

# The Beal's Conjecture and Fermat's Last Theorem

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**Abstract:**  $A^x + B^y = C^z$ , where  $A, B, C, x, y$  and  $z$  are non-zero integers with  $x, y, z \geq 3$ , then  $A, B$  and  $C$  have a common prime factor equivalently. The equation  $A^x + B^y = C^z$  has no solutions in non-zero integers and pairwise coprime integers  $A, B, C$  if  $x, y, z \geq 3$ . The conjecture was formulated in 1993 by Andrew Beal. If proved, \$1, 000, 000 prize award. The conjecture was formulated in 1993 by Andrew Beal a banker and amateur Mathematician, while investigating generalization of Fermat's Last theorem. Since 1997, Beal has offered a monetary prize for a peer-reviewed proof of this conjecture or a counter example.

**Keywords:** Beal's conjecture, non-zero integers, pairwise coprime factor, Boscomplex theorem, Fermat's last theorem

## 1. Proof

From Beal's equation  $A^x + B^y = C^z$ , where  $x, y, z \geq 3$ .

$$\text{Let } x = y = z = n. \quad (1)$$

$$\Rightarrow A^n + B^n = C^n \quad (2),$$

where  $n = 3$  (minimum power) according to Beal.

[1] Let  $A = 3, B = 6, C = 9$  {pairwise coprime integers},  $n = 3$  (a constant index).

**3, 6 and 9 have a common prime factor 3.**

$$\text{From } A^n + B^n = C^n$$

$$\Rightarrow 3^3 + 6^3 = 9^3$$

$$27 + 216 = 729$$

$$\text{But } 243 \neq 729.$$

$$\text{Ratio, } r \text{ of RHS: } \text{LHS} = 729/243 = 3.$$

$$\text{Thus } 3(243) = 729.$$

$$\Rightarrow r = C^n / (A^n + B^n) \quad (3).$$

$$\text{Or } r = C^z / (A^x + B^y) \quad (4),$$

**The general form of the ratio.**

$$\Rightarrow r(243) = 729$$

$$\text{Hence } r(27 + 216) = 729$$

$$\text{Thus } r(3^3 + 6^3) = 9^3$$

$$\Rightarrow r(A^n + B^n) = C^n \quad (5)$$

$$\therefore n(A^n + B^n) = C^n \quad (6)$$

where  $x = y = z = r = n$ .

### Check

$$\text{From } r(A^x + B^y) = C^z$$

$$\Rightarrow 3(3^3 + 6^3) = 9^3$$

$$3(27 + 216) = 729$$

$$3(243) = 729$$

$$729 = 729.$$

$$\text{From (5), } r(A^n + B^n) = C^n$$

$$r(A^x + B^y) = C^z \quad (7), \text{ where } x \neq y \neq z \neq r \neq n.$$

$$\Rightarrow A^x + B^y = (C^z)/r \quad (8)$$

$$\text{Thus } A^x = (C^z)/r - B^y \quad (9)$$

$$\therefore A = \sqrt[x]{(C^z)/r - B^y}. \quad (10)$$

Similarly, from (8),

$$B^y = (C^z)/r - A^x. \quad (11)$$

$$\therefore B = \sqrt[y]{(C^z)/r - A^x}. \quad (12)$$

From (7),

$$C = \sqrt[z]{r(A^x + B^y)}. \quad (13)$$

### Note:

- 1) The general form of  $r(A^n + B^n) = C^n$  is written as  $r(A^x + B^y) = C^z$  or  $A^x + B^y = (C^z)/r$ . This is Boscomplex Theorem.
- 2) Equations (10), (11) And (13) are the general solutions to  $A^x + B^y = C^z$ .
- 3) Therefore, Boscomplex Theorem states that  $r(A^x + B^y) = C^z$ , if  $A, B, C, x, y,$  and  $z$  are non-zero integers with  $x, y, z \geq 3$ ,

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then A, B and C have a common prime factor equivalently.

4)  $C > A$ , and B.

