Some New Results on K-even Sequential Harmonious Labeling of Graphs

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Abstract: ***Singh and Varkey introduced the odd sequential graphs. Gayathri and Hemalatha introduced even sequential harmonious labeling of graphs and also k-even sequential harmonious labeling of graphs. Here, we investigate some new results on k-even sequential harmonious labeling of graphs. In this paper, we have shown that the graphs P_n^3, Alternate triangular snakes and Alternate quadrilateral snakes are k-even sequential harmonious graphs.***

Keywords: Path, Alternate triangular snake, Alternate quadrilateral snake

1. Introduction

All the graphs in this paper are finite, simple and undirected. The symbols V(G) and E(G) denote the vertex set and the edge set of a graph G.

The cardinality of the vertex set is called the order of G. The cardinality of the edge set is called the size of G.A graph with p vertices and q edges is called a (p, q) graph.

In[1], Gayathri and Hemalatha says that a labeling is an even sequential harmonious labeling if there exist an injection f from the vertex set V to {0, 1, 2, .., 2q} such that the induced mapping f^+ from the edge set E to {2, 4, 6, …, 2q} defined by

$$f^+(uv)=\begin{cases} f(u)+f(v), \text{ if } f(u)+f(v) \text{ is even} \\ f(u)+f(v)+1, \text{ if } f(u)+f(v) \text{ is odd are distinct.} \end{cases}$$

A graph G is said to be an even sequential harmonious graph if it admits an even sequential harmonious labeling.

Here, we have introduced k-even sequential harmonious labeling by extending the above definition for any integer k≥1.We say that a labeling is an k-even sequential harmonious labeling if there exist an injection f from the set V to {k-1, k, k+1, .., k+2q-1}such that the induced mapping f^+ from the set E to {2k, 2k+2, 2k+4, …, 2k+2q-2} defined by

$$f^+(uv)=\begin{cases} f(u)+f(v), \text{ if } f(u)+f(v) \text{ is even} \\ f(u)+f(v)+1, \text{ if } f(u)+f(v) \text{ is odd are distinct.} \end{cases}$$

A graph G is said to be an k-even sequential harmonious graph if it admits an k-even sequential harmonious labeling.

In this paper, we investigate some new results on k-even sequential harmonious labeling of graphs. Throughout this paper, k denote any positive integer ≥ 1. For brevity, we use k-ESHL for k-even sequential harmonious labeling.

2. Main Results

Definition: 2.1

By a graph P_n^3, we mean the graph obtained from P_n joining each pair of vertices at distance 3 in P_n.

Theorem: 2.2
The graph P_n^3, (n≥4) is a k-even sequential harmonious graph.

Proof:
Let the vertices of P_n^3 be $\{v_i : 1 \le i \le n\}$ and the edges of P_n^3 be$\{ v_iv_{i+1} : 1 \le i \le n-1 \}$U
$\{ v_iv_{i+3} : 1 \le i \le n-3\}$. Which are denoted in Fig.2.2(a).

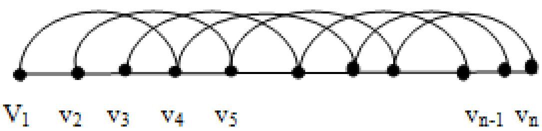

Figure 2.2(a): P_n^3 with ordinary labeling

We first, label the vertices of P_n^3 as follows,
Define f: $V(P_n^3) \rightarrow \{k-1, k, k+1, .., k+2q-1\}$ by
$f(v_i) = 2(i-1)-1+k$, $1 \le i \le n$.

Then the induced edge labels are
$f^+(v_iv_{i+1}) = 4i-4+2k$, $1 \le i \le n-1$.
$f^+(v_iv_{i+3}) = 4i+2k-2$, $1 \le i \le n-3$.

Clearly, the edge labels are even and distinct,
$f^+(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}$.

Hence, the graph P_n^3, (n≥4) is a k-even sequential harmonious graph.

2-ESHL of P_9^3 is shown in Fig.2.2(b).

