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Pre-School Teacher Knowledge of Mathematical Reasoning: A Grounded Theory Study

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Abstract: ***Mathematics is a body of knowledge that is key in transferring skills such as ability to think logically, critically, having increased investigative, resourceful, and creative and problem solving skills. However, there is little evidence to back up this claim especially that, early childhood mathematics education has not been part of main stream education system in our country for a long time. The current study sought to examine teacher knowledge of mathematical reasoning in order to help our teachers' interpret the curriculum framework and the early learning standards and development effectively and efficiently. Mixed methods were used 49 in-service teachers took part in this study. 25 had started at level 2 thus diploma entrant and expected to be at the university for three years, 24 had started at level 1, and thus they entered the university with a certificate and learnt for a full year before going into level 2 and expected to be at the university for four years. Findings varied as answers ranged from opportunities available, how to improve mathematics reasoning, motivation related to the importance /significance of mathematics reasoning. Although, in other cases the findings were consistent with what other researchers had said on some of the games, words and questions a teacher can use to promote reasoning but not consistent with the definition of a game and a word. The same was the case with opportunities abundant for promoting reasoning in early learners. The null hypothesis is accepted while the hypothesis is rejected. With some findings consistent with literature while others are based on students perceptions on what they feel are opportunities, games, words, questions that they feel can promote mathematical reasoning. The study recommended more training in subject matter and pedagogy because the more knowledge, skills and values teachers are exposed to the better amidst the debate of whether reasoning can be taught or it is innate.***

Keywords: Mathematical reasoning, teacher knowledge, early childhood mathematics education, early learning standards and development

1. Introduction

Mathematics is a body of knowledge that is key in transferring skills such as ability to think logically, critically, having increased investigative, resourceful, and creative and problem solving skills. And young children possess a remarkably broad, complex, and sophisticated – albeit informal – knowledge of mathematics (NAP, 2015). Children's early knowledge of mathematics is surprisingly important and it strongly predicts later success in mathematics. Mathematical thinking reaches beyond competence with numbers and shapes to form a foundation for general cognition and learning (Cresswell & Speelman, 2020). Children need a robust foundation in mathematics knowledge in their earliest years. However, mathematics is generally not taught well to young children (Copley, 2010) and has not been the focus of the main education stream (pre-school act, 1957; Zambia Pre-school Association, 1972) and there are varying understandings concerning mathematical thinking in children (Piaget, 1962; Sarama & Clements, 2009). Preschool educators tend not to support mathematics learning and when they do it is often of a low quality (Reikeras, 2020; Mwape, 2018 in press).

2. Statement of the Problem

Mathematical reasoning is one of the approaches needed to survive in the 21st century (Rizki & Priatna, 2019; Valle, 2018)). It is also a key to unlocking other skills such as creativity and innovation, critical thinking and problem solving, collaboration, communication, construction and exploration in children as well as students. But it seems in-service teachers entering university education is not aware of mathematics reasoning as an approach to early learning and development. If this construct of mathematics education is not attended to and if teachers lack this knowledge base, the children being taught by such teachers may lag behind. Teachers need advanced subject matter and pedagogy to be competent in advancing 21st skills (Cresswell & Speelman, 2020) although there is no sufficient conclusive evidence that mathematics enhances higher order cognitive functions (Fadel, 2014 cited in Cresswell & Speelman, 2020)

3. Research Questions

a) What awareness do teachers have concerning mathematics reasoning as an aspect under approaches to learning and development in early years?
b) What reasoning opportunities do they think are in abundant for young children?
c) What are some of the words and questions do they think should be emphasized to promote logic and reasoning in children?

4. Research Objectives

a) Assess teacher's awareness of mathematics reasoning as an aspect under approaches to learning and development in early years
b) Examine teacher knowledge of the reasoning opportunities that are in abundant for young children.
c) Find out some of the words and questions that they think should be emphasized to promote logic and reasoning in children.

5. Purpose of the Study

The purpose of the study was to examine pre-school teacher knowledge of mathematics reasoning as an aspect under approaches to learning and development in early years, so as

to help ECE teachers interpret mathematics reasoning as an approach to learning other academics as stated forth in the early learning development standards (ELDS 2016).

6. Research Hypothesis

Students are aware of the approaches to learning in early learning and development

7. Theoretical Framework

The study used research and development (R&D) theory that was used by Mumu & Tanujaya (2019). R & D is used to achieve new knowledge that might be used to create new technology, products, services, or systems. In this stud, it was envisaged that the results might help teachers to interpret the ELDs. And when that is found, it may be one of the solutions to arrest underperformance in mathematics that has been recorded at all levels of the education system in the country (NNF, 2020)

8. Literature Review

8.1 Mathematical reasoning knowledge

As ECE is gaining ground globally and nationally (Walsh et al, 2010; OECD, 2001), there is need to examine teacher knowledge of numeracy especially mathematics reasoning as an approach to learning in early years. Teacher awareness seems to be qualified by the extent to which they could both use their content knowledge in mathematics and modify their instructional choices (Turkan, 2016). However, the majority of teachers are not trained on how to use numeracy especially mathematics reasoning as an approach to learning in play based pedagogy (Chimfwembe, 2020; Nagisa, 2019). For them to use it as an approach they need advanced content knowledge of mathematics such as functions, trigonometric functions and identities, calculus-introduction to differentiations, applications of the derivative, anti-derivatives, rates of change, areas, volumes of solids of revolution and the definite integral, methods of substitution-, financial mathematics, statistical analysis, combinatorics, proof by mathematical induction, vectors with application to projectile motion, 3 dimensional vectors, vector equation of lines, complex numbers, calculus-further integration techniques with partial fractions and integration by parts, mechanics-application of calculus to mechanics with simple harmonic motion, modelling motion without and with resistance, projectiles and resistant motion and concepts of mathematics modelling (Cresswell & Speelman, 2020).

