

The Advance Method for the Optimum Solution of a Transportation Problem

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Abstract: *Transportation problem is one of the predominant areas of operations research, widely used as a decision making tool in engineering, business management and many other fields. In this paper, we present. "The Advance method for the optimum solution of a Transportation Problem." is proposed to find an optimum solution. The algorithm for proposed method discussed in this paper gives an initial as well as either optimal solution or near to optimal solution. The Numerical examples are provided to illustrate the proposed algorithm. It can be seen that the proposed algorithm gives an optimal solution nearly comparable to MODI's method in less time period.*

Keywords: Transportation, optimality, basic feasible solution, linear programming, optimal solution

1. Introduction

The transportation problem and cycle canceling methods are classical in optimization. The usual attributions are to the 1940's and later. However, Tolsto (1930) was a pioneer in operations research and hence wrote a book on transportation planning which was published by the National Commissariat of

Transportation of the Soviet Union, an article called Methods of ending the minimal total kilometrage in cargo-transportation planning in space, in which he studied the transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost cycle in its residual graph. He might have been the first to observe that the cycle condition is necessary for optimality. Moreover, he assumed, but did not explicitly state or prove, the fact that checking the cycle condition is also sufficient for optimality.

The transportation problem is concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the transportation cost between each

Source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from sources to destinations that minimizes the total transportation cost.

There are various types of transportation models and the simplest of them was first presented by Hitchcock (1941). It was further developed by Koopmans (1949) and Dantzig (1951). Several extensions of transportation model and methods have been subsequently developed. Transportation Problem (TP) is based on supply and demand of commodities transported from several sources to the different destinations. The sources from which we need to

transport refer the supply while the destination where commodities arrive referred the demand. It has been seen that on many occasion, the decision problem can also be formatting as TP. In general we try to minimize total transportation cost for the commodities transporting from source to destination.

There are two types of Transportation Problem namely (1) Balanced Transportation Problem and (2) Unbalanced Transportation Problem.

Definition of Balanced Transportation Problem: A Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand.

Definition of Unbalanced Transportation Problem: A Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

Basic Feasible Solution

Any solution $X_{ij} \geq 0$ is said to be a feasible solution of a transportation problem if it satisfies the constraints. The feasible solution is said to be basic feasible solution if the number of nonnegative allocations is equal to $(m+n-1)$ while satisfying all rim requirements, i.e., it must satisfy requirement and availability constraint. There are three ways to get basic feasible solution.

- 1) North West Corner Rule
- 2) Minimum Cost Method or Matrix Minima Method
- 3) Vogel's Approximation Method or Regret Method

Optimal Solution: A feasible solution of transportation problem is said to be optimal if it minimizes the total cost of transportation. There always exists an optimal solution to a balanced transportation problem. We start with initial basic feasible solution to reach optimal solution which is obtained from above three methods. We then check whether the number of allocated cells is exactly equal to $(m+n-1)$, where m and n are number of rows and columns respectively. It

works on the assumption that if the initial basic feasible solution is not basic, then there exists a loop. Here, we explain the MODI'S method to attain the optimality.

Algorithm of MODIFIED DISTRIBUTION (MODI) METHOD

Step: 1 For an initial basic feasible solution with (m+n-1) occupied (basic) cells, calculate u_i and v_j values for rows and columns respectively using the relationship $C_{ij} = u_i + v_j$ for all allocated cells only. To start with assume any one of the u_i or v_j to be zero.

Step :2 For the unoccupied (non-basic) cells, calculate the cell evaluations or the net evaluations as $\Delta_{ij} = C_{ij} - (u_i + v_j)$.

Step: 3

- a) If all $\Delta_{ij} > 0$, the current solution is optimal and unique.
- b) If any $\Delta_{ij} = 0$, the current solution is optimal, but an alternate solution exists.
- c) If any $\Delta_{ij} < 0$, then an improved solution can be obtained; by converting one of the basic cells to a non basic cells and one of the non basic cells to a basic cell. Go to step IV.

