $\|\psi_k(t)\|^2 = 1$ where $\psi_k(t)$ is the k^{th} derivative of ψ and

$$C_k = \left(\int_{-\infty}^{\infty} \left| \frac{d^k}{dt^k} \left[\psi(t) \right] \right|^2 dt \right)^{-\frac{1}{2}} \tag{6}$$

Such that $\int_{-\infty}^{\infty} |\psi_k(t)|^2 dt = 1, \forall \quad k = 1, 2, 3........., 7$.

[Refer Table-I]

Since we know that the Fourier transform of e^{-t^2} i.e.

$$\mathscr{F}\left(e^{-t^2}\right) = \sqrt{\pi} e^{-\omega^2/4} \text{ and } \mathscr{F}\left(\frac{1}{1+t^2}\right) = \pi e^{-|\omega|}$$

Hence by using convolution, we have

$$\hat{\Psi}(\omega) = \mathscr{F}\left(e^{-t^2} \cdot \frac{1}{1+t^2}\right) = \pi\sqrt{\pi} \int_{-\infty}^{\infty} e^{-\tau^2/4} \cdot e^{-|\omega-\tau|} d\tau$$

$$= \pi^2 \left[e^{1-\omega} \left(erf(\omega/2 - 1) + 1 \right) - e^{1+\omega} \left(erf(\omega/2 + 1) - 1 \right) \right]$$

Where $erf(\alpha) = \frac{2}{\sqrt{\pi}} \int_0^{\alpha} e^{-\xi^2} d\xi$

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$\therefore$

$$\hat{\Psi}_k(\omega) = \mathscr{F}(\psi_k(t)) = (i\omega)^k \pi^2 \left[e^{1-\omega}\left(erf\left(\omega/2-1\right)+1\right) - e^{1+\omega}\left(erf\left(\omega/\right.\right.\right.$$

$$\because \ \mathscr{F}\left(\frac{d^n\psi(t)}{dt^n}\right) = (i\omega)^n \mathscr{F}(\psi(t))$$

It has been verified that for the above functions $\psi_k(x)$,

3.1 $\int_{-\infty}^{\infty} |\psi_k(x)|^2 dx = 1 < \infty$, $\forall \quad k = 1,2,3.........,7$.

32. $C_{\psi_k} = \int_{-\infty}^{\infty} \frac{\left|\hat{\Psi}_k(\omega)\right|^2}{\omega} df < \infty$ **and** $\int_{-\infty}^{\infty} \psi_k(t)\,dt = 0$

$\forall \quad k = 1,2,3.........,7$. [Refer Table-I]

Moreover $\int_{-\infty}^{\infty} t^n \psi_k(t)\,dx = 0$, $\forall \quad k = 2,3.........,7$

and $n = 1,2,3.......(k-1)$.

Hence the functions $\psi_k(x)$, $\forall \quad k = 1,2,3.........,7$ forms a family of continuous wavelets. Here after this family is named as raees wavelet family, shortly rsw associated with numbers 1, 2……7, according to their order. The graphical representation of the family is given from Fig-1.1 to Fig-1.7.

4. Analysis of Elementary Periodic Signals Using new Wavelets

These examples show how to obtain spectral information of a signal using continuous wavelet transform analysis. Signal Processing Toolbox in Matlab is needed to run the example. The tools best suited for a spectral analysis of signals are those based on the FFT. While wavelets are not specifically designed for spectral analysis, we can recover some spectral information using wavelet analysis.

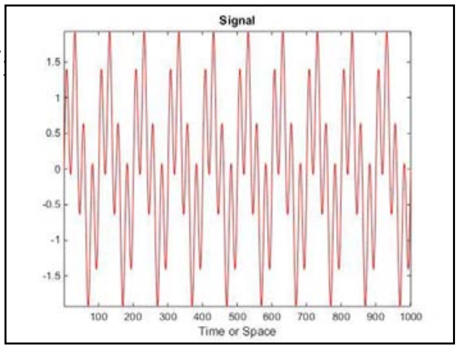
In this example, we perform both Fourier analysis and wavelet analysis of various elementary periodic signals. Then, we compare the spectral information found in each of them.

Example 1: Analysis of a Sum of Two Periodic Signals:

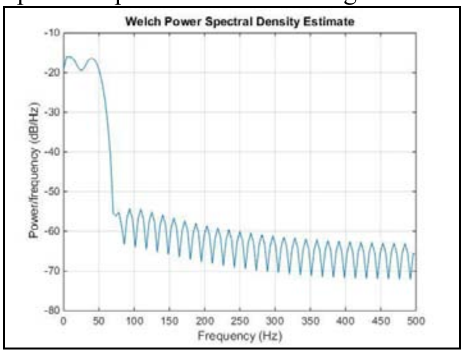
We now define a periodic signal, which is a sum of two sines of frequencies F1 = 10 and F2 = 40. We will compute the power spectral density (PSD) estimate using spectral estimation and the continuous wavelet transform using the wavelet rsw4.