Mathematical reasoning might be a challenge to teach and use in teaching for teachers who receive weak preparation for teaching mathematics, which may in part be due to their having a low level of mathematics knowledge, attitude and perception prior to their choosing to pursue early childhood mathematics teaching as a profession (Chimfwembe, 2020; Cresswell & Speelman, 2020 Vvob, 2017-2021; Lithner & Berqvist, 2012;). Because content knowledge is a prerequisite for knowing how to teach the content, increasing the mathematics knowledge of early childhood educators needs to be a priority (DfE, 2013 cited in The NRICH Primary Team).

Mathematics should be established in play, this could help in interpreting how numeracy can be used as an approach to learning (ELDs, 2016) since early child hood mathematics education is relatively new in the country (Chimfwembe, 2020; Kalinde 2020 & Nagisa, 2019) especially in centres annexed to primary government schools. Even in the private sector most emphasis was on the content that is now in the standard ECE curriculum (Payne et al, 1975) and yet no research has been undertaken to assess teacher awareness of mathematics reasoning and opportunities that are available to cultivate reasoning skills in children. According to ELDs (2016) mathematical reasoning is stated as a specific aspect under the sub-domain numeracy which is one of the approaches to child development and learning in early years.

Reasoning is an increasing sophisticated capacity for logical thought and actions, such as “analyzing, proving, evaluating, explaining, inferring, justifying and generalizing (Australian Curriculum mathematics-ACARA, 2016).” This means that mathematics reasoning at which ever level can be integrated with other developmental domains for modern scientific investigations (NAP, 2015).Generalizing and justifying has been the focus of most curricula and learning materials (Stylianders, 2010 in Vale 2017).

8.2 Opportunities that promote logic and reasoning

Mathematics reasoning can be viewed from some of the following perspectives: Creative mathematically founded reasoning (CMR) and initiative reasoning (IR) (Mumu & Tanujaya, 2019). CMR is divided into global and local creative reasoning. While IR is divided into two types of reasoning thus, algorithm and memorized. The two types of reasoning under IR are further divided into delimiting, guided and familiar. The two types of reasoning under CMR calls for reflection and argumentation in addition to teacher setting situations that are original and acceptable to mathematical foundations (Boesen, Lither and Palm, 2010). For example, in early childhood children’s play and interests are the foundation of their first mathematical and scientific investigations (Dooley, 2014 & Bjork Lund, 2020). Therefore, children should be given opportunities to solve problems arising from meaningful contexts (Sterner, 2020, Segerby, 2020, Vale 2017, Mueller, 2014 & Chimfwembe, 2020). Meaning the teacher should find mathematics in play or play should be found in mathematics using didactic approaches (Frit Stratmaun et al, 2016 in Bjork Lund, 2020 & Blazkova, 2013).The teacher should have sufficient subject matter and pedagogical content Knowledge to promote the construct of mathematics reasoning. While the reasoning types under IR calls for teacher employing strategies that can help children to recall by memory an answer. The strategy implementation consists only of writing such an answer with less consideration for the process. But teachers of the young children should bear in mind that, teaching involving young children entail certain challenges (Muchipa, 2020) that cannot simply be solved by adopting research/teaching designs that are used with older children. Actually children act differently in their everyday

situations than they do in experiment situations (Donaldson, 1978).

8.3 Words and questions that promote logic and reasoning

Reasoning comes naturally (Iumen 2020). Although you do not have to make it happen, you can make it happen in different ways. One can think positively or negatively, you can think with your heart and you can think with rational judgement. You can think strategically, analytically, mathematically and scientifically.

Logic is the ability to systematic think or steps to solve a problem or derive a pattern. It uses propositional variables which are often letters, to represent propositions. The foundation of logic are its proposition, or statement. The proposition is either accurate (true) or not accurate (false). Premises are the propositions used to build the argument. The argument is then built on the premises. Then an inference is made from the premises. Finally, a conclusion is drawn (Meleen, 2020). But if you are a teacher of adults' logic and adolescence are not terms often linked together. And yet as adolescents move into formal operational, they begin to become capable of reasoning without manipulatives. However the odds are that in any given classroom, you have students who use both stages of operational thinking thus, concrete and formal thinking in this case. The trick is to make the logic process as hands on, and physical as possible for students. Make Logic kinesthetic so that students have a physical movement to associate with the steps in the logical reasoning process. The teacher can also ask deeper questions that should call for comprehension, application, analysis, synthesis, evaluation, deduction, induction, abduction, refutation, balanced thinking, multiple perspectives, causal reasoning, ethical reasoning and creative thinking.

8.4 Operational definition of words

Mathematics in play is defined as the games, opportunities, contexts, words, questions, examples, ways that the teacher should make use of to ensure that mathematics reasoning is used as an approach to teaching and learning in early learning and development (Elin 2020).

Authentic: This means that the idea the responded gave is related to what is in literature or what others have talked about.

Social and emotional development: This means the ideas the respondent gave are related more to the social and emotional development domain than the cognitive domain.
Motivation: This means that ideas the student gave are related to the motivation domain and less to numeracy but maybe numeracy education

Function- This means that, a student mentioned something that was close to significance of mathematics reasoning and not as an opportunity that is in abundant to promoting mathematical reasoning.

Others-This means that the researcher could not classify what the respondent had written.

9. Methodology

Mixed methods were used. This was in line with the philosophy of realism.

9.1 Grounded theory

Grounded theory is used in naturalistic research that is used primarily to generate theoretical explanation of social processes in specific contexts (Noble, 2016 & Currie, 2009, Glaser & Strauss, 1967 cited in Tie, Birks & Francis, 2019). Both qualitative and quantitative data generation techniques can be used in a grounded theory (Tie, 2019). This depends on how participants constructed meaning in relation to the area of inquiry which is assessment of teachers' awareness of the construct of mathematics reasoning as an approach to learning in early childhood. Teachers confirmed at the beginning that they had no knowledge of mathematics taught at the university that can enable them to teach mathematics effectively in their classrooms, they do not even have an idea about the activities to use so that their lessons can be playful and that they have never had access to ELDs. Hence, the reason they have come for training so that they can be assisted. Because they have heard that, the institution is offering a degree program that is focused on play based pedagogy which is the in pedagogy under discussion now globally (Bjork Lund, Heuvel- Panhuizen, Kullberg, 2020). Therefore, the researcher should just teach them. The students' philosophy was not in tandem with teaching mathematics in higher learning institutions. Teaching in higher learning should be problem –solving, inquiry, investigation, practical activity etc. based unlike teacher directed as they proposed.