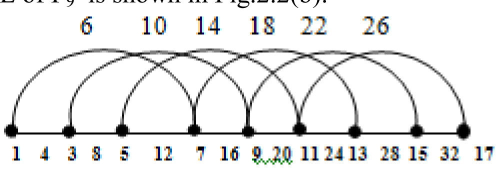

Figure 2.2 (b): 2-ESHL of P_9^3

Definition: 2.3
An Alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i.

That is, every alternate edge of a path is replaced by a cycle C_3.

Theorem: 2.4
Alternate triangular snakes is a k-even sequential harmonious graph.

Proof:
Let $A(T_n)$ be a alternate triangular snake.

Let the vertices of $A(T_n)$ be $\{v_i:1 \le i \le n\}$ U $\{u_i:1 \le i \le n/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd $\}$ U$\{w_i:1 \le i \le (n-2)/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd $\}$and the edges of $A(T_n)$ be$\{ v_iv_{i+1}:1 \le i \le n-1 \}$ U $\{ u_iv_{2i}:1 \le i \le n/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd $\}$ U $\{u_iv_{2i-1}:1 \le i \le n/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd$\}$ U$\{w_iv_{2i}:1 \le i \le (n-2)/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd$\}$ U$\{w_iv_{2i+1}:1 \le i \le (n-2)/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd $\}$. Which are denoted in Fig.2.4(a).

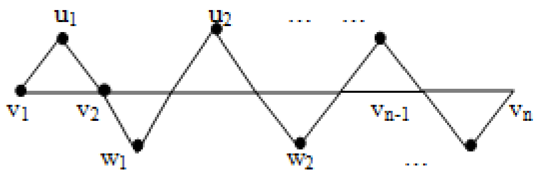

Figure 2.4(a): A (T_n) with ordinary labeling

We first, label the vertices of $A(T_n)$ as follows,
Define $f : V(AT_n) \to \{k-1, k, k+1, .., k+2q-1\}$ by

$f(v_i) = 3i-1+k, \quad 1 \le i \le n$

$$f(u_i) = 3i-2+k \quad \begin{cases} 1 \le i \le n/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

$$f(w_i) = 3i-k+1 \quad \begin{cases} 1 \le i \le (n-2)/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

Then the induced edge labels are

$f^*(v_iv_{i+1}) = 6i-6+2k, \quad 1 \le i \le n-1$

$$f^*(u_iv_{2i-1}) = 6i-2k \quad \begin{cases} 1 \le i \le n/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

$$f^*(u_iv_{2i}) = 6i+2k \quad \begin{cases} 1 \le i \le n/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

$$f^*(w_iv_{2i}) = 6i+4k \quad \begin{cases} 1 \le i \le (n-2)/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

$$f^*(w_iv_{2i+1}) = 6i-2+2k \quad \begin{cases} 1 \le i \le (n-2)/2, & \text{if n is even} \\ 1 \le i \le (n-1)/2, & \text{if n is odd} \end{cases}$$

Clearly, the edge labels are even and distinct, $f^+(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}$.

Hence, the graph $A(T_n)$is a k-even sequential harmonious graph.

2-ESHL of $A(T_7)$ is shown in Fig.2.4(b).

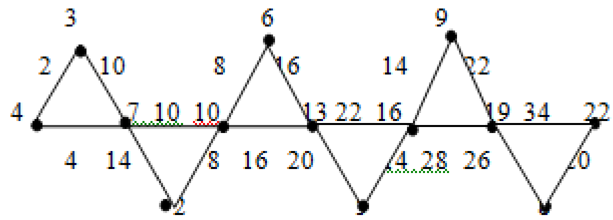

Figure 2.4(b): 2-ESHL of $A(T_7)$

Definition: 2.5
An Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_iw_i respectively and then joining v_i and w_i.

That is, every alternate edge of a path is replaced by a cycle C_4.

Theorem: 2.6
Alternate quadrilateral snakes is a k-even sequential harmonious graph.