First, we build and plot the analyzed signal.

$$x = \sin(2\pi tF1) + \sin(2\pi tF2).$$



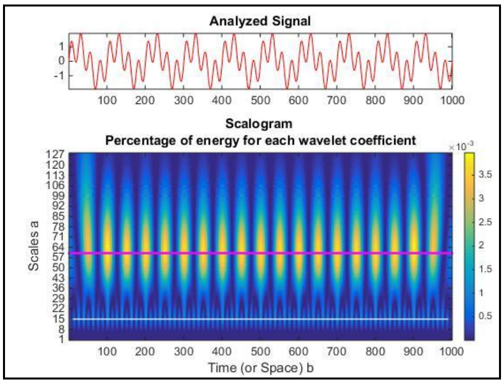
Then, compute and plot the PSD of the signal.



Using an interpolated function of the PSD function and locating the two first local maxima of this function, we see approximations of the two "true" frequencies as 7.8 and 39.1.

Using the function scal2freq in Matlab, we compute the correspondence table of scales and frequencies for the rsw4 wavelet. Then, we find the scales corresponding to the frequencies F1 = 10 and F2 = 40.

Now, we compute the continuous wavelet transform of the signal, and then, plot the scalogram of wavelet coefficients and two horizontal lines corresponding to the scales Sca_1=60 and Sca_2=15 linked to the frequencies F1 and F2. We can see that these lines are associated to the local maxima of energy in the scalogram. Note that the component with the lowest frequency contains the most energy.



Example 2: Analysis of a More Complicated Signal:

Now we design a more complicated signal, which is a piecewise sine defined on three adjacent intervals: frequency F1 = 10 for the intervals [0 0.25] and [0.75 1] which correspond to the indices 1:250 and 750:1000, and frequency F2 = 40 for the interval [0.25 0.75] which corresponds to the indices 251:749.

We build the analyzed signal,

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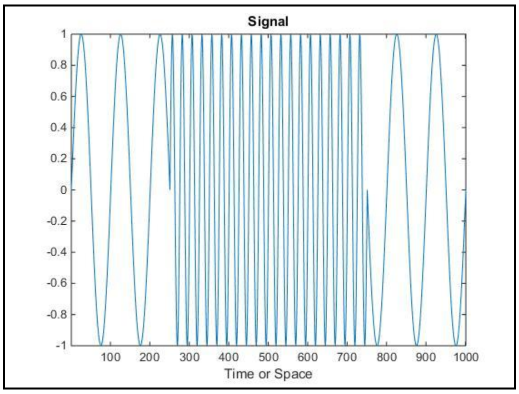
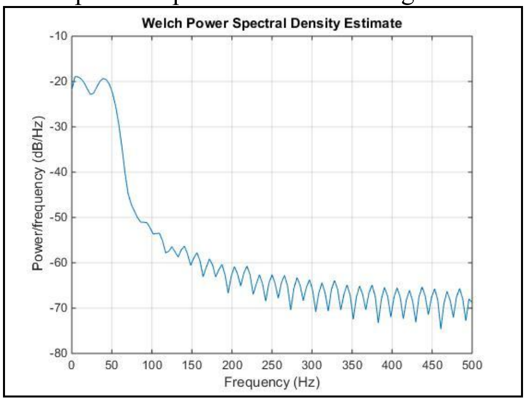
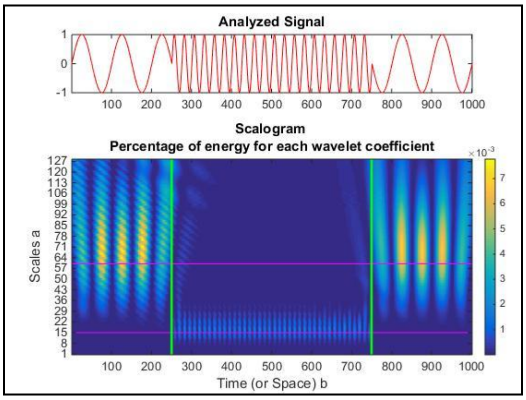
x = sin (2πtF1) for ((t<0.25) + (t>0.75)) + sin (2πtF2) for (t>0.25).*(t<0.75);



Now we compute and plot the PSD of the signal.



The spectrum is very similar to that achieved for the case of the sum of two sines. It gives no information on the location of any events in time or space. Now, compute the continuous wavelet transform of the signal and plot the scalogram of wavelet coefficients and two horizontal lines corresponding to the scales Sca_1 and Sca_2 linked to the frequencies F1 and F2. We will also draw the two vertical lines that separate the intervals.



Note that the wavelet analysis works very effectively to detect time or space events. The intervals with the different frequencies are clearly detected.