9.2 Population, sample, sampling procedure and techniques

The population consistent of ECE in-service teachers. While the sample had 49 respondents. 25 respondents had started at level 2 thus diploma entrant and expected to be at the university for three years, 24 had started at level 1, and thus they entered the university with a certificate and learnt for a full year before going into level 2 and were expected to be at the university for four years. Both groups were in year 2. Teachers were purposively sampled. Because this was in line with the purpose of the study. All teachers are expected to interpret national documents and develop and implement plans efficiently and effectively (MOE, 1996). The sampling criteria was exploratory in nature as there seemed to have been no study in existence on the interpretation of the ELDs and development of mathematics lesson plans in early childhood education using play as a pedagogy at university.

9.3 Data collection techniques

This study used class exercise, internet, literature and observation to collect data. Data was collected from December to January 2020/2021.The researcher started with gathering data from ECE books and the internet on mathematics in play- games, context, words, examples and

ways. Then gave a class exercise and observed the students reaction. The area of focus was numeracy and to be specific mathematical reasoning as an aspect under approaches to learning and assessed whether students are aware about this. Then examined teachers' awareness of opportunities that existed to teaching mathematics reasoning as an approach and later found out the words that they can use. This was done by teachers answering six questions in a period of two hours. Then the researcher collected the papers and marked. Then proceeded with the teaching plan of going through the questions, using problem solving, inquiry, investigations and practical activities (Mary & Yankelewitz 2014).

9.4 Coding, data analysis & presentation

Questionnaires were numbered and themes identified. Comparative analysis was used. Papers from the pre-test were marked and results analysed using simple computations. While answers from an interview were coded according to themes. This were later compared and interpreted before embarking on a teaching plan. Data was presented in tables, themes, percentages and frequencies.

9.5 Limitations

Since bases of knowledge are vast, this research did not look at contemporary research on early childhood mathematics content domains of number, measurement, geometry, algebra, statistics and probability; processes such as communication, problem solving, connections and representations; Other aspects such as curiosity, interest, persistence, creativity, imitation and emulation and advanced content domains such as functions, trigonometric functions and identities, calculus-introduction to differentiations, applications of the derivative, anti-derivatives, rates of change, areas, volumes of solids of revolution and the definite integral, methods of substitution-, financial mathematics, statistical analysis, combinatorics, proof by mathematical induction, vectors with application to projectile motion, 3 dimensional vectors, vector equation of lines, complex numbers, calculus-further integration techniques with partial fractions and integration by parts, mechanics-application of calculus to mechanics with simple harmonic motion, modelling motion without and with resistance, projectiles and resistant motion and concepts of mathematics modelling and how they can be taught in a play ful manner.

9.6 Ethical Considerations

This was part of the big project that the researcher is part of that is assessing mathematics performance in Lusaka and the country in general. Consent was sought from the head of academics and the permanent secretary. Students were informed about the purpose of the pre-test, interviews and observation.

10. Findings, interpretations & discussions

10.1 Teacher awareness of mathematical reasoning as an aspect in early learning and development.

The following bar graph gives a summary of teacher awareness of mathematical reasoning as an aspect in early learning and development.

Table 1: Level 2 teacher knowledge of mathematical reasoning as an aspect

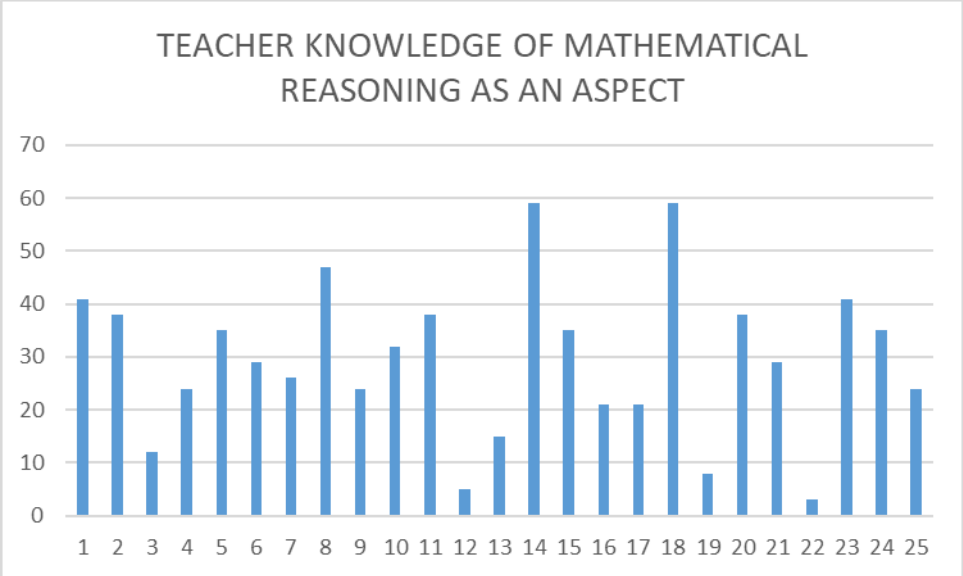

Table 1 above shows that only 02 out of 25 students thus, 08% had knowledge of mathematics as an aspect in early learning and development while 23 thus, 92% were not aware