Proof:
Let $A(Q_n)$ be a alternate quadrilateral snake.
Let the vertices of $A(Q_n)$ be $\{v_i:1 \le i \le n\}$ U $\{u_i:1 \le i \le n$, if n is even and $1 \le i \le n-1$, if n is odd$\}$ U $\{w_i:1 \le i \le n-2$, if n is even and $1 \le i \le n-1$, if n is odd$\}$and the edges of $A(Q_n)$ be $\{ v_iv_{i+1}:1 \le i \le n-1 \}U\{ u_{2i-1}u_{2i}:1 \le i \le n/2$, if n is even and $1 \le i \le (n-1)/2$, if n is odd$\}$ U $\{v_iu_i:1 \le i \le n$, if n is even and $1 \le i \le n-1$, if n is odd$\}$ U$\{w_{2i-1}w_{2i}:1 \le i \le (n-2)/2$, if n is even and$1 \le i \le (n-1)/2$, if n is odd$\}$ U$\{v_iw_{i-1}:2 \le i \le n-1$, if n is even and $2 \le i \le n$, if n is odd$\}$. Which are denoted in Fig.2.6(a).
u_1 u_2 … …

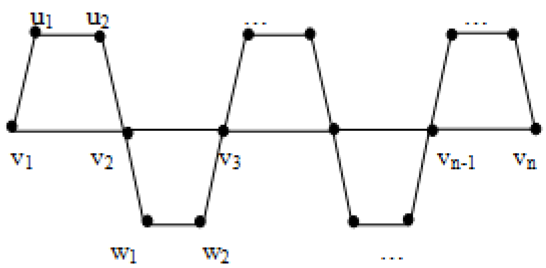

Figure 2.6(a) : $A(Q_n)$ with ordinary labeling

We first, label the vertices of $A(Q_n)$ as follows,
Define $f : V(AQ_n) \to \{k-1, k, k+1, .., k+2q-1\}$ by

$f(v_i) = 4i-6+k \quad , 1 \le i \le n$

$$f(u_i) = 4i-4+k \quad \begin{cases} 1 \le i \le n, & \text{if n is even} \\ 1 \le i \le n-1, & \text{if n is odd} \end{cases}$$

$$f(w_i) = 4i+k-1 \quad \begin{cases} 1 \le i \le n-2, & \text{if n is even} \\ 1 \le i \le n-1, & \text{if n is odd} \end{cases}$$

Then the induced edge labels are

$f^+(v_i v_{i+1}) = 8i-8+2k, \quad 1 \le i \le n-1$

$$f^+(u_{2i-1}u_{2i}) = 8i+2k \quad \begin{cases} 1 \le i \le n/2, \text{ if } n \text{ is even} \\ 1 \le i \le (n-1)/2, \text{ if } n \text{ is odd} \end{cases}$$

$$f^+(v_i u_i) = 8i-2k \quad \begin{cases} 1 \le i \le n, \text{ if } n \text{ is even} \\ 1 \le i \le n-1, \text{ if } n \text{ is odd} \end{cases}$$

$$f^+(w_{2i-1}w_{2i}) = 8i+2+2k \quad \begin{cases} 1 \le i \le (n-2)/2, \text{ if } n \text{ is even} \\ 1 \le i \le (n-1)/2, \text{ if } n \text{ is odd} \end{cases}$$

$$f^+(v_i w_{i-1}) = 8i-2+4k \quad \begin{cases} 2 \le i \le n-1, \text{ if } n \text{ is even} \\ 2 \le i \le n, \text{ if } n \text{ is odd} \end{cases}$$

Clearly, the edge labels are even and distinct,

$f^+(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}$.

Hence, the graph $A(Q_n)$ is a k-even sequential harmonious graph.

2-ESHL of $A(Q_6)$ is shown in Fig.2.6(b)

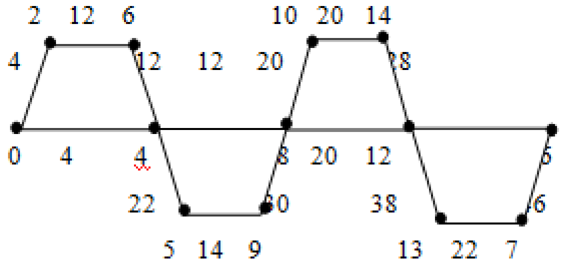

Figure 2.6(b): 2-ESHL of $A(Q_6)$

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