$\therefore$  From  $n(A^n + B^n) = C^n$

$$\Rightarrow A^n + B^n = C^n/n$$

$$A^n = (C^n)/n - B^n \quad (14)$$

$$\therefore A = \sqrt[n]{(C^n/n - B^n)} \quad (15)$$

$$\Rightarrow A = \sqrt[3]{(C^3/3 - B^3)} \quad (16)$$

$$\text{Similarly, } B = \sqrt[n]{(C^n/n - A^n)} \quad (17)$$

$$\Rightarrow B = \sqrt[3]{(C^3/3 - A^3)} \quad (18)$$

$$\text{And } C = \sqrt[n]{n(A^n + B^n)} \quad (19)$$

$$\Rightarrow C = \sqrt[3]{3(A^3 + B^3)} \quad (20)$$

Therefore, equations (15), (17) and (19) are the solutions to the equation  $A^x + B^y = C^z$ , where  $x = y = z = r = n$ .

## 2. Worked Examples

**For example;**

[1] (i) To find B value in the above example when  $A = 3$ ,  $C = 9$ ,  $n = 3$ ; given  $A^x + B^y = C^z$ .

**Solution**

Given;  $A = 3$ ,  $C = 9$ ,  $n = 3$ , and  $B = ?$

From Boscomplex Theorem,  $r(A^x + B^y) = C^z$

$$\Leftrightarrow n(A^n + B^n) = C^n; x = y = z = r = n.$$

$$\Rightarrow B = \sqrt[n]{(C^n/n - A^n)}$$

$$B = \sqrt[3]{(9^3/3 - 3^3)}, n = 3$$

$$= \sqrt[3]{(729/3 - 27)}$$

$$B = \sqrt[3]{216} = 6 \text{ as above.}$$

ii) To find the value of A in the above example when  $B = 6$ ,  $C = 9$ ,  $n = 3$ .

**Solution**

From  $A = \sqrt[n]{(C^n/n - B^n)}$

$$= \sqrt[3]{(9^3/3 - 6^3)}$$

$$= \sqrt[3]{(729/3 - 216)} = \sqrt[3]{243 - 216}$$

$$A = \sqrt[3]{27} = 3 \text{ as above.}$$

(iii) To find C value in the above example when  $A = 3$ ,  $B = 6$ , and  $n = 3$ .

**Solution**

From Boscomplex theorem,

$$r(A^x + B^y) = C^z$$

$$\Leftrightarrow n(A^n + B^n) = C^n$$

$$\Rightarrow C = \sqrt[n]{n(A^n + B^n)}$$

$$= \sqrt[3]{3(3^3 + 6^3)}$$

$$C = \sqrt[3]{3(27 + 216)} = \sqrt[3]{729} = 9 \text{ as above.}$$

[2] Let  $A = 5$ ,  $B = 10$ ,  $C = 15$ ,  $n = 3$ .

From (2),  $A^n + B^n = C^n$

$$\Rightarrow 5^3 + 10^3 = 15^3; \text{ where } 5, 10 \text{ and } 15 \text{ have coprime factor } 5.$$

$$\Rightarrow 5^3 + 10^3 = 15^3$$

$$125 + 1,000 = 3375$$

$$\text{But } 1125 \neq 3375$$

Hence ratio,  $r = 3375/1125 = 3$ , it holds ( i.e  $r = n = 3$  ).

$$\Rightarrow r(1125) = 3375$$

Hence  $r(125 + 1,000) = 3375$

$$r(5^3 + 10^3) = 15^3$$

$$\Rightarrow r(5^n + 10^n) = 15^n \Leftrightarrow 3(5^3 + 10^3) = 15^3$$

But  $r = n$ .

$$\therefore 3375 = 3375$$

Hence  $n(5^n + 10^n) = 15^n$ .

$\therefore n(A^n + B^n) = C^n$  as in [1] above.

Therefore, the values of A, B and C are calculated as in example 1 (i), (ii) and (iii) above.

**i.e.;** From  $A = \sqrt[n]{(C^n/n - B^n)}$ ,

$B = \sqrt[n]{(C^n/n - A^n)}$ ,  $C = \sqrt[n]{n(A^n + B^n)}$

From  $A = \sqrt[n]{(C^n/n - B^n)}$

$$A = \sqrt[3]{15^3/3 - 10^3}$$

A = 5 as above.

From  $B = \sqrt[n]{(C^n/n - A^n)}$

$$B = \sqrt[3]{15^3/3 - 5^3}$$

B = 10 as above

From  $C = \sqrt[n]{n(A^n + B^n)}$

$$C = \sqrt[3]{3(5^3 + 10^3)}$$

C = 15 as above.

