Regular Interval Valued Fuzzy Soft Matrices

Dr. N. Sarala[1], M. Prabhavathi[2]

[1]Department of Mathematics, A.D.M .College for Women (Auto), Nagapattinam, India)

[2]Department of Mathematics, E.G.S. Pillay Arts & Science College, Nagapattinam, India)

Abstract: ***In this paper, we define regular interval valued fuzzy soft matrices as a generalization of regular fuzzy soft matrices and obtain the structure of Row space, column space of an Interval valued fuzzy soft matrices (IVFSM).***

Keywords: Soft set, fuzzy soft set, inter fuzzy soft matrix, Interval valued fuzzy soft set, Interval valued fuzzy soft Matrix, Regular Interval valued fuzzy soft Matrix.

1. Introduction

We deal with interval – valued fuzzy matrices (IVFM) that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0,1]. Recently the concept of IVFM a generalization of fuzzy matrix was introduced and developed by shymal and pal[5], by extending the max. min operations on fuzzy algebra F=[0,1], for elements, a, b$\in$F, a+b= max{a,b} and a-b=min{a,b}. In [2], Meenakshi and Kaliraja have represented and IVFM as an interval matrix of its lower and upper limit fuzzy matrices. In [3], Meenakshi and Jenita have introduced the concept of K-regular fuzzy matrix and discussed about inverses associated with a K-regular fuzzy matrix as a generalization of results on regular fuzzy matrix developed in [1]. A matrix A $\in F_n$, the set of all nxn fuzzy matrices is said to be right(left) K-regular if there exists x(y)$\in F_{n,}$ such that A^k x A = $A^k(AYA^k=A^k)$, x(y) is called a right(left) K-g inverses of A, Where K is a positive Integer. Recently we have expended characterization of various inverses of K-Regular interval – valued fuzzy matrices in [6] . [20] Dr.N.Sarala and M.Prabhavathi have proposed the union and intersection of interval valued fuzzy soft matrix. [13] P.Rajarajeswari and P.Dhanalakshmi have introduced interval valued fuzzy soft matrix, its types with examples and some new operation of an the basis of weights. In [21] Dr.N.Sarala and M.Prabhavathi have introduced the definition of „AND" and „OR" operation of interval valued fuzzy soft matrix

In this paper, we define regular interval valued fuzzy soft matrices as a generalization of regular interval valued fuzzy soft matrices and obtain the structure of Row space, column space of an Interval valued fuzzy soft matrices (IVFSM).

2. Preliminaries

Soft set 2.1 [9]

Suppose that U is an initial Universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair (F,E) is called a soft set over U where F is a mapping given by F :E$\rightarrow$P(U). Clearly a soft set is a mapping from parameters to P(U)and it is not a set, but a parameterized family of subsets of the Universe.

Fuzzy soft set 2.2 [10]

Let U be an initial Universe set and E be the set of parameters, let A $\subseteq$ E. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: A$\rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Fuzzy soft Matrices 2.3 [12]

Let U ={$c_1,c_2,c_3\ldots c_m$} be the Universe set and E be the set of parameters given by E ={$e_1,e_2,e_3\ldots e_n$}. Let A $\subseteq$ E and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E). Then fuzzy soft set (F,A) in a matrix form as A_{mxn} =$[a_{ij}]_{mxn}$ or A=$[a_{ij}]$ i=1,2,…m, j=1,2,3,…n

$$[a_{ij}] = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \text{ Where } a_{ij} = \mu_j(c_i) \text{ represents the membership of in } C_i \text{ fuzzy set } F(e_j). \\ 0 & \text{if } e_j \notin A \end{cases}$$

Interval valued fuzzy soft set 2.4 [11]

Let U be an initial Universe set and E be the set of parameters, let A $\subseteq$ E. A pair (F,A) is called Interval valued fuzzy soft set over U where F is a mapping given by F: A$\rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U.

Definition 2.1.

An Interval-Valued Fuzzy Matrix(IVFM) of order mxn is defined as A=$(a_{ij})_{mxn}$, Where a_{ij}= $[a_{ijL},a_{ijU}]$, the ij^{th} element of A is an interval representing the membership value. All the elements of an IVFM are intervals and all the intervals are the subintervals of the interval [0,1].

