# Investigative Study of the Behavior of Lotka-Volterra Model of COVID-19

**Athira Manikandan**

Aerospace Engineering Graduate from Karunya Institute of Technology Sciences

**Abstract:** ***Several mathematical models have been fashioned since its inception to predict the future of coronavirus disease (COVID-19) and its transmission rate among the vast population. These models have had far-reaching consequences, in helping the organizations and governments take appropriate action to curb the pandemic. This paper presents an analytical study of a new mathematical model of COVID-19 that helps us in understanding the transmission levels of virus infection based on Lotka-Volterra (LV) modeling. Mathematical models can simulate the effects of various factors such as temperature and humidity levels on the disease. Considering only one patient, and studying the pattern of interfaces between cells in the host's body, will enable us to discuss all the factors that work to spread the infection to vast population groups. Fraction calculus is one of the chief models recently implemented to simulate the spread of disease among the population, vital to explain the behaviors of ecological models. This paper aims to study the LV models of COVID-19 with fractional order.***

**Keywords:** COVID-19, Lotka-Volterra (LV)

## 1. Introduction & Literature Review

The predator-prey model could be defined as a representation of the interaction between different species of animals residing in the same bio-network, such that the population of each set of these species relies on the birth or death rate and the successful interactions with the other species. As per Restrepo, J., & Sánchez, C. (2010)., the model is established based on the following assumptions:

- Prey population has an immeasurable quantity of food
- Interaction between the predators and prey populations is proportional to the product of the two populations.
- Survival of the predators depends on prey. A stands for prey's rate of birth, β-Predator's rate of death, γ-Population growth rate and δ –population decrease of each of the species.

Meanwhile, the Lotka-Volterra model adopts two inputs that increase and deducts prey or predators in certain intervals. Lotka-Volterra Equations are the differential equation which is modeled to establish the relationships that exist between predator and prey population and illustrate the interactions between two species, i. e., a predator and a prey.

In 1921, with the Lotka model pairing the first order non-linear differential equations, and Vio Volterra working independently, equating the relationship between the two species (Egerton, 2015)., it was made possible to analyze the relationship between two species in the ecosystem. Heberman, R. (2007) described the association between plants, animals, and their environment using a mathematical model which was developed based on the observed population data of a variety of species from humans to algae in the lake. In addition, Lambert (Lambert. M. et al, 2010) underlined that population modeling is an application of differential equations in comprehending the fluctuations in the population of species in an ecosystem. Furthermore, it was stated that the modeling is considered the changes in the population levels caused by the interaction of an organism with its physical environment. Li (Li, H., 2011) studied the Lotka-Volterra model with multiple species and approximated a numerical solution using a MATLAB function. On the other hand, Vaidyanathan (Vaidyanathan, S., 2010) used Sundarapandian's theorem (2002) to find the solution to the problem of the Lotka-Volterra model, by studying two and three species predator-prey models. Accordingly, Dayar (Dayar, T., & Mikeev, L., 2010) suggested the modifications and combinations of the traditional way to have quick, efficient, consistent, and precise results for a few parameter scales of the Lotka-Volterra model. Additionally, Korobeinikove A., & Wake G. C (1999) also examined the model of two animals residing in the same territory and their dependence on each other for survival. The model assumed that the number of prey is inversely proportional to that of the predators. The rate of change is assumed to be constant if the predator population gets wiped out.

In the past two years, several scientists have published papers on the COVID-19 virus, discussing the transmission rate of the virus and proposals on the treatment. This paper presents an analytical study of a new mathematical model of COVID-19 that helps us in understanding the transmission levels of virus infection based on Lotka-Volterra (LV) modeling. Mathematical models can simulate the effects of various factors such as temperature and humidity levels on the disease. Considering only one patient, and studying the pattern of interfaces between cells in the host's body, will enable us to discuss all the factors that work to spread the infection to vast population groups. Fraction calculus is one of the chief models recently implemented to simulate the spread of disease among the population, vital to explain the behaviors of ecological models. This paper aims to study the LV models of COVID-19 with fractional order.