**[3] Let A = 10, B = 20, C = 30**

From  $A^n + B^n = C^n$

$\Rightarrow 10^3 + 20^3 = 30^3$ , coprime factor is 5.

$$1,000 + 8,000 = 27,000$$

$$9,000 \neq 27,000$$

$\therefore r = 27,000/9000 = 3$ , it holds (i.e.  $r = n = 3$ )

$$\Rightarrow r(9,000) = 27,000$$

$$r(1,000 + 8000) = 27,000$$

$$r(10^3 + 20^3) = 30^3$$

$$r = n, \quad 3 = n$$

$$\Rightarrow n(10^n + 20^n) = 30^n$$

$\therefore n(A^n + B^n) = C^n$ , it holds.

$\therefore A = \sqrt[n]{(C^n/n - B^n)}$ ,  $B = \sqrt[n]{(C^n/n - A^n)}$ , and  $C = \sqrt[n]{n(A^n + B^n)}$  are the solutions to the equation  $A^x + B^y = C^z$  when  $x = y = z = n$ .

$$\Rightarrow A = \sqrt[3]{(30^3/3 - 20^3)} \quad B = \sqrt[3]{(30^3/3 - 10^3)} \quad C = \sqrt[3]{3(10^3 + 20^3)}$$

A = 10 as above, B = 20 as above, C = 30 as above.

**[4] Let A = 6, B = 12, C = 18, n = 3.**

From  $A^x + B^y = C^z$ ,

But  $r = C^z / (A^x + B^y)$

$$\Rightarrow r = \frac{18^3}{6^3 + 12^3} = 5832 / (216 + 1728) = 5832 / 1944$$

$$r = 3$$

$$\therefore r = n = 3$$

$\therefore$  the value of A = 6 is given by  $A = \sqrt[n]{(C^n/n - B^n)}$

$$A = \sqrt[3]{(18^3/3 - 12^3)} = \sqrt[3]{(5832/3 - 1728)} = 1944 - 1728 = \sqrt[3]{216} = 6 \text{ as above.}$$

Similarly  $B = \sqrt[n]{(C^n/n - A^n)}$ ,

$$\Rightarrow B = \sqrt[3]{(5832/3 - 6^3)} = \sqrt[3]{(5832/3 - 216)} = 1944 - 216 = \sqrt[3]{1728} = 12 \text{ as above.}$$

From  $C = \sqrt[n]{n(A^n + B^n)}$

$$\Rightarrow C = \sqrt[3]{3(6^3 + 12^3)} = \sqrt[3]{3(216 + 1728)} = \sqrt[3]{3(1944)} = \sqrt[3]{5832} = 18 \text{ as above.}$$

**B. WHEN  $r \neq n = 3$ .**

**[5] Let A = 6, B = 9, C = 12, n = 3.**

From  $A^n + B^n = C^n$ ,  $x = y = z = n$ ,

$$6^3 + 9^3 = 12^3, \text{ coprime factor} = 3.$$

$$216 + 729 = 1728$$

$$945 \neq 1728$$

$r = 1728/945 = 1.8285714$ . It doesn't hold (i.e.  $r \neq n = 3$ )

**NB: If  $r \neq n = 3$ , express the ratio as a fraction since it's a rational number.**

$$\Rightarrow \text{ratio, } r = 1728/945 = 64/35$$

$$\Leftrightarrow 64/35 = C^n/A^n + B^n$$

$$\Rightarrow r = C^n/A^n + B^n = 64/35.$$

$\therefore r < n$ .