For A = (a_{ij}) = ($[a_{ijL,}a_{ijU}]$) and B = (b_{ij}) = ($[b_{ijL},b_{ijU}]$) of order mxn their sum denoted as A+B defined as,

$$A+B = (a_{ij}+b_{ij}) = ([(a_{ijL}+b_{ijL}),(a_{ijU}+b_{ijU})]) \ldots(2.1)$$
$$= ([\max\{a_{ijL,}b_{ijL}\}, \max\{a_{ijU},b_{ijU}\}])$$

For A = $(a_{ij})_{mxn}$ and B = $(b_{ij})_{nxp}$ their product denoted as AB is defined as,

$$AB = (c_{ij}) =\sum^n_{k=1}(a_{ik}b_{ik}) i=1,2,\ldots,m \text{ and } j=1,2,\ldots,p$$
$$=[\sum^n_{k=1} (a_{ikL}b_{ikL}),\sum^n_{k=1} (a_{ikU}b_{ikU})] \; i=1,2,\ldots,m \text{ and } j=1,2,\ldots,p \ldots(2.2)$$

In particular if $_{ainL}$ = a_{ijU} and b_{ijL}= b_{ijU} then (2,2) reduces to the standard max.min composition of fuzzy matrices [1].

A $\leq$ B if and only if $a_{ijL} \leq b_{ijL}$ and $a_{ijU} \leq b_{ijU}$

In [2], representation of an IVFM as an interval matrix of its lower and upper limit fuzzy matrices in introduced and discussed the regularity of an IVFM in terms of its lower and upper limit fuzzy matrices.

Definition 2.2

For a pair of Fuzzy Matrices $E=(e_{ij})$ and $F=(f_{ij})$ in $F_{m,n}$ such that $E \leq F$, it can be defined that the interval matrix is denoted as $[E,F]$, whose ij^{th} entry is the interval with lower limit e_{ij} and upper limit f_{ij}, that is $([e_{ij},f_{ij}])$. In particular for $E=F$, IVFM $[E,E]$ reduces to the fuzzy matrix $E \in F_{mn}$

For $A=(a_{ij})=([a_{ijL},a_{ijU}]) \in (IVFM)_{mn}$, let us define $A_L=(a_{ijL})$ and $A_U=(a_{ijU})$. Clearly A_L and A_U belongs to $F_{m,n}$ such that $A_L \leq A_U$ and from Definition (2.2) A can be written as $A=[A_L,A_u]$

For $A \in (IVFM)_{mn}$, A^T, R (A), C (A) denotes the transpose of A, row space of A, column space of A respectively.

Interval valued fuzzy soft matrix 2.5[13]

Let $U=\{c_1,c_2,c_3\ldots c_m\}$ be the Universe set and E be the set of parameters given by $E=\{e_1,e_2,e_3\ldots e_n\}$. Let $A \subseteq E$ and (F,A) be a interval valued fuzzy soft set over U, where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U. Then the Interval valued fuzzy soft set can expressed in matrix form as

$\cdot_{mxn}=[a_{ij}]_{mxn}$ or $\tilde{A}=[a_{ij}]$ i= 1,2,…m, j=1,2,…n

$$\text{Where } a_{ij} = \begin{cases} [\mu_{jL}(c_i), \mu_{jU}(c_i)] & \text{if } e_j \in A \\ [0,0] & \text{if } e_j \notin A \end{cases}$$

$[\mu_{jL}(c_i), \mu_{ju}(c_i)]$ represents the membership of c_i in the Interval valued fuzzy set F (e_j).

Note that if$\mu_{jU}(c_i)=\mu_{jL}(c_i)$ then the Interval- valued fuzzy soft matrix (IVFSM) reduces to an FSM

Example: 2.1

Suppose that there are four houses under consideration, namely the universes $U=\{h_1,h_2,h_3,h_4\}$, and the parameter set $E=\{e_1,e_2,e_3,e_4\}$ where e_i stands for "beautiful", "large", "cheap", and "in green surroundings" respectively. Consider the mapping F from parameter set $A=\{e_1,e_2\} \subseteq E$ to all interval valued fuzzy subsets of power set U. Consider an interval valued fuzzy soft set (F,A) which describes the "attractiveness of houses" that is considering for purchase. Then interval valued fuzzy soft set (F,A) is

(F, A)= {F(e_1)={(h_1, [0.6,0.8]), (h_2, [0.8,0.9]), (h_3, [0.6,0.7]), (h_4,[0.5,0.6])}
F(e_2)={(h_1, [0.7,0.8]), (h_2, [0.6,0.7]), (h_3, [0.5,0.7]), (h_4, [0.8, 0.9])}