Table 2: Level 1 teacher knowledge of mathematical reasoning as an aspect

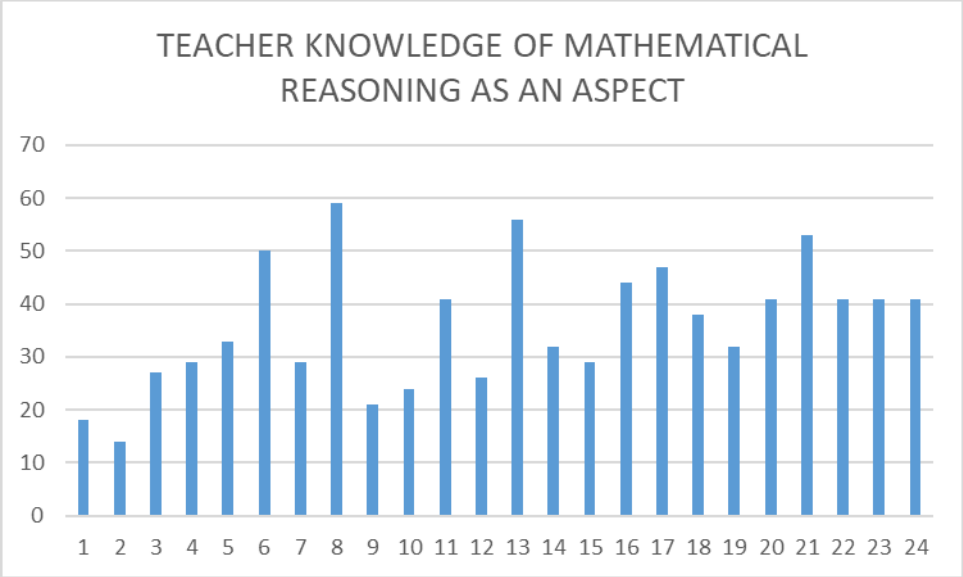

Table 2 above show that 04 respondents thus, 17% were aware of mathematical reasoning as an aspect while the 20 respondents thus 83% were not aware.

10.2 Mathematical reasoning opportunities in abundance for young children

Table 3 shows findings in five categories thus authentic, social and emotional development related motivation-related, function-related and other mathematical opportunities in abundance for young children.

Table 3: Some mathematical reasoning opportunities in abundance for young children

Authentic	Social and Emotional related	Motivation related	Function related	Others
Recording children's predictions peer playing	Focusing on your strength	Wanting to go back to school	To develop cognitive	The brain is choosing to sustain certain kinds of connections and at the same time deciding not to maintain connections or other areas
Hands on experiences (charts, graphs, field trips)	Surrounding oneself with people who have abundance mentality	"Clever", "excellent", "very good"	Physical and mental development	
Minds on experiences thus (storytelling, riddles, rhymes, tongue twisters and fairy tales)	Some are abandoned because of peer pressure	Travelling and eating	They get to solve problems easily	
Making predictions based on previous experiences	Feeling ashamed		They get to think fast	
Peer playing			They will know a lot of thing	
			And they will not	
			Children will discover themselves about each one thinks	
			Help learners to know how each one differs to other	
			Early childhood education fosters children's reasoning and problem solving	

As can be seen from the table 3, the responses on this question varied. Some respondents in this study mentioned recording children's predictions, hands on experiences (charts, graphs, field trips) and minds on experiences thus (storytelling, riddles, rhymes, tongue twisters and fairy tales), making predictions based on previous experiences and peer playing as mathematical reasoning opportunities in abundance for young children.

This was consisted with what Copley, 2010 & Bjork Lund, 2020 mentioned as some mathematical reasoning opportunities. While others gave social and emotional development related, unclassified, motivation and importance – related answers.

Some of the responses that most students gave are true, although others were not in line with the definition of mathematical reasoning (Mumu & Tanuyaya, 2019). Some opportunities like riddles are an introduction to the notion of

logic and mathematical language. While making predictions is also key in understanding the notion of statistics and approximations. Other opportunities mentioned were mental mathematics. This opportunity is also consistent with NNF (2020) where the hundred cell calculations has been mentioned as one of the approaches to teaching numeracy and to promoting mental mathematics. While other opportunities were giving children open ended materials to explore which was consistent with ECLKC (2020) & Education Review (2021);providing children with plenty of time which was consistent with Chimfwembe (2020) & Nagisa (2019); with plenty of intriguing manipulatives that are open ended such as coloured rods, beads and many others. However, to some extent, these findings were not consistent with opportunities given by Copley (2010) such as classifying and labelling sets of objects, solving problems, observing others, listening to others directly but may be implied.

Further the respondents in this study recommended that during clean-up the teachers should provide activities that are challenging to enable learners practice skills such as counting objects, comparing magnitudes, exploring pattern shapes and spatial relations, filling different –size containers with water, sand, counters and crayons. This was also consistent with Copley (2010); Clements & Sarama (2017). However, these findings are from the researcher's compilation of what all the respondents said. But the picture was different from responses from one questionnaire to another as can be seen from simple statistics below:

From 49 respondents, only 8 out of 49 respondents thus 16 % got 3/5 from the question on reasoning opportunities in abundance for children. While the rest got 2 out of 5 and below thus 94% failed this question. This finding also differs from Machana (2013) finding when she assessed teacher's challenges in the teaching of mathematics at foundation stage. Her findings were that teachers had challenges teaching children who were multilingual (Chimfwembe, 2020), teachers also had problems because they had no addition time as most of the time was spent doing other institutional and national duties. And the last challenge was that teachers were employing whole class teaching which was not recommended for early childhood learners. Muchana's recommendation is consistent with Blazkova (2013). But not consistent with the current study because in this study the researcher was trying to find a best way of helping this students on how to interpret the ELDs in line with the concept of mathematical reasoning.

Results were further sorted according to levels and analyzed, 3/5 was taken to be yes while 2/5 was taken to be no. Six respondents got 3/5 but this just represented 6 out of 25 respondents from level 2. This was equal to 24% while the rest thus, 19 out of 25 respondent got less than 3, thus 76% of the respondents failed to mention the reasoning opportunities that are in abundant for young children. The picture was even worse with level 1 students were only 2 out of 24 students thus 8% got 3/5 questions right and the rest thus 92% failed.