From  $A^x + B^y = C^z \Leftrightarrow A^n + B^n = C^n$

$$\Rightarrow r(A^x + B^y) = C^z \Leftrightarrow r(A^n + B^n) = C^n \quad (21)$$

$$\therefore A^n + B^n = (C^n)/r \quad (22)$$

$$\text{Hence } A^n = (C^n)/r - B^n \quad (23)$$

$$\therefore A = \sqrt[n]{(C^n)/r - B^n} \quad (24)$$

$$\text{OR } A = \sqrt[n]{C^n/[C^n/(A^n + B^n)] - B^n}$$

$$\therefore A = \sqrt[n]{A^n} \quad (25)$$

Similarly, from equation (22),  $A^n + B^n = (C^n)/r$

$$B = \sqrt[n]{(C^n)/r - A^n} \quad (26)$$

$$\text{OR } B = \sqrt[n]{C^n/[C^n/(A^n + B^n)] - A^n}$$

$$B = \sqrt[n]{(A^n + B^n) - A^n}$$

$$\therefore B = \sqrt[n]{B^n} \quad (27)$$

Also from equation (4),  $r(A^n + B^n) = C^n$

$$C = \sqrt[n]{r(A^n + B^n)}$$

$$\text{Hence } C = \sqrt[n]{C^n/(A^n + B^n)}(A^n + B^n)$$

$$\therefore C = \sqrt[n]{C^n} \quad (28)$$

**NB: Equations (24) - (28) are used to solve  $A^x + B^y = C^z$ , where  $x = y = z = n$ ,  $r \neq n = 3$ , without replacing  $r$  with  $n$  or  $3$  (the minimum index or power).**

**Therefore, the numbers  $A = 6$ ,  $B = 9$ ,  $C = 12$  can be calculated by substituting one another in the formulae above respectively.**

$$\Rightarrow A = \sqrt[n]{(C^n)/r - B^n}$$

$$\therefore A = \sqrt[3]{12^3 / (64/35) - 9^3} = 6 \text{ as above.}$$

$$\text{OR } A = \sqrt[n]{A^n} = \sqrt[3]{6^3} = \sqrt[3]{216}$$

$$\therefore A = 6 \text{ as above.}$$

$$B = \sqrt[n]{(C^n)/r - A^n} = \sqrt[3]{12^3 / (64/35) - 6^3} = 9 \text{ as above.}$$

$$\text{OR } B = \sqrt[n]{B^n} = \sqrt[3]{9^3}$$

$$\therefore B = 9 \text{ as before.}$$

$$C = \sqrt[n]{r(A^n + B^n)} = \sqrt[3]{64/35(6^3 + 9^3)} = 12 \text{ as above.}$$

$$\text{OR } C = \sqrt[n]{C^n} = \sqrt[3]{12^3}$$

$$\therefore C = 12 \text{ as before.}$$

**[6] Let  $A = 15$ ,  $B = 45$ ,  $C = 90$ ,  $n = 3$ .**

From  $A^n + B^n = C^n$

$$15^3 + 45^3 = 90^3$$

$$3,375 + 91,125 = 729,000$$

$$94,500 \neq 729,000$$

$$\text{ratio, } r = 729,000/94,500$$

$$r = 54/7$$

$$\Rightarrow r = C^z / (A^x + B^y) = 729000/94500 = 54/7.$$

$$\Leftrightarrow r = C^n / (A^n + B^n) = 729000/94500 = 54/7.$$

### Example

**(i) To find the value of  $A$  above when  $B = 45$ ,  $C = 90$ ,  $n = 3$ .**

**Solution:**

$$\text{From } A = \sqrt[n]{(C^n)/r - B^n}$$

$$A = \sqrt[3]{(90^3)/45/7 - 45^3}$$

$$\therefore A = 15 \text{ as given above.}$$

**(ii) To find the value of  $B$  above when  $A = 15$ ,  $C = 90$ ,  $n = 3$ .**

$$\text{From } B = \sqrt[n]{(C^n)/r - A^n} = \sqrt[3]{\{[90^3 / (54/7)] - 15^3\}}$$

$$B = 45 \text{ as above.}$$

**(iii) To find the value of  $C$  above when  $A = 15$ ,  $B = 45$ ,  $n = 3$ .**

**Solution:**

$$\text{From } C = \sqrt[n]{r(A^n + B^n)}$$

$$C = \sqrt[3]{54/7(15^3 + 45^3)}$$

$$C = \sqrt[3]{54/7(3375 + 91125)} = 90 \text{ as above.}$$

**C. WHEN  $x = y = z = n > 3$ .**

Let  $n = 4$ ,  $A = 3$ ,  $B = 6$ ,  $C = 9$ .