We would represent this Interval valued fuzzy soft set in matrix form as

$$\begin{bmatrix} [0.6,0.8] & [0.7,0.8] & [0.0,0.0] & [0.0,0.0] \\ [0.8,0.9] & [0.6,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.6,0.7] & [0.5,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.5,0.6] & [0.8,0.9] & [0.0,0.0] & [0.0,0.0] \end{bmatrix}$$

Definition 2.3
A matrix $A \in (IVFM)_n$ is said to be right k-regular if there exist a matrix $X \in (IVFM)_n$, such that $A^k XA=A^k$, for some positive integer k. X is called a right k-g inverse of A. Let $A\{1_r{}^k\}=\{X/\ A^k XA = A^k\}$.

Definition 2.4
A matrix $A \in (IVFM)_n$ is said to be left k-regular if there exist a matrix $Y \in (IVFM)_n$, such that $AYA^K=A^k$, for some positive integer k. Yis called a left k-g inverse of A. Let $A\{1_\ell{}^k\}=\{Y/\ A^kYA = A^k\}$.

In particular for a fuzzy matrix A, $a_{ijL}=a_{ijU}$ then definition (2.3) and definition(2.4) reduce to right left k-regular fuzzy matrices respectively found in [3].

In the sequel we shall make use of the following results on IVFM found in [2] and [4].

Lemma 2.1

For $A=[A_L,A_U] \in (IVFM)_{mn}$ and $B=[B_L,B_U] \in (IVFM)_{np}$, the following hold.

(i) $A^T=[A_L{}^T,A_U{}^T]$
(ii) $AB=[A_LB_L,A_UB_U]$.

Lemma 2.2
For $A,B \in (IVFM)_m$
(i) $R(B) \subseteq R(A) \Leftrightarrow B=XA$ for some $X \in (IVFM)_m$
(ii) $C(B) \subseteq C(A) \Leftrightarrow B=XY$ for some $Y \in (IVFM)_n$

Theorem 2.1
For $A,B \in (IVFM)_n$, with R(A) =R(B) and $R(A^k)=R(B^k)$ then A is right K-regular IVFM $\Leftrightarrow$ B is right k-regular IVFM

Theorem 2.2
For $A,B \in (IVFM)_n$, with C(A) =C(B) and $C((A^k)=C(B^k)$ then A is left K-regular IVFM $\Leftrightarrow$ B is left k-regular IVFM.

3. Regular Interval Valued Fuzzy Soft Matrices

In this section ,we define regular interval valued fuzzy soft matrices and obtain the structure of row spaces , column spaces of an interval valued fuzzy soft matrices(IVFSM).

Definition: 3.1
Let $\tilde{A}$ is called Regular interval valued fuzzy soft matrix, then there exists $\tilde{X} \in (IVFSM)_{mn}$
Such that $\tilde{A}\,\tilde{X}\,\tilde{A} = \tilde{A}$.

Lemma:3.1:
For $\tilde{A} = [\tilde{A}_L , \tilde{A}_U] \in (IVFSM)_{mn}$ and $\tilde{B} = [\tilde{B}_L , \tilde{B}_U] \in (IVFSM)_{np}$, The following hold.
(i) $\tilde{A}^T = [\tilde{A}_L{}^T , \tilde{A}_U{}^T]$
(ii) $\tilde{A}\,\tilde{B} = [\tilde{A}_L\,\tilde{B}_L , \tilde{A}_U\,\tilde{B}_U]$

(iii) $\tilde{A} + \tilde{B} = [\tilde{A}_L + \tilde{B}_L, \tilde{A}_U + \tilde{B}_U]$
(iv) $[\alpha+\beta](\tilde{A} + \tilde{B}) = [\alpha\tilde{A}_L + \beta \tilde{B}_L, \alpha \tilde{A}_U + \beta \tilde{B}_U]$, for $\alpha \le \beta \in$ FSM
(v) $[\alpha, \beta]\tilde{A} = [\alpha\tilde{A}_L, \beta\tilde{A}_U]$, for $\alpha \le \beta \in$ FSM
(vi) $\alpha\tilde{A} = [\alpha\tilde{A}_L, \alpha \tilde{A}_U]$, for $\alpha \in$ FSM

proof:

(i) this directly follows from the definition.
(ii) Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] = [\mu_{AL}, \mu_{AU}]$
$\tilde{B} = [\tilde{B}_L, \tilde{B}_U] = [\mu_{BL}, \mu_{BU}]$

Then
$\tilde{A}\tilde{B} = [[\sum_{k=1}^{n} (\mu_{AL} \cdot \mu_{BLk}), \sum_{k=1}^{n} (\mu_{AUK} \cdot \mu_{BUk}),$
$= [\tilde{A}_C \tilde{B}_C, \tilde{A}_U \tilde{B}_U]$

$\tilde{A}_C \tilde{B}_C$ and $\tilde{A}_U \tilde{B}_U$ are the max. Min composition of matrices in FSM_{np}.

(iii) Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] = [\mu_{AL}, \mu_{AU}]$
$\tilde{B} = [\tilde{B}_L, \tilde{B}_U] = [\mu_{BL}, \mu_{BU}]$.
$\tilde{A} + \tilde{B} = (a_{ij} + b_{ij}) = [(\mu_{AL}, \mu_{BL}), (\mu_{AU}, \mu_{BU})]$
$= [\tilde{A}_L + \tilde{B}_L, \tilde{A}_U + \tilde{B}_U]$

(iv) Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] = [\mu_{AL}, \mu_{AU}]$.
$\tilde{B} = [\tilde{B}_L, \tilde{B}_U] = [\mu_{BL}, \mu_{BU}]$.
$[\alpha, \beta] (\alpha \tilde{A} + \beta \tilde{B} = (\alpha a_{ij} + \beta b_{ij})$
$= [(\alpha \mu_{AL}, \beta \mu_{BL}), (\alpha \mu_{AU}, \beta \mu_{BU})]$
$= [\alpha \tilde{A}_L + \beta \tilde{B}_L, \alpha \tilde{A}_U + \beta \tilde{B}_U]$.

(v) Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] = [\mu_{AL}, \mu_{AU}]$
$(\alpha, \beta) \tilde{A} = [\alpha, \beta] [\mu_{AL}, \mu_{AU}]$
$= [\alpha \mu_{AL}, \beta\mu_{AU}]$
$= [\alpha \tilde{A}_L, \beta \tilde{A}_U]$ for $\alpha, \beta \in$ FSM.

(vi) Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] = [\mu_{AL}, \mu_{AU}]$.
$\alpha \tilde{A} = [\alpha\mu_{AL}, \alpha\mu_{AU}]$.
$= [\alpha \tilde{A}_L, \alpha \tilde{A}_U]$ for $\alpha \in$ F.
Hence the lemma.

Theorem : 3.1
Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] \in (IVFSM)_{mn}$.
Then $\tilde{A}$ is regular IVFSM $\Leftrightarrow$ $\tilde{A}_L$ and $\tilde{A}_U \in FSM_{mn}$ are regular.

Proof:
Let $\tilde{A} \in (IVFSM)_{mn}$.
If $\tilde{A}$ is regular IVFSM, Then there exists $\tilde{X} \in (IVFSM)_{mn}$.
Such that $\tilde{A}$ X $\tilde{A} = \tilde{A}$.
Let $\tilde{X} = [\tilde{X}_L, \tilde{X}_U]$ with $\tilde{X}_L, \tilde{X}_U \in FSM_{mn}$.
then
$\Rightarrow [\tilde{A}_L, \tilde{A}_U] [\tilde{X}_L, \tilde{X}_U] [\tilde{A}_L, \tilde{A}_U] = [\tilde{A}_L, \tilde{A}_U]$
$\Rightarrow \tilde{A}_L \tilde{X}_L \tilde{A}_L = \tilde{A}_L$ and $\tilde{A}_U \tilde{X}_U \tilde{A}_U = \tilde{A}_U$
$\Rightarrow \tilde{A}_L$ is regular and $\tilde{A}_U$ is regular FSM_{mn}

Thus $\tilde{A}$ is regular IVFSM $\Rightarrow \tilde{A}_L$ and $\tilde{A}_U \in FSM_{mn}$ are regular .

Conversely,

Suppose $\tilde{A}_L$ and $\tilde{A}_U$ are regular, then $\tilde{A}_L \tilde{X}_L \tilde{A}_L = \tilde{A}_L$ and $\tilde{A}_U \tilde{X}_U \tilde{A}_U = \tilde{A}_U$ for some $\tilde{X}_L$ and
Since $\tilde{A}_L \le \tilde{A}_U$, it is possible to choose atleast are $\tilde{X} \in \tilde{A}_L$ {1} and $\tilde{Y} \in \tilde{A}_U$ {1} such that $\tilde{X} \le \tilde{Y}$.