## 2. Numerical Problem

Volterra equations, the system of differential equations demonstrates the interaction of competing or predator-prey populations, may be written in the following form,

$$\frac{dP}{dt} = aP - bPQ + eQ$$

$$\frac{dQ}{dt} = (c - e - d)\,Q + bPQ$$

Together with the initial conditions

$$Q = Q_0 \text{ and } P = P_0 \text{ at time } t = 0$$

We assume that the virus is the predator that preys on humans. The mutation process of the virus within the body of the host or among the infected populations works in a similar fashion as the feeding process in the LV model.

$$Healhty\ Population = P\ at\ time\ t$$
$$Infected\ Population = Q\ at\ time\ t$$
$$Rate\ of\ infection = b, such\ that\ b > 0$$
$$Entry\ rate\ of\ individuals = a, such\ that\ a > 0$$
$$Entry\ rate\ of\ infected\ individuals = c, such\ that\ c > 0$$
$$Death\ rate = d, such\ that\ d > 0$$
$$Cure\ rate = e, such\ that\ e > 0$$

The specific values of the constants determine the solution of the differential equation which depends on and will often affect a stable cyclic change of the populations. However, this is not sufficient to help the predators survive, which will then fall. Consequently, more preys survive, which will then increase. Hence, this will lead to a cyclic increase in the predator-prey populations between the two extreme limits, and the system has two different equilibrium points.

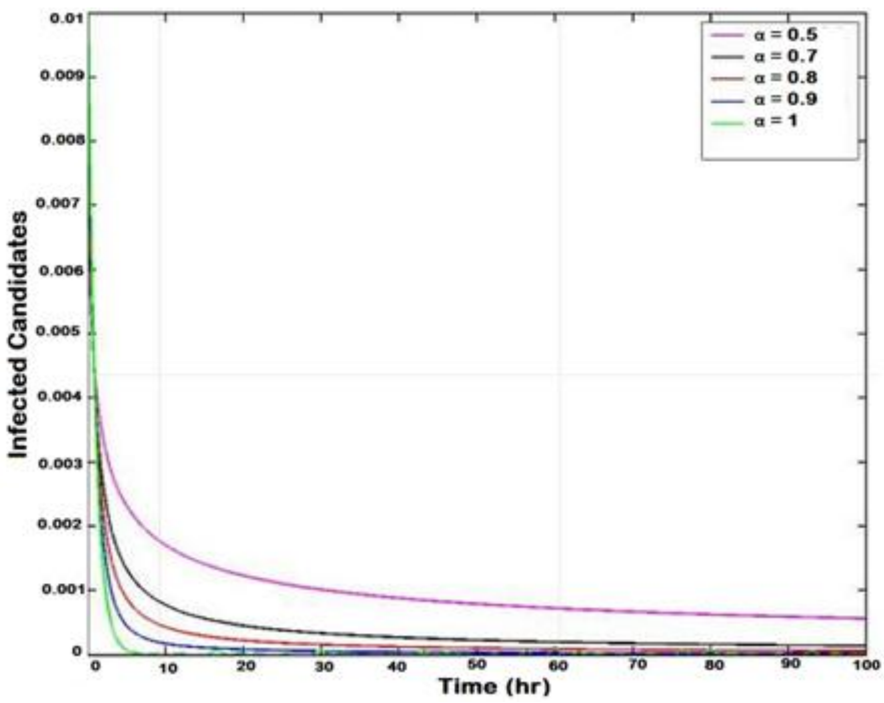
## 3. Result

As per the statistics, it is approximated,

$$Entry\ rate\ of\ healthy\ individuals, a = 0.012$$
$$Rate\ of\ infection, b = 0.01$$
$$Entry\ rate\ of\ infected\ individuals, c = 0.0001$$
$$Rate\ of\ death, d = 0.02$$
$$Rate\ of\ cute, e = 0.98$$

The system is numerically integrated using the before mentioned values of the systems' parameters, with orders α=1, α=0.9, α=0.8, α=0.7, and α=0.5. The summary of the simulation results is illustrated in the following figure.
The initial condition for the figure is: (1) $(y_1, y_2) = (0.01, 0.01)$.