This study has brought in subject matter based challenges in students thus, mathematics reasoning though there still remains varying views, and available opportunities to boost it (Sarama & Clements, 2009, Klein and Wakeley, 2004; Wilson and Cooney, 2002 cited in Yildiz & GoK Colak, 2019) has opposed to management based challenges such as planning, implementation and evaluation processes of in-class and out-class mathematics activities and parent involvement in such activities (Yildiz & GoK Colak, 2019).

The students in this study did not even like the idea of going through the questions immediately after answering them and problem solving but enjoyed practical activities. Although others said, preparation of activities took them a lot of time even if the researcher guided by giving them activities to do.

10.3 Useful games that promote reasoning

The respondents in this study mentioned memory, puzzle, bottle cap shooting, molding, number, matching, field, who am I, traditional-"isolo", "ichiyato" and many others, snakes and ladder, fire on the mountain, count walk, the missing number, building blocks, sand play, water play, music & dance, bouncing sums, stew poll, sharing equation eagle as some of the games that a teacher should use to promote mathematical reasoning in children. This findings are consistent with Copley (2010) & Jenny (2011). Although Copley (2010) mentioned building models out of cubes for example 1+2=3 (odd), 5+10= 15(odd), 7+2=9 (odd); Guess my cage, investigation such as letting a child who says earthworms have eyes carry an experiment, record by creating charts and graphs to justify own beliefs, justifying rules about numbers for example, even numbers added to odd numbers is equal to an odd number as some of the games to promote reasoning in children. In line with the same, Jenny (2011) said that a game should have rules, strategies and outcomes that are defined by mathematical parameters. It should have two or more players who should take turns, each competing to achieve a winning situation of some kind, each able to exercise some choice about how to move at any time through playing. According to *ibid, snakes and ladder although it was popular among the respondents is not a game because winning relies totally on chance. The players make no decisions, nor do they have to think further than counting. There is no interaction between players-nothing that one player does affect other players turns in any way.* (Gough, 1999).

Adding on to Jenny's sentiments, Oldfield (n.D) quoted by Jenny (2011) said that,

> *A mathematical activity should involve a challenge, usually against one or more opponents, are governed by a set of rules and have a clear underlying structure, normally have a distinct finishing idea and lastly have specific mathematical cognitive objectives.*

10.4 Context where to emphasize reasoning

Individual respondents gave the following answers:

> *playing games (consistent with Jenny, 2011), during class activities (, in homes by parents, in group discussions, in class exercises-give children numbers to add /subtract, to draw the correct objects, during story- telling time, when playing number games and other games that stimulate mental capacities, at*

school- because it is where a child most interacts with its friends, during play time, circle time and most of the time during teaching, when giving instructions, all the time, in individual activities-when the teacher has given work to the learners to provide answers individually, through song, at an early age-when the child starts to grasp information.

This finding is consistent with other key documents such as NCTM&NAEYC (2000) & ELDs (2016). This is because mathematics is one of the few school subjects that to its content does not differ that much across the word (Norqvist, 2016). Although their might be differences regarding the way this uniform subject is taught as teachers are more directed by syllabuses, various textbooks and more or less responsibility is left with the student.

10.5 Words and questions to promote logic and reasoning in Children

Respondents gave words like *how, why, arrange, order, match, how, where, find, which, who, when, what, share, explain, sort, add, subtract, remove and compare* as some of the words that they can use in class to promote reasoning in children.

There were also variations in the way they responded as some gave the following answers:

by giving children games such as puzzle games, cheers so that they think critically and they will be able to reason, to reward for example, very good, well done, keep it up, or give a clap/clap for him, encourage them by talking good about them-give it a try, motivation words, asking learners questions-such as what happened, what colour is a banana, how many oranges are in the basket, how are you, what is your name, where do you stay, asking them to count numbers, introducing games, by using meaningful words when children are doing the clean-up in class the teacher should help the children to classify the objects. Like put away the building blocks, or pick all the papers on the floor and throw in the bin, identify objects, differentiate objects, selecting colour red objects.

Situations were described such as,

a teacher should use meaningful words to reinforce different mathematical reasoning, by using real objects when teaching also using real life experiences and also giving clear instructions when explaining certain concepts, careful planning classroom experiences, sentences, questions, instructions/commands were given-throw the ball, catch the ball, tell us, do it social and moral development words like please, sorry, appreciate, thank you, ask may I, excuse me, playing together, sharing responsibilities and how to motivate learner emerged and not actual words like *or, not, because, some, all, never, probably* as described by (Copley, 2010) that are used as logic words.

05 respondent left this question unanswered. 04 from level 01 and from level 02. Also unclassified words/statements like

imagination or imagine, think, count, before, after, between, describe games, promote nurture effective and active learning, tongue twister words, using proverbs words, inductive, deductive, allowing children to give references, giving children remedial work every day, putting children into groups and discuss on a question, learning through trial and error, looking at one thing at a time, allowing for different learning styles, stimulate children's curiosity and thinking skills.

Questions that were given also differed as can be seen below:

Table 4: Some questions that can promote mathematical reasoning according to levels

Level 1	Level 2
1) What is the next numeral between 2 & 4? (patterns)	1) Who is older between you and your sister? (number & measurement)
2) Which shape has 3 sides and all sides are equal? (number & geometry)	2) Where is the box of toys? (spatial sense)
3) Which one is bigger number 9&8? (number quantity)	3) How many blue and white pencils are in the box? (number-counting)
4) Mention the last month of the year: (measurement & number)	4) What is 2+1 (operations)
5) Where were you born? (measurement & number)	5) What do you count sweets before sharing with friends? (divisions, fractions, general)
6) In which grade you are? (number)	6) What do you do when you want to put objects?
7) How many are you in the family? (number)	7) Ask learners to tell stories about themselves (general)
8) What would you have done if it were you? (General)	8) Count numbers from 1 to 10 (number)
9) How do you add numbers using objects?	9) Write anything you can think of in your book? (General)
10) Why do you think the mother was happy with what the daughter did? (General)	10) How does this work?
11) What is your name? (General/language & literacy development)	11) How can you do this?
12) Where is your school? (General/language & literacy development)	12) What happened?