The wavelet procedure we used was to use both the continuous wavelet transform and the function scal2freq to compute a correspondence table between values of scales and frequencies. This table depends on the selected wavelet. Then, we searched the scale corresponding to the frequency or vice versa.

This procedure produces approximated values; however they are generally good enough when applied to signals that are not too complex. This procedure is also very efficient at detecting time or space events

5. Figures and Tables

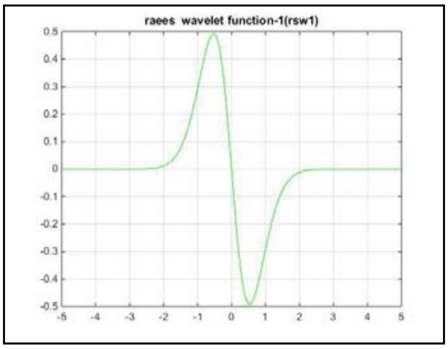


Figure 1.1

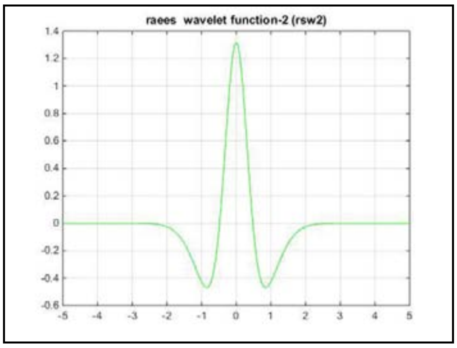


Figure 1.2

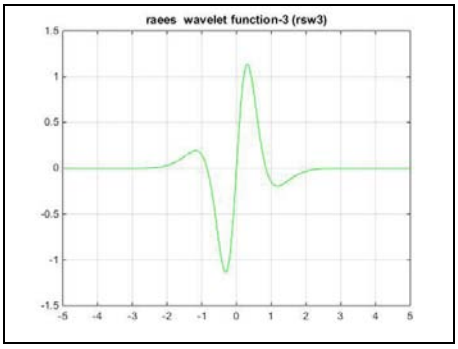


Figure 1.3

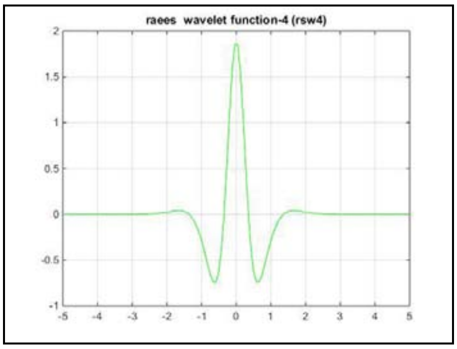


Figure 1.4

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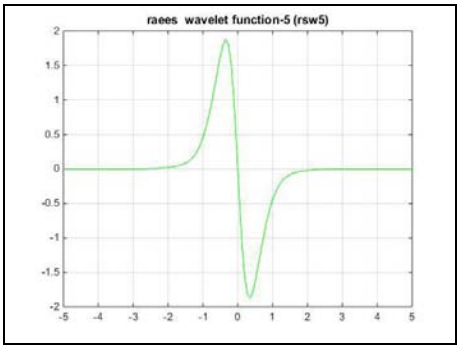

Figure 1.5

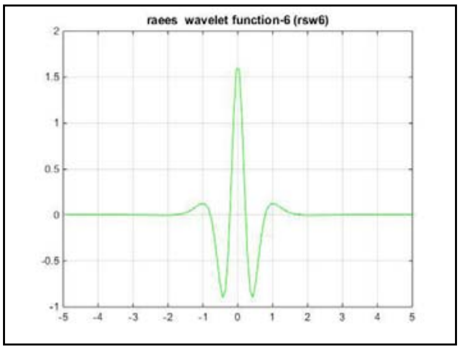

Figure 1.6

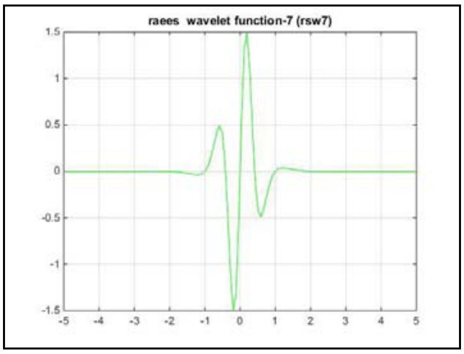

Figure 1.7

Table 1

Order of the Wavelet (k)	C_{ψ_k}	C_k
1	–119.07533914891	1.601616579725480
2	431.3058522364652	9.112718044441433
3	–3441.0125590885	95.9742033130829
4	46109.58291987393	1596.419728602841
5	–935295.5508285811	38948.93123589542
6	27136072.04053418	1330506.195338057
7	–1081545882.025461	61348820.76589487

6. Conclusion

In this paper we introduce a new Continuous Wavelet family (rsw) and are used in Analysis of elementary periodic signals.

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