$$\text{From } A^x + B^y = C^z$$

$$\Rightarrow A^n + B^n = C^n$$

$$\text{Thus } 3^4 + 6^4 = 9^4; x = y = z = n = 4$$

$$81 + 1296 = 6561$$

$$\text{But } 1377 \neq 6561$$

$$\therefore \text{ratio, } r = C^z / A^x + B^y$$

$$= C^n / (A^n + B^n)$$

$$\Rightarrow r = 6561 / 1377$$

$$\therefore r = 81/17.$$

$$\text{From } r (A^x + B^y) = C^z$$

$$\Rightarrow r (A^n + B^n) = C^n$$

$$\text{Hence } A^n + B^n = C^n / r$$

$$\text{Thus } A^n = (C^n) / r - B^n$$

$$\therefore A = \sqrt[n]{(C^n) / r - B^n}$$

$$\text{Similarly, } B^n = (C^n) / r - A^n$$

$$\therefore B = \sqrt[n]{(C^n) / r - A^n}$$

$$\text{And } C^n = r (A^n + B^n),$$

$$\therefore C = \sqrt[n]{r (A^n + B^n)}$$

**Therefore, the solutions to the equation  $A^x + B^y = C^z$  are;**

**$A = \sqrt[n]{(C^n) / r - B^n}$ ,  $B = \sqrt[n]{(C^n) / r - A^n}$ , and  $C = \sqrt[n]{r (A^n + B^n)}$ , where  $x = y = z = n > 3$ .**

$\therefore$  The values of  $A = 3$ ,  $B = 6$ , and  $C = 9$  are calculated as below.

**(i) To find A value 3 above when  $b = 6$  and  $c = 9$ .**

$$\text{From } A = \sqrt[n]{(C^n) / r - B^n}$$

$$A = \sqrt[4]{(9^4) / (81/17) - 6^4}$$

$$= \sqrt[4]{6561 (81/17) - (1296)}$$

$$\therefore A = 3 \text{ as above.}$$

**(ii) To find B value 6 above when  $A = 3$ ,  $C = 9$ ,  $n = 4$**

$$\text{From } B = \sqrt[n]{(C^n) / r - A^n} = \sqrt[4]{(9^4) / (81/17) - 3^4} = 6 \text{ as above.}$$

**(iii) To find C value 9 above when  $A = 3$ ,  $B = 6$ ,  $n = 4$ .**

$$\text{From } C = \sqrt[n]{r (A^n + B^n)} = \sqrt[4]{81/17 (3^4 + 6^4)} = 9 \text{ as above.}$$

**(b) Let  $x = y = z = n = 5$ ;  $A = 3$ ,  $B = 6$ ,  $C = 9$ .**

$$\text{From } A^x + B^y = C^z$$

$$\Rightarrow 3^5 + 6^5 = 9^5$$

$$243 + 7776 = 59049$$

$$8019 \neq 59049$$

$$\text{From ratio, } r \text{ of } C^n: A^n + B^n = C^n / A^n + B^n$$

$$r = 59049 / 8019 = 81/11$$

$$\therefore r = 81/11$$

$$\text{From } r (A^n + B^n) = C^n$$

$$\Rightarrow A^n + B^n = (C^n) / r$$

$$\therefore \text{the solutions to the equation } A^x + B^y = C^z$$

$$\Leftrightarrow A^n + B^n = C^n \text{ where } x = y = z = n, n > 3, r \neq n, \text{ are;}$$

$$A = \sqrt[n]{(C^n) / r - B^n}, B = \sqrt[n]{(C^n) / r - A^n}, C = \sqrt[n]{r (A^n + B^n)}.$$

**Prove;**

$$\text{From } A = \sqrt[n]{(C^n) / r - B^n}, \text{ given; } B = 6, C = 9, n = 5$$

$$\Rightarrow A = \sqrt[5]{(9^5) / (81/11) - 6^5}$$

$$A = \sqrt[5]{59049 (81/11) - 7776} = 3$$

$$\therefore A = 3 \text{ as above.}$$

$$B = \sqrt[n]{(C^n) / r - A^n}, A = 3, C = 9, n = 5.$$

$$\Rightarrow B = \sqrt[5]{(9^5) / (81/11) - 3^5}$$

$$= \sqrt[5]{59049 (81/11) - 243}$$

$$\therefore B = 6 \text{ as above.}$$

$$\begin{aligned}
 C &= \sqrt[n]{r(A^n + B^n)} \\
 &= 81/11 (3^5 + 6^5) \\
 C &= \sqrt[5]{81/11 (243 + 7776)} \\
 &= \sqrt[5]{81/11 (819)} = 9 \\
 \therefore C &= 9 \text{ as above.}
 \end{aligned}$$