Let us define the interval valued fuzzy soft matrix $\tilde{Z} = [\tilde{X}, \tilde{Y}]$.
Then by the definition of regular.
$\tilde{A} \tilde{Z} \tilde{A} = [\tilde{A}_L, \tilde{A}_U] [\tilde{X}, \tilde{Y}] [\tilde{A}_L, \tilde{A}_U]$
$= [\tilde{A}_L, \tilde{A}_U]$
$= \tilde{A}$.
Thus $\tilde{A}$ is regular
Hence the theorem.

Example : 3.1
Let us consider $\tilde{A} = [\tilde{A}_L, \tilde{A}_U] \in (IVFSM)_B$.

When $\tilde{A}_L = \begin{bmatrix} 0.3 & 0.1 & 0.5 \\ 0.6 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.2 \end{bmatrix}$

$\tilde{A}_U = \begin{vmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{vmatrix}$

Then $\tilde{A}_L \le \tilde{A}_U$

Here $\tilde{A}_U$ is regular being idempotent and $\tilde{A}_L$ is not regular.
There is no $\tilde{X} \in (IVFSM)_3$ such that, $\tilde{A}_L \tilde{X}_L \tilde{A}_L = \tilde{A}_L$.
$\therefore \tilde{A}$ is not regular for, it A is regular, then for some $\tilde{X} \in (IVFSM)_3$, $\tilde{A} \tilde{X} \tilde{A} = \tilde{A}$
$\Rightarrow \tilde{A}_L \tilde{X}_L \tilde{A}_L = \tilde{A}_L$ is regular which is not possible.

Theorem : 3.1
Let $\tilde{A} = [\tilde{A}_L, \tilde{A}_U]$ be an $(IVFSM)_{mn}$
Then
(i) $\mathcal{R}(A) = [\mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U)] \in (IVFSM)_{In}$
(ii) $\mathcal{C}(A) = [\mathcal{C}(\tilde{A}_L), \mathcal{C}(\tilde{A}_U)] \in (IVFSM)_{Im}$

proof :
(i) since $\tilde{A} \in (IVFSM)_{mn}$, any vector $x \in \mathcal{R}(A)$ is of the form $x = y\,\tilde{A}$ for some y $\in (IVFSM)_{Im}$ that is, x is an interval valued vector with n components.
let is compute $x \in \mathcal{R}(A)$ as follows. x is a linear combination of the low of
$\tilde{A} \Rightarrow x = \sum_{j=i}^{m} \alpha_i, \tilde{A}_{i*}$ when $\tilde{A}_{i*}$ is the i[th] low of $\tilde{A}$.
Eguating the j_{th} component on both sides yield.
$x = \sum_{j=i}^{m} \alpha_i . a_{ij}$
$(a_{ij}) = [\mu_{AL}, \mu_{AU}]$.
$xj = \sum_{i=1}^{m} \alpha_i . [\mu_{AL}, \mu_{AU}]$.
$= \sum_{i=1}^{m} [\alpha_i \mu_{AL}, \mu_{AU}]$.
$= \sum_{i=1}^{m} (\alpha_i \mu_{AL}), \sum_{i=1}^{m} (\alpha_i \mu_{AL})]$
$= [x_{jL}, x_{jU}]$.
x_{jL} is the j[th] component of $x_L \in \mathcal{R}(\tilde{A}_L)$ and x_{jL} is the j[th] component of $x_U \in R(\tilde{A}_U)$.
Hence $x = [x_L, x_U]$.

Therefore, $\mathcal{R}(\tilde{A}) = [\mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U)]$

(ii) For $\tilde{A} = [\tilde{A}_L), \tilde{A}_U]$, the transpose of A is $\tilde{A}^T = [(\tilde{A}^T_L), (\tilde{A}^T_U]$.
by using (i) we get
$\mathcal{C}(\tilde{A}) = \mathcal{R}(\tilde{A}^T) = [\mathcal{R}(\tilde{A}_L^T), \mathcal{R}(\tilde{A}_U^T)]$
$= [\mathcal{C}((\tilde{A}_L), \mathcal{C}(\tilde{A}_U)]$
Hence the theorem.

