Common Fixed Point Theorem in Fuzzy Metric Spaces for Compatible Maps

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Abstract: ***The FPT itself attractive combination of investigation, topology moreover geometry. Over the last few decades the theory of FP has appeared as especially dominant as well as significant instrument inside the learn of NonLP. In particular, FPT have been applied in a variety of diverse fields as biology, chemistry, economics, engineering, game theory and physics. The situation is also likely to evaluates some existing complications beginning science and technology, where one is concerned with a system of differential, integral and FE***

Keywords: Fuzzy metric space, Compatible maps, nonstop, completeness, uniqueness, invertible, summable

1. Introduction

In genuine world, the many-sided quality by and large emerges from uncertainly as equivocalness. The PT has been age old and powerful instrument to deal with uncertainly, yet it very well may be connected just to the circumstances whose attributes depend on irregular procedures, i.e., process in which the event of occasions is entirely controlled by shot. Uncertainly may emerge because of PI about the issue, or because of data which isn't CD, or because of natural imprecision in the dialect with which the issue is characterized or because of receipt of data from in excess of one source. FST is an EMT to deal with the uncertainly emerging because of uncertainty. In 1965, Lotfi A-Zadeh [101] propounded the FST in his section.

Our aim of this chapter is to find some more results for compatible map of type (β) in FMS.

For the sake of completeness, we recall some definition and known results in FMS, which are used in this chapter.

Definition 1.1.1: Let X be any set. A FS in X is a function with area X and values in $[\text{zero}, \text{one}]$.

Definition 1.1.2: A binary operation $\star: [\text{zero}, \text{one}] \times [\text{zero}, \text{one}] \rightarrow [\text{zero}, \text{one}]$ is continuo zero, one us $\parallel t - norm \parallel$ with star is fulfilling the subsequent situation:
1.1.2(a) $\star$ is comme & asse,
1.1.2 (b) $\star$ is continuous,
1.1.2 (c)$a \star 1 = a for all a \in [\text{zero}, \text{one}]$
1.1.2 (d) $a \star b \ \leq \ c \star d whenever a \leq c and b \leq d,$
for all$a, b, c, d \in [\text{zero}, \text{one}]$
Examples of $t - norm$ are $a \star b \ = min\{a, b\} and a \star b = ab.$

Definition 1.1.3: A 3-tuple $(X, M, \star)$ is a FMS whenever X is an AS$\star$ is continuous $\parallel t - norm \parallel$and M is FA on $X \times X \times [\text{zero}, +\text{infinite})$ fulfilling, every point $x , y , z \ \in X$ and $s , t > \ zero,$ the FC :
1.1.3 (a) $M(x, y, t) nonnegtive$
1.1.3 (b) $M(x, y, 0) \ = \ 0$
1.1.3 (c) $M(x, y, t) \ = 1 \ iff x = y$
1.1.3 (d) $M(x, y, t)$ is comme
1.1.3 (e) $M(x, y, t) \ \star \ M(y, z, s) \ \leq \ M(x, z, t + s)$
1.1.3 (f) $M(x, y,\cdot) : \ (0, \infty^{+}) \rightarrow \ [0,1] is continuous.$

We note that, $M(x, y, t)$ can be realized as the measure of closeness with connecting x&yw.r.tt. It be identified to $M(x, y,\cdot)$ is ND$\forall x, y \in X$. Let $M(x, y,\star)$ be a FMS for $t > \ 0$, the OB

$$Ball(space , metric , variable) \text{ equal } \{y \in X: M((space , metric , variable)) > 1 - r\}.$$

Now, the collection $\{B((space , metric , variable)): x \in X, 0<r<1, t>0$ is a NBDs for a topology τon X induced by the FMS. I.e topology is Housdroff and FC.

Example 1.1.4suppose (universal, distance) be a MS. Define $a \star b =$ LUB of a and b and $M(space , metric , variable) = \frac{t}{t+dis (x,y)} \forall x, y \in X$ and all $t > \ 0$. subsequently $(X, M,\star)$ is a FMS. It is call the FMSI through d.

Definition 1.1.5A sequence $\{x_n\}$ in a FMS$(X, M,\star)$be known as converges to x if f for each $\varepsilon > 0$ and each $t > 0, n_0 \in N$s.t$M(x_n, x, t) > 1 - \varepsilon for all n \geq n_0$.
Definition 1.1.6 A $\{x_n\}$ be in a FMS$(X, M,\star)$be known to be a CSC to x iff each $\varepsilon > 0$ and each $t > 0, n_0 \in N$s.t$M(x_m, x_n, t) > 1 - \varepsilon for all m, n \geq n_0$.