**D. WHEN  $x \neq y \neq z$ .****(b) Let  $x = 3$ ,  $y = 4$  and  $z = 5$ , given  $A = 3$ ,  $B = 6$  and  $C = 9$ .**

$$\begin{aligned}
 \text{From } A^x + B^y &= C^z, \\
 3^3 + 6^4 &= 9^5 \\
 27 + 1296 &= 59049 \\
 1323 &\neq 59049 \\
 \text{From ratio, } r &= C^z/A^x + B^y \\
 \Rightarrow r &= 59049/1323 = 2187/49. \\
 \therefore r &= 2187/49 \\
 \text{From } A^x + B^y &= C^z, \\
 \Rightarrow r(A^x + B^y) &= C^z
 \end{aligned}$$

**Hence the above values of A, B and C are calculated from (10), (12) and (13) respectively as below. i.e;**

$$\begin{aligned}
 A &= \sqrt[x]{[(C^z)/r - B^y]} \\
 &= \sqrt[3]{[9^5/(2187/49) - 6^4]} \\
 \therefore A &= 3 \text{ as above.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Similarly, } B &= \sqrt[y]{[(C^z)/r - A^x]} \\
 &= \sqrt[4]{[9^5/(2187/49) - 3^3]} \\
 \therefore B &= 6 \text{ as above.}
 \end{aligned}$$

$$\begin{aligned}
 \text{From } r(A^x + B^y) &= C^z, \\
 \Rightarrow C &= \sqrt[z]{r(A^x + B^y)} \\
 \therefore C &= \sqrt[5]{[2187/49 (3^3 + 6^4)]} \\
 &= \sqrt[5]{2187/49 (27 + 1296)} \\
 \therefore C &= 9 \text{ as above.}
 \end{aligned}$$

**(b) Let  $A = 15$ ,  $B = 30$ ,  $C = 60$ ;  $x = 4$ ,  $y = 3$ ,  $z = 6$ .**

$$\begin{aligned}
 \text{From } r(A^x + B^y) &= C^z \\
 \text{ratio, } r &= C^z/A^x + B^y \\
 \Rightarrow r &= 60^6/(15^4 + 30^3) \\
 &= 46,656,000,000/50,625 + 27,000 \\
 &= 46,656,000,000/77,625 \\
 &= 13,824,000/23 \\
 &= 601043.47826087
 \end{aligned}$$

**NB: substitute r value as a fraction for accuracy.**

$$\therefore r = 13,824,000/23.$$

Hence the values of A, B and C above are calculated as below.

$$\begin{aligned}
 \text{From } A &= \sqrt[x]{(C^z)/r - B^y} \\
 &= \sqrt[4]{60^6/(13824000/23) - 30^3} \\
 &= \sqrt[4]{46,656,000,000 \times 23/13,824,000 - 27,000} \\
 &= \sqrt[4]{77625 - 27,000} = \sqrt[4]{50,625} = 15 \\
 \therefore A &= 15 \text{ as above.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Similarly, } B &= \sqrt[y]{(C^z)/r - A^x} \\
 &= \sqrt[3]{60^6/(13,824,000)/23 - 15^4} \\
 &= \sqrt[3]{77625 - 50,625} = \sqrt[3]{27,000} = 30 \\
 \therefore B &= 30 \text{ as above.}
 \end{aligned}$$

$$\begin{aligned}
 \text{From } C &= \sqrt[z]{r(A^x + B^y)}, \\
 &= \sqrt[6]{13,824,000/23 (15^4 + 30^3)} \\
 &= \sqrt[6]{13,824,000/23 (50,625 + 27,000)} \\
 &= \sqrt[6]{46,656,000,000} = 60 \\
 \therefore C &= 60 \text{ as above.}
 \end{aligned}$$

**E. FOR  $x = y = z = \infty$  (INFINITY).**

$$\text{From } A^x + B^y = C^z,$$

$$\Rightarrow A^\infty + B^\infty = C^\infty$$

$$\text{Thus the ratio, } r = C^z / (A^x + B^y)$$

$$= C^\infty / (A^\infty + B^\infty)$$

$$= \infty / (\infty + \infty) = \infty / 2\infty = 1/2$$

$$\therefore r = 1/2 \text{ or } 0.5.$$

**NB: to find r, use  $2\infty = \infty$  according to the formula for finding r, not the reduced form  $\infty = \infty$ , since  $r < C^z$ .**