Theorem : 3.2
For $\tilde{A}$, $\tilde{B}$ Є $(IVFSM)_{mn}$.
(i) $\mathcal{R}(\tilde{B}) \leq (\tilde{A}) \Leftrightarrow \tilde{B} = \tilde{X}\tilde{A}$ for some x Є $(IVFSM)_m$.
(ii) $\mathcal{C}(\tilde{B}) \leq \mathcal{C}(\tilde{A}) \Leftrightarrow \tilde{B} = \tilde{A}\tilde{Y}$ for some Y Є $(IVFSM)_n$.

Proof :
Let $\tilde{A} = [\tilde{A}_L , \tilde{A}_U]$ and $\tilde{B} = [\tilde{B}_L , \tilde{B}_U]$
Since $\tilde{B} = \tilde{X}\tilde{A}$, for some $\tilde{X}$ Є (IVFSM)
Put $\tilde{X} = [\tilde{X}_L , \tilde{X}_U]$. Then by lemma 3.1

(iii) $\tilde{B}_L = \tilde{X}_L \tilde{A}_L$ and $\tilde{B}_U = \tilde{X}_U \tilde{A}_U$.
Hence lemma 2·4 $\mathcal{R}(\tilde{B}_L) \subseteq \mathcal{R}(\tilde{A}_L)$ and $\mathcal{R}(\tilde{B}_U) \subseteq \mathcal{R}(\tilde{A}_U)$
By theorem 3.1 (i) $\mathcal{R}(\tilde{B}) = [\mathcal{R}(\tilde{B}_L), \mathcal{R}(\tilde{B}_U)] \subseteq [\mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U)] = \mathcal{R}(\tilde{A})$
Thus $\mathcal{R}(\tilde{B}) = \mathcal{R}(\tilde{A})$
Conversely $\mathcal{R}(\tilde{B}) \subseteq \mathcal{R}(\tilde{A})$
$\Rightarrow \mathcal{R}(\tilde{B}_L) \subseteq \mathcal{R}(\tilde{A}_L)$ and $\mathcal{R}(\tilde{B}_U) \subseteq \mathcal{R}(\tilde{A}_U)$ (by theorem 3.1, (i))
$\Rightarrow \tilde{B}_L = \tilde{Y}\tilde{A}_L$ and $\tilde{B}_U = \tilde{Z}\tilde{A}_U$ (by lemma 2.3)
Then $\tilde{B} = [\tilde{B}_L , \tilde{B}_U]$
$= [\tilde{Y}\tilde{A}_L , \tilde{Z}\tilde{A}_U]$
$= [\tilde{Y}, \tilde{Z}] [\tilde{A}_L , \tilde{A}_U]$ (by lemma 3.1)
$= \tilde{X} [\tilde{A}_L , \tilde{A}_U]$ where $\tilde{X} [\tilde{Y},\tilde{Z}]$ Є $(IVFSM)_{mn}$.
$= \tilde{X}\tilde{A}$
$\tilde{B} = \tilde{X}\tilde{A}$.

(i) This can be proved along the same lines as that of (i) that hence omitled .

Theorem : 3.3
For $\tilde{A}$ Є $(IVFSM)_{mn}$, $\tilde{B}$ Є $(IVFSM)_{mn}$. the following hold .
$\mathcal{R}(\tilde{A}\tilde{B}) \subseteq (\tilde{A})$ are $\mathcal{C}(\tilde{A}\tilde{B}) \subseteq \mathcal{C}(\tilde{B})$.

Proof :
Let $\tilde{A} = [\tilde{A}_L , \tilde{A}_U]$ and $\tilde{B} = [\tilde{B}_L , \tilde{B}_U]$
$\tilde{A}^T = [\tilde{A}_L^T , \tilde{A}_U^T]$ and $\tilde{B}^T = [\tilde{B}_L^T , \tilde{B}_U^T]$
Then by lemma 3.1
$AB = [\tilde{A}_L \tilde{B}_L , \tilde{A}_U \tilde{B}_U]$

By theorem 3.1
$\mathcal{R}(\tilde{A}\tilde{B}) = ([\tilde{A}_L \tilde{B}_L , \tilde{A}_U \tilde{B}_U])$
$= [\mathcal{R}(\tilde{A}_L \tilde{B}_L) , \mathcal{R}(\tilde{A}_U \tilde{B}_U)] \subseteq [\mathcal{R}(\tilde{A}_L) , \mathcal{R}(\tilde{A}_U)]$
$= (\tilde{A})$. (by lemma 3.1)