A FMS$(X, M,\star)$ is known's to be complete if each CS in it conto a point in it.

Definition 1.1.7SM A and S of a FMS$(X, M,\star)$ are is knowns to be compatible if and only if $M(ASx_n, SAx_n, t) \rightarrow 1$ for all $t > \ 0$, i.e$\{x_n\}$ is a in X s.t$Sx_n , Ax_n \ \rightarrow \ p$of various $p \in X as n \rightarrow \infty$.

Definition 1.1.8SM A and S of a FMS$(X, M,\star)$are said to be compatible of type $(\beta) \leftrightarrow M(AAx_n, SSx_n, t) \rightarrow 1$each$t > \ 0$, i.e$\{x_n\}$ is a in X s.t $Sx_n , Ax_n \ \rightarrow \ p$and $p \in X as n \rightarrow \infty$.

Definition 1.1.92-map A and B from a FMS$(X, M,\star)$ into itself are known to be WC if they commute at their CP i.e., $Ax \ = \ Bx$ implies $ABx \ = \ BAx$each $x \in \ X$.

Remark 1.1.10 The concept of CM of type (β) is more general then the concept of CM in FMS.

Definition 1.1.11suppose A and S be2-SM of a FMs$(X, M, \star)$ then A and S is said to be a WC if $M(ASx_n, SAx_n, t) \leq M(Sx_n, Ax_n, t)$ for all x in X .

It can be seen that CMs$(ASx = SAx \ \forall\ x \in X)$ are CM but opposite is not right.

Lemma 1.1.12 In a FMS$(X, M, \star)$ limit of a sequence is exclusive.

Lemma 1.1.13 Let $(X, M, \star)$ be a FMS. Then for all $x, y \in X M(x, y, .)$ is a NDF.

Lemma 1.1.14suppose t $(X, M, \star)$ be a FMS. If $\exists k \in (0,1)$s.tFA$x, y \in\ X, M(x, y, kt) \geq M(x, y, t)\ \forall t > 0, then x\ =\ y$.

Lemma 1.1.15A $\{x_n\}$ in a FMS$(X, M, \star)(X, M, \star)$. If $\exists$ a number $k \in (0,1)$ such that
$M(x_{n+2}, x_{n+1}, kt) \geq\ M(x_{n+1}, x_n\ , t)\ \forall\ t >\ 0\ and n \in\ N$
after that $\{x_n\}$ is a CS in X.

Lemma 1.1.16 The only $t - norm\ \star$ satisfying $r \star r = r$ for all $r \in [0,1]$ is the min $t - norm$ that is
$a \star b = min\{a, b\}$ for all $a, b \in [0,1]$.

1.2 Common Fixed Point Theorem for Compatible Maps of Type (β) and Type (α)

In this section we prove a CFPT for CM of type (β) and type (α) in FMS. In fact we prove the following theorem.

Theorem 1.2.1suppose $(X, M, \star)$ be a FMS and let $A, B, S, T, P and Q$ be mappings from X into itself s.t the following conditions are satisfied:
1.2.1(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,
1.2.1(b)$AB\ = BA, ST\ =\ TS, PB = BP, QT = TQ$,
1.2.1(c) either P or AB is continuous,
1.2.1(d) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,
1.2.1(e) $\exists k \in (0,1)$ such that for every $x, y \in X and t >\ 0$

$$M^2(Px, Qy, kt) \geq M^2(ABx, STy, t) \star M^2(Px, ABx, t) \star M^2(Qy, STy, t)\ \star M^2(Px, STy, t) * M^2(ABx, ABx, t)$$

To show A, B, S, T, P and Q have a UCFP in X.