Step: 4 select the cell corresponding to most negative cell evaluation. This cell is called the entering cell. Identify a closed path or a loop which starts and ends at the entering cell and connects some basic cells at every corner. It may be noted that right angle turns in this path are permitted.

Step:5 Put a + sign in the entering cell and mark the remaining corners of the loop alternately with - and + signs, with a plus sign at the cell being evaluated. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest one with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. This quantity is added to all the cells on the path marked with plus sign and subtract from those cells mark with minus sign. In this way the unoccupied cell under consideration becomes an occupied cell making one of the occupied cells as unoccupied cell. Repeat the whole procedure until an optimum solution is attained i.e. Δ_{ij} is positive or zero. Finally calculate new transportation cost.

2. Algorithm for Proposed Method

Step: 1 Construct the transportation matrix from given transportation problem.

Step: 2 Select the minimum odd cost from all the cost in matrix.

Step: 3 Subtract the selected least odd cost only from odd cost in matrix, now there will be atleast one one non zero and remaining all cost become even.

Step: 4 Compare the minimum of supply or demand whichever is minimum then allocate the min supply or demand at the place of minimum value of related row or column.

If tie at the place of minimum value in supply or demand then allocate at the maximum of supply or demand is observed.

Step: 5 After performing step-4, delete the row or column for further allocation where supply from a given source is depleted or the demand for a given destination is satisfied.

Step: 6 Repeat step-4 and step-5 unless and until all the demands are satisfied and all the supplies are exhausted.

3. Numerical Examples (Proposed Method):

1) Consider the following cost minimizing transportation problem.

	D_1	D_2	D_3	Supply
S_1	11	9	6	40
S_2	12	14	11	50
S_3	10	8	10	40
Demand	55	45	30	Total=130

By applying the proposed Method, Allocations are obtained as follows,

	D_1	D_2	D_3	Supply
S_1	11	9	10	40
S_2	50	14	30	50
S_3	10	8	35	40
Demand	55	45	30	Total=130

The Total cost from these allocations is 1200 units.

2) Consider the following cost minimizing transportation problem,

	D_1	D_2	D_3	D_4	Supply
S_1	13	18	30	8	8
S_2	55	20	25	40	10
S_3	30	6	50	10	11
Demand	4	7	6	12	Total=29

By applying the proposed Method, Allocations are obtained as follows,

	D_1	D_2	D_3	D_4	Supply
S_1	4	18	30	4	8
S_2	55	4	25	5	10
S_3	30	6	50	8	11
Demand	4	7	6	12	Total=29

The Total cost from these allocations is 412 units.

3) Consider the following cost minimizing transportation problem,

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	Total=34

By applying the proposed Method, Allocations are obtained as follows,

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	Total =34

The Total cost from these allocations is 743 units.

4) Consider the following cost minimizing transportation problem,

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	3	4	6	8	9	20
S_2	2	10	1	5	8	30
S_3	7	11	20	40	3	15
S_4	2	1	9	14	16	13
Demand	40	6	8	18	6	Total =78

By applying the proposed Method, Allocations are obtained as follows,

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	3	4	6	8	9	20
S_2	2	10	1	5	8	30
S_3	7	11	20	40	3	15
S_4	2	1	9	14	16	13
Demand	40	6	8	18	6	Total =78

The Total cost from these allocations is 267 units.

4. Comparison

Comparison of total cost of Transportation Problem of above examples between MODI method and Proposed method is:

Table No:	Problem Dimension	MODI'S Method	Proposed Method
1	3×3	1320	1200
2	3×4	412	412
3	3×4	743	743
4	4×5	267	267

5. Conclusion

In this Paper, we have developed the algorithm is very helpful as having less computations and also required the short time of period for getting the optimal solution.

Also in this paper we have described the comparison between the MODI'S Method and the Proposed method and also the solution is nearly same according to the MODI'S method.

From this method we are getting an optimal solution without solving the Initial Basic Feasible Solution. Thus the Proposed method Provide an optimal solution nearly to the MODI'S Method.

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