Developing language, including vocabulary is a key task for young children. Because this allows children to learn from and with others and also lay the critical foundation for success (Wasik & Hindman, 2013). The findings from this study indicate that, most teachers understood the questions but did not know what the "word" meant. In linguistics, a word of a spoken language can be defined as the smallest sequence of phonemes that can be uttered in isolation with objective or practical meaning (wiki, 2020).

However, the questions they gave were different from Copley (2010)'s questions of

Are you sure? How do you know? Why do you think…? What else can you find that works like this? What would happen if…? I wonder how this could be changed. What would the pattern be? What if…? I wonder why…? Perhaps it's because…

The teachers in this study also mentioned some ways in which teachers could continue modelling reasoning language in everyday experiences. They gave the following answers:

Give children problems to solve, give remedial work, ask children questions and allow them to also ask- for example ask them what they did in the previous lesson before they start a new lesson, just to find if they can remember what they learnt.

Other teachers also said that,

Children should be encouraged to speak correctly at all times. A teacher should not use jargon words to the learners. By making a daily programme, for example, children should be given a task each day. By giving them tasks to go and do at home and also through everyday interactions, songs and use of playing games, role play and free play

The responses that the teachers gave differed with the following ways: By using phrases such as

If that is right, then…or I want to find out if…or so I will try… or that can't be, because if it were then…; pausing before responding, re-organising data to search for patterns, recording what happens after each trial in an experiment, thinking aloud and expressing own thoughts about the problems and listening to children's problems (Payne et al, 1975 &; Smith, 2013; Shaw & Blake 1998).

11. Conclusions & recommendations

This research has covered a broad spectrum of literature on mathematics reasoning as an approach to learning other academics as stated forth in the early learning development standards (ELDS 2016). Particular attention has been on teacher knowledge of mathematical reasoning using university in-service students as a sample to help interpret the ELDs. Findings showed that most students had less teacher knowledge. The study further assessed teacher's awareness of mathematics reasoning as an approach to learning. Findings on this question varied because answers ranged from opportunities available, how to improve mathematics reasoning, motivation related to the importance /significance of mathematics reasoning.

This study further examined teacher knowledge of the reasoning opportunities that are in abundant for young children and some of the games that teachers think should be emphasized to promote logic and reasoning in children. The findings on these research question were consistent with what others had said on reasoning opportunities and some games a teacher can use to promote reasoning but not consistent with the definition of game. Because among the games mentioned, some were not expected to be considered as games.

Lastly, the study sought to find some of the words and questions that they thought should be emphasized to promote logic and reasoning in children. The findings on this study varied as it's always the case when one gives an open ended question (Desa & Reimers 2019**).** There was no common answer. However, since the purpose of the study was to examine teacher knowledge of mathematics reasoning in lieu of interpreting the ELDs an open ended question was sufficient because this one is not biased to one response and usually enables learners to think critically, make an attempt to answer a question in detail which is one of the key skills in the 21st century era (Wasik & Hindman, 2020).

Findings have varied but generally respondents had less than 50% teacher knowledge of mathematical reasoning hence, the null hypothesis is accepted while the research objective is rejected. With some findings consistent with literature while others are based on students perceptions on what they feel are opportunities, games, words, questions that they feel can promote mathematical reasoning. In the ELDs mathematics reasoning has been cited as a specific aspect under numeracy. This aspect is to be used as an approach to learning in early learning and development.

To avoid a variation in the answers, this study recommends that,

- Research should probe further to reduce on the variance in responses and increase on the amount of information - concerning places where reasoning activities may be emphasised- collected per respondent to help this students to interpret the ELDs effectively and sufficiently.
- Reasoning activities maybe emphasised during routine activities for example, when teachers give directions such as, *those people not wearing red may get their jackets to go outside or those boys and girls who did not have a turn yesterday may go and play* (Copley 2010; Shaw & Blake 1998).
- Teachers should also use open-ended prompts, provide opportunities for children to express their ideas and to receive feedback from adults and peers on what they have shared.
- A much cognitive psychology and neuroscience research base need to be developed at university T as there is no conclusion that mathematics enhances higher order cognitive functions.
- A course in advanced mathematics should be one of the courses to be introduced at university T because this is one of the areas of mathematics where logic and reasoning maybe emphasized (Cresswell & Craig 2020).
- The methodology course should include mathematical reasoning opportunities abundant for young children, games to promote mathematics reasoning, words & questions that a teacher can use to foster children's reasoning and other ways of modelling reasoning languages in everyday experiences.

- Tasks that include the cognitive reflection test and Wason selection task should be included when assessing teacher knowledge of mathematical reasoning because they are of particular interest and yet they have typically and reliably eluded participants in all studies and studies where they have been included the results have been uncorrelated with general intelligence, education levels and other demographic information (Cresswell & Craig 2020).
- There should be a broad spectrum of educational research focusing on different content -number, measurement, geometry, algebra, functions, statistics, probability (NCTM & NAEYC 2000; NNF, 2020).
- Under mathematics methods-writing a weekly schedule, daily plan, integrated lesson plan in teaching and learning mathematics among the youngest children in the educational system (ECLKC, 2021) should be emphasized on by teachers.
- Approaches such as exploring and construction (Hannikainen and Munter, 2018;Solem and Reikaras, 2017;Schimitt et al 2018), pretend/imaginative/make-belief/socio dramatic/role (Vygotsky, 1978;Jung& Li, 2015;Storli & Sandseter, 2015), rule based (Lai et al, 2018 & Ramain et al, 2019), independent (Becker et al, 2018 & Hannikainen and Munter, 2018) and interaction play (Engdahl, 2011& Bjorklund, 2008 should be the focus of early childhood mathematics.
- Future study should examining students' awareness of curiosity and interest, persistence, creativity, imitation and emulation as aspects under approaches to learning and development (ELDs, 2016).