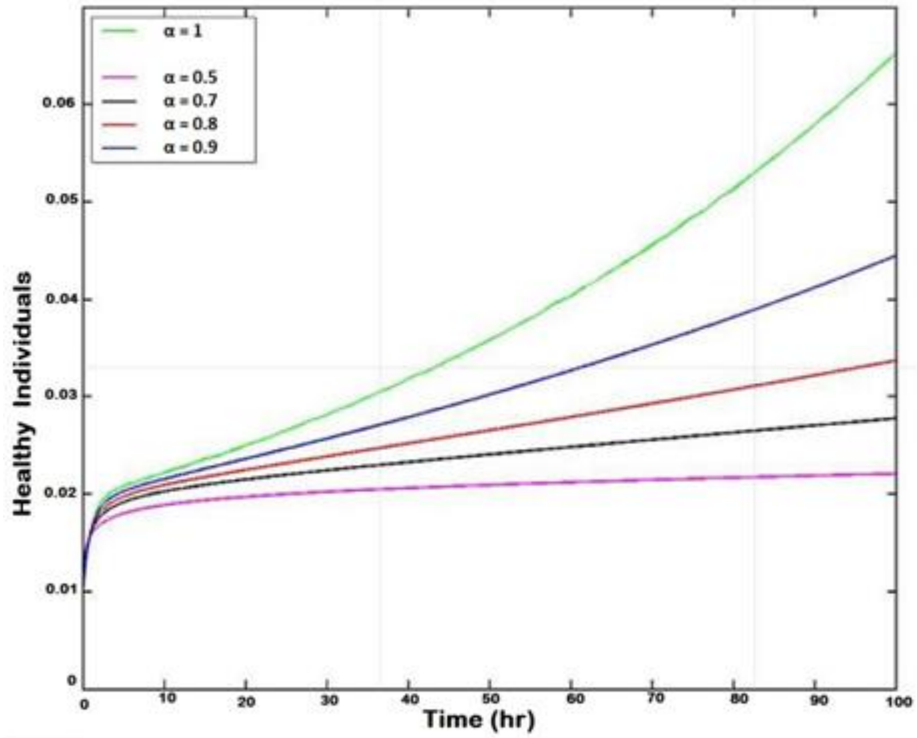
Plots of the system for a=0.012, b=0.01, c=0.0001, d=0.02, e=0.98 for different order and initial conditions of $(y_1, y_2) = (0.01, 0.01)$.






Paper ID: SR211110105514 DOI: 10.21275/SR211110105514 

**ISSN: 2319-7064**
**SJIF (2020): 7.803**


In comparison to the integer-order complement, the result shows that it has converged to the co-existence point. The cause for this interesting outcome is because the oscillations in its integer-order case get eliminated as a consequence of the memory effect in the fractional case. Hence, the system with the fractional case is more precise and accurate to describe the changing aspects of the proposed Lotka-Volterra model of COVID-19. Efficiency of the Fractional parameters to control the flattening of the curves of the infected candidates is clearly depicted in the results of the simulation. These models will help suggest better strategies to improve public health by mitigating or slowing down the transmission of the disease.

## 4. Conclusion

This work has examined some dynamical behaviors in the models of COVID-19 based on Lotka-Volterra. The proposed simulations involve the fractional-order case and its integer-order complement. The actuality of the non-negative solution of the fractional model has been confirmed. The effect of the fractional parameter on the transmission of the COVID-19 infection has been displayed in the numerical simulation results.