$\mathcal{R}(\tilde{A}\tilde{B}) \subseteq \mathcal{R}(\tilde{A})$
$\mathcal{C}(\tilde{A}\tilde{B}) = \mathcal{R}(\tilde{A}\tilde{B})^T = \mathcal{R}(\tilde{B}^T\tilde{A}^T) \subseteq \mathcal{R}(\tilde{B}^T)$
$= \mathcal{C}(\tilde{B})$
$\mathcal{C}(\tilde{A}\tilde{B}) = \mathcal{C}(\tilde{B})$
Hence the theorem .

References

[1] Kim, K.H. and Roush, F.N., Generalized fuzzy matrices, Fuzzy sets and systems, 4(1980), 293 – 315.
[2] Meenakshi, AR., and Kaliraja,M., Regular Interval – valued Fuzzy Matrices, Advances in Fuzzy Mathematics, 5 (1)(2010), 7 – 15.
[3] Meenakshi, AR., and Jenita, p., Generalized Regular Fuzzy Matrices, Iranian Journal of fuzzy Systems, 8 (2), (2011) 133 – 141.
[4] Meenakshi, AR., and Poongodi, P., Generalized Regular Interval Valued Fuzzy Matrices, International Journal of Fuzzy Mathematics and Systems, 2 (1) (2012), 29 – 36.
[5] Shyamal. A.K. and Pal.M., Interval – Valued Fuzzy Matrices, Journal of Fuzzy Mathematics, 14 (3) (2006) 582 – 592.
[6] Meenakshi, AR., and Kaliraja,M., g-inverses of Interval – valued Fuzzy Matrices, j in Fuzzy Mathematics,vol.45 ,No.1,2013,83-92
[7] Meenakshi, AR., and Kaliraja,M.,inverses of k- Regular Interval – valued Fuzz Matrices, Advances in Fuzzy Mathematics,
[8] Shyamal, A.K., and Pal. (2006).Interval valued fuzzy matrices, Journal of Fuzzy Mathematics, Vol.14(3), 582 -592.
[9] P.K. Maji, R. Biswas and A.R. Roy, fuzzy soft set, The Journal of Fuzzy Mathematics, 9(3) (2001) 589 -602.
[10] P.K. Maji, R. Biswas and A.R. Roy, Intuitionistic fuzzy soft set, The Journal of Fuzzy Mathematics, 12 (2004) 669 – 683
[11] Y. Yang and J. Chenli,Fuzzy soft matrices and their applications, Part I, Lecture Notes in computer science, 7002, 2011, pp: 618 – 627.
[12] L.A. Zadeh, Fuzzy sets, Information and control, 8 (1985) 338 – 353.
[13] P. Rajarajeswari and P.Dhanalakshmi Interval - valued fuzzy soft matrix theory, Annals of Pure and Applied Mathematics, Vol. 7 (2014) 61 – 72.
[14] A . Ben Israel and T . N . E . Greville, generalized Inverses : Theory and applications, Wiley , new York , 1974.
[15] K. H.Kim and F . W. Roush, Generalized Fuzzy Matrices,Fuzzy Sets and Systems, 4(1980) 293-315.
[16] AR. Meenakshi , Fuzzy Matrix Theory and Applications, MJP Publishers, Chennai, 2008.
[17] C . J . Klir and T . A . Folger, Fuzzy Sets, Uncertainty and Information, Prentice hall of India, New Delhi,1988.
[18] AR. Meenakshi , and P.Jenita, Generalized regular Fuzzy Matrices, Iranian journal of Fuzzy systems, 8(2) (2011) 133-141.
[19] AR. Meenakshi , On regularity of block fuzzy matrices, Int .J. Fuzzy math., 12(2004) 439-450.
[20] D.r.N.Sarala and M.Prabhavathi an application of Interval valued fuzzy soft matrix in medical Diagnosis. IOSR Journal of Mathematics Vol.11,Issue 1 ver.VI (Jan-Feb.2015), pp: 1-6
[21] D.r.N.Sarala and M.Prabhavathi An a Application of Interval Valued Fuzzy soft matrix in decision making problem International Journal of Mathematics trends and technology (IJMTT) v 21 (1):1-10May2015.