$\therefore$  The solutions to the equation  $A^x + B^y = C^z$  are given by;

$$\text{From (10), } A = \sqrt[x]{(C^z)/r - B^y}$$

$$\Rightarrow A = \sqrt[\infty]{(C^\infty)/r - B^\infty},$$

$$A = \sqrt[\infty]{(\infty)1/2 - \infty} = \sqrt[\infty]{(2\infty - \infty)} = \sqrt[\infty]{\infty} = \infty.$$

$$\therefore A = \infty, \text{ root of infinity} = \infty.$$

Similarly, from (12),

$$B = \sqrt[y]{(C^z)/r - A^x}$$

$$\Rightarrow B = \sqrt[\infty]{(C^\infty)/r - A^\infty}$$

$$= \sqrt[\infty]{\infty / (1/2) - \infty}$$

$$= \sqrt[\infty]{2\infty - \infty} = \sqrt[\infty]{\infty} = \infty$$

$$\therefore B = \infty.$$

Similarly, from (13),

$$r(A^x + B^y) = C^z$$

$$\Rightarrow C = \sqrt[z]{r(A^x + B^y)}$$

$$= \sqrt[\infty]{1/2(A^\infty + B^\infty)}$$

$$= \sqrt[\infty]{1/2(\infty + \infty)}$$

$$= \sqrt[\infty]{1/2(2\infty)} = \sqrt[\infty]{\infty} = \infty$$

$$\therefore C = \infty$$

$\therefore A = \infty, B = \infty, \text{ and } C = \infty, \text{ are the general solutions to the equation } A^x + B^y = C^z, \text{ where } x = y = z = \infty.$

### 3. Conclusion

- Equation  $A^x + B^y = C^z$  has solutions  $A = \sqrt[x]{(C^z)/r - B^y}$ ,  $B = \sqrt[y]{(C^z)/r - A^x}$  and  $C = \sqrt[z]{r(A^x + B^y)}$  in non-zero integers and pairwise prime integers A, B and C if  $x, y, z \geq 3$ . Hence disproving the Beal's conjecture.
- (a) It should be noted that the equation  $n(A^n + B^n) = C^n$  is obtained from the general equation  $r(A^x + B^y) = C^z$  when  $x = y = z = r = n$ , and is used when  $n = 3$  (minimum power).
  - If  $r \neq n$ , the general equation  $r(A^x + B^y) = C^z$  must be used. The value of r should be expressed as a fraction for accuracy if it's not a whole number.
  - The value of C must be greater than the values of A and B. That is, it holds if  $C > A$ , and B.
- It's noted that this approach to Beal's conjecture acts as a general approach to solve any set of equations of this kind if the condition in 2 (c) above is full-filled. **Hence it is a proof to Fermat's Last conjecture. Therefore, this approach solves both the Beal's conjecture and the Fermat's Last Theory.**

### Author Profile



**Adriko Bosco** is an independent researcher from Terego District Uganda, he has many articles in Mathematics including the millennium Mathematics and the Hilbert David's 23 Mathematics problems. The career objective of the author is to work for the improvement of Mathematics in Uganda and the whole world. He Otumbari Secondary school, St. Joseph's college Ombaci. He got a certificate in ICT from Makerere University in Uganda. The author taught Physics, Chemistry, Biology and Mathematics in many Secondary schools in Uganda and South Sudan for more than seventeen years. He served as a head-teacher of Wulu Secondary from 2008-2010. He has many manuscripts in Mathematics since May 2020 including; 1. Riemann hypothesis 1859, 2. Goldbach's conjecture, 3. Birch and Swinnerton--Dyer Conjecture, 4. The Beal conjecture, 5. Twin prime conjecture, 6. Fermat's Last Theory, 7. Diophantine equations, 8. Solvability of a Diophantine equation, 9. Arbitrary quadratic forms, 10. Reciprocity laws and Algebraic number fields, 11. Deal with Pi ( $\pi$ ) and Euler's constant, e, 12. He has derived some formulae for solving Arithmetic mean, and so on. He is preparing a Mathematics book entitled "THE BOOK OF WISDOM AND GINIUSENESS". The author is also a song writer.