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References

[1] Child Development and Early Learning: A Foundation for Professional Knowledge and Competencies: Washington, dc; National Academies of Science, Engineering and medicine@www.nap.edu>resource accessed on 24/12/2020

[2] Allen LR & Kelly BB editors (2015) Committee on Sciences of Children Birth to Age 8: Deepening and Broadening the foundation for Success; Board on Children, Youth and Families; Institute of Medicine; National Research Council. Transforming the workforce for children birth through age 8: A unifying foundation. Washington (DC): National Academies Press.2015 July 23.4, Child development and early learning… @www.ncbi.nlm.nih.gov>nbk310550 accessed on 24/12/2020

[3] Jennie, Liz and Bernard (2014) Reasoning: Identifying Opportunities.UK: The NRICH Primary Team, @https://nrich.maths.org/10990 accessed on 24/12/2020

[4] Digital Chalkboard (2009-2020) Mathematics: Kindergarten through grade twelve (k-12) standards for mathematics practice. California; Department of Education, the Regents of the university of California and the California mathematics project @1.3, 4 21st Century skill::CCSS Mathematics:k-12 standards for… accessed on 25/12/2020

[5] MOE (2016) Early Learning and Development Standards for Zambia. Lusaka: CDC

[6] MOESVTEE (2013) Early Childhood Education Syllabi (3-6 yeas) Level 1-3. Lusaka: CDC

[7] MOE (2015) Early Childhood Education Teachers Diploma Syllabus. Lusaka: CDC.

[8] Noble H (2016) What is Grounded theory?/ Evidence Based Nursing @ebn.bmj.com>content accessed on 25/12/2020

[9] Tie YC, Meanie Birks, and Karen Francis (2019) Grounded theory research: A design framework for novice researcher SAGE open Med.2019;7:2050312118822927.doi:10.1177/2050312118822927@ www.ncbi.nlm.nih.gov> articles accessed on 25/12/2020

[10] Vvob (2017-2021) Zambia –STEEL: Supporting early childhood teachers on play-based learning pedagogies.@www.vvob.org>programmes>za accessed on 25/12/2020

[11] Turkan Sultan (2016) in-service teachers reasoning about scenarios of teaching mathematics to English language learners, The mathematics Enthusiast:volume.13: No1, Article9 @ https://scholarworks.umt.edu/tme/vol13/iss1/9 accessed on 26/12/2020

[12] Tomas Bergqvist & Johan Lithner (2012) Mathematical reasoning in teacher s presentations. June 2012. The journal of Mathematical behavior 31(2):252-269.DOi:10.1016/j, jmathb.2011.12.002 accessed on 26/12/2020

[13] Bjork Lund, C., Van den Heuvel- Panhuizen, M & Kullberg, A. Research on early childhood mathematics teaching and learning. ZDM Mathematics Education 52, 607-619 (2020). https://doi.org/10.1007/s11858..020..01177..3 accessed on 27/12/2020

[14] Therese Dooley, Elizabeth Dunphy, Gerry Shiel, Deirdre Butler, Dolores Corcoran, Therese Farrell, Siun Nic Mhuiri, Maura O'Connor, Joe Travers, Bobby Perry (2014) Mathematics in Early Childhood and Primary Education (3-8 years): Teaching and Learning. Dublin: NCCA@ncca.ie>media...resear... accessed on 27/12/2020

[15] Segerby, C & Chronaki, A (2018) Primary Students' Participation in Mathematics reasoning: coordinating reciprocal teaching and systematic functional linguistics to support reasoning in the Swedish Context. EDeR… Educational Design Research, 2 (1), 1-32.@ http://dx.doi.org/10.15460/eder.2.11150 accessed on 27/12/2020

[16] Gorel Sterner, Ulrika Wolff & Ola Helenius (2019) Reasoning about Representations: Effect of an Early Maths Intervention, Scandinavian journal of Education Research, 64:5, 782-800, DOI:10.1080/00313831.2019.1600579@https://doi.org/10.1080/00313831.2019.1600579 @

www.tandfonline.com>doi>full accessed on 28/12/2020

[17] Vale Colleen; Bragg Leicha A; Widjaja, Wanty; Herbert Sandra; Loong, Esther Yook-Kin (2017) Children's Mathematical Reasoning: Opportunities for Developing understanding and creative thinking. Adelaide: Australian Association of Mathematics Teachers (AAMT) @eric.ed.gov>. accessed on 28/12/2020

[18] Mueller Mary & Dina Yankelewitz (2014) Teacher Promoting Student Mathematical Reasoning: Investigations in Mathematics Learning, The Research Council on Mathematics Learning, volume 7, No 2 @files.eric.ed.gov>full-text accessed on 28/12/2020

[19] Nagisa Nakawa (2019) Current Situation of Zambian Children's Guided – Play in an Early Childhood Education Pre-mathematics classroom: A case study: Conference: 27th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology. South Africa: Durban@www.researchgate.net >net publication accessed on 29/12/2020

[20] Lungu S & Matafwali B (2020) Play-Based Learning in Early childhood Education (ECE) Centres in Zambia: A teacher Perspective. European Journal of Education Studies, 7(12) @ scholar.google.com>citations accessed on 29/12/2020

[21] Jeinne Mumu & Benidiktus Tanuyaya (2019) Measure Reasoning Skill of Mathematics Students. International Journal of Higher Education, 8(6), Doi:10.5430/whe.v8n6p85@www.sciedu.ca>downloa d PDF accessed on 28/12/2020

[22] Dana Charles Mc Coy, Stephanie Simmons Zuilkowski, Hirokazu Yoshikawa & Gunther (2017), Early Childhood Care and Education and School Readiness in Zambia, journal of Research on Educational Effectiveness, 10:3, 482-506, DOI: 10.1080/19345747.2016.1250850 accessed on 30/12/2020

[23] Kalinde Bibi an (2016) Cultural Play songs in early childhood education in Zambia: in and outside of classroom practice. Thesis submitted in partial fulfilment for the degree doctor of music at the university of Pretoria (published article)@repository.up.ac.za> handle accessed on 30/12/2020

[24] Tiwine Muchipa (2020) Starting early: Improving child development through play in Zambia.@www.unicef.org>Zambia> stories. accessed on 30/12/2020

[25] Geoffrey Kazembe Tambulukani Thesis Submitted … @ lollenautafoundation.eu>upl..accessed on 15/11/2020

[26] Meleen Michele (2020) Examples of Logic: 4 main Types of Reasoning@ examples.yourdictionary.com>exa. accessed on 31/12/2020

[27] Kay Currie (2009) Using Survey Data to assist theoretical sampling in grounded theory research, October 2009. Pub Med; Nurse Researcher17 (1):24-33. DOI:10.7748/nr2009.10.17.1.24.c7336 accessed on 01/01/2020

[28] Douglas H. Clements & Julie Sarama (2017) blog on play, mathematics and false dichotomies posted in general, parents, teacher educators and teachers. *Development and Research in Early Mathematics Education @dreme.stanford.edu>news>play-m…* accessed on 01/01/2020

[29] Elin Reikeras (2020) Relations between play skills and mathematical skills in toddlers. ZDM Mathematics Education 52, 703-716 (2020).https://doi.org/10.1007/s11858-020-01141-1or link.springer.com>article accessed on 01/01/2020

[30] Jae Eun Lee (2017) Pre-school teachers pedagogical content knowledge in Mathematics. August 2017. International Journal of Earlychildhood 49(4) Doi:10.1007/s 13158-017-0189-1 @eric.ed.gov accessed on 01/02/2020

[31] Math Riddles: Try to answer these brain teasers and math riddles @ www.mathwarehouse.com>riddles accessed on 01/02/2021

[32] Head Start/ ECLKC-Early Childhood Learning & Knowledge Centre (2020) Approaches to Learning: School Readiness @eclkc.oh.acf.hhs.gov>article>app accessed on 01/02/2021

[33] Education review office (2021) A guide to children's early mathematics Learning@ ero.gov.nz/publication accessed on 02/02/2021

[34] Chimfwembe Getrude-Gondwe, *Play-based pedagogy in ECE Centres: An emphasis on Module Improvement for expanse students*, " International Journal of Science and Research (IJSR), https://www.ijsr.net/search index results paper id. php? Id=SR201213022753, Volume 9, issue 12, December 2020.774-778 [12] accessed on 08/01/2021

[35] Chimfwembe Getrude, *Classroom practices that exist when University In-service Students Teach informal geometry*, " International Journal of Science and Research (IJSR), https://www.ijsr.net/search index-results-paper id. php? Id=SR201213040558, Volume 9, issue 12, December 2020.779-786/ accessed on 07/01/2021

[36] Getrude Chimfwembe-Gondwe, " *Pre-school Teachers Pedagogical Content Knowledge for Teaching Informal Geometry: A case of In-service University Students*, "International Journal of Science and Research (IJSR), https://www.ijsr.net/search_index_results_paperid.php ?id=SR 201224234848, volume 9 issue 12, December2020, 1565-1583 accessed on 07/01/2021

[37] Guven, Y & GoK Colak, F(2019). Difficullties in Early Childhood Education Teachers' in Mathematics Activities, Acta Didactica Napocensa12(1), 89-106, DOI:10.24193/adn.12.1.6@www.researchgate.net>pub lication accessed on 07/01/2021

[38] Maphetla Magdelin Machaba (2013) Teacher Challenges in the teaching of mathematics at foundation stage. Doctoral thesis submitted in accordance with the requirements for the degree of doctor of education in the subject early childhood education at the University of south Africa@uir.unisa.ac.za>handle>thes... accessed on 07/01/2021

[39] Li, Y & Schoenfield A.H, Problematizing Teaching and Learning Mathematics as" given" in STEM

education, IJ STEM Education 6, 44(2019).https://doi.org/10.1186/s40594-019-0197-9.DOI:https://doi.org/101186/s40594-019-0197-9@stemeducationjourna.springeropen.com>… accessed on 07/01/2021

[40] Susan Sperry Smith (2013) Early Childhood Mathematics. Boston: Pearson.

[41] Jean M. Shaw & Sally S. Blake (1998) Mathematics for Young Children. London Prentice Hall international.

[42] Copley V.J (2010). The young child and mathematics. Reston: NCTM and NAEYC

[43] Desa S C & Stian Reimers, Comparing the use open and closed questions for web-based measures of the continued-influence effect. Behav-res 51, 1426-1440 (2019). https://doi.org/10.3758/s13428-018-1066-z/link.springer.com>article accessed on 10/01/2021

[44] Barbara Wasik & Annemarie, Realising the Promise of Open- Ended Questions, December 2013. The Reading Teacher 67(4) DOI: 10.1002/trtr.1218/www.resaerchgate.net>publication accessed on 10/01/2021

[45] Norqvist M (2016) On Mathematics Reasoning-being told or find out. Umwea, Sverige: Department of mathematics and mathematical statistics @www.diva..portal.org> smash>record accessed on 10/01/2021

[46] Cresswell C, Craig P Speelman CP (2020) Does mathematics training lead to better logical thinking and reasoning? A cross-sectional assessment from students to Professors. PLoS ONE 15(7): e0236153.https://doi.org/10.137/journal.pone.0236153 Sydney. Department of Mathematics and Statistics @.https://journals.plos.org>article>jo… accessed on 10/01/2020

[47] GRZ (n.D) The Day Nurseries Act: Chapter 313 of the laws of Zambia. Lusaka: Ministry of Legal Affairs.@www.parliament.gov.zm>acts accessed on 26/01/2020

[48] Mwape S (2018) Zambia bans unregistered pre-schools @ www.lusakatimes.com>2018/01/23 accessed on 26/01/2020

[49] Colleen Vale (2018) 7 reasons why Mathematics is important for 21st Century thinking. Monash University@www.monash.edu>articles>seven... Accessed on 26/01/2020

[50] Ruzena Blazkova (2013) Didactics of Mathematics 1-Pedagogical fakulta Mu @ www.ped.muni.cz>interma accessed on 15/10/2020