Proof: Let $x_0\ \in X$, then from 1.2.1(a) we have $x_1, x_2\ \in X$ s.t

$$Px_0\ = STx_1 and Qx_1\ = ABx_2$$

Inductively, we CS$\{x_n\} and\ \{y_n\}$ in X s.t for $n \in N$

$$Px_{2n-2}\ =\ STx_{2n-1} = y_{2n-1} and Qx_{2n-1}\ =\ ABx_{2n}\ = y_{2n}$$

Step 1 Put $x =\ x_{2n} and y = x_{2n+1}$ in 1.2.1(e) then we have

$$M^2(Px_{2n}, Qx_{2n+1}, kt) \geq M^2(ABx_{2n}, STx_{2n+1}, t) \star M^2(Px_{2n}, ABx_{2n}, t) \star M^2(Qx_{2n+1}, STx_{2n+1}, t)\ \star M^2(Px_{2n}, STx_{2n+1}, t) * M^2(ABx_{2n}, ABx_{2n}, t)$$

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq M^2(y_{2n}, y_{2n+1}, t) \star M^2(y_{2n+1}, y_{2n}, t) \star M^2(y_{2n+2}, y_{2n+1}, t)\ \star M^2(y_{2n+1}, y_{2n+1}, t) * M^2(y_{2n}, y_{2n}, t)$$

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq M^2(y_{2n}, y_{2n+1}, t) \star M^2(y_{2n+2}, y_{2n+1}, t)$$

From lemma 2.1.13 and 2.1.14 we have

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq M^2(y_{2n}, y_{2n+1}, t)$$

That is

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$$

likewise WH

$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$
TWH
$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$
$M(y_{n+1}, y_{n+2}, t) \geq M\left(y_n, y_{n+1}, \frac{t}{k}\right)$
$M(y_n, y_{n+1}, t) \geq M\left(y_0, y_1, \frac{t}{k^n}\right) \rightarrow\ 1\ as n \rightarrow\ \infty$,
and hence $M(y_n, y_{n+1}, t) \rightarrow\ 1\ as n \rightarrow\ \infty\ for all t > 0$.

For each $\epsilon > 0\ and t > 0$, we can choose $n_0 \in N$ such that
$M(y_n, y_{n+1}, t) >\ 1 - \epsilon for all n > n_0$.

FA$m, n \in N$ we suppose that $\geq n$. i.e

$M(y_n, y_m, t)\ \geq$
$M\left(y_n, y_{n+1}, \frac{t}{m-n}\right)\ \star\ M\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right)\ \star$
$\dots . \star\ M\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$
$M(y_n, y_m, t) \geq\ (1 - \epsilon) \star\ (1 - \epsilon) \star \dots \dots \star\ (1 - \epsilon)(m - n times$
$M(y_n, y_m, t) \geq\ (1 - \epsilon\ )$
And hence $\{y_n\}$ is a CS in X.

Since $(X, M, \star)$ is complete, $\{y_n\}$ con to some point $z \in X$. Also its SSC to the SP$z \in X$.

That is
$\{Px_{2n+2}\} \rightarrow z and \{STx_{2n+1}\} \rightarrow\ z$ 1.2.1 (i)
$\{Qx_{2n+1}\} \rightarrow\ z and \{ABx_{2n}\} \rightarrow z$ 1.2.1(ii)
Case 1 if AB is nonstop
if AB is nonstop, we have

$$(AB)^2x_{2n}\ \rightarrow\ ABz and ABPx_{2n}\ \rightarrow\ ABz$$

As (P, AB) is CP of type (β), we have
$M(PPx_{2n}, (AB)(AB)x_{2n}, t)\ =\ 1, for all t > 0$
Or
$M(PPx_{2n}, ABz, t)\ =\ 1$
Therefore, $PPx_{2n}\ \rightarrow ABz$.
Step 2 Put $x\ = (AB)x_{2n} and y = x_{2n+1}$ in 1.2.1(e) we have

$$M^2(P(AB)x_{2n}, Qy, kt) \geq M^2(AB(AB)x_{2n}, STx_{2n+1}, t) \star M^2(P(AB)x_{2n}, AB(AB)x_{2n}, t)$$
$$\star M^2(Qx_{2n+1}, STx_{2n+1}, t)\ \star M^2(P(AB)x_{2n}, STx_{2n+1}, t) *$$
M2ABABx2n,ABABx2n,t

Taking $n \rightarrow \infty$ we get

$$M^2((AB)z, z, kt) \geq M^2((AB)z, z, t) \star M^2((AB)z, (AB)z, t) \star M^2((AB)z, z, t)\ \star M^2((AB)z, z, t) * M^2((AB)z, z, t)$$

$$M^2((AB)z, z, kt)\ \geq\ M^2((AB)z, z, t)\ \star M^2((AB)z, z, t)$$

i.e$M\big((AB)z,z,kt\big) \geq M\big((AB)z,z,t\big)$
TF by lemma 2.1.14 we have

$ABz = z$. 1.2.1(iii)

Step -3 Put $x = z and y = x_{2n+1}$ in 2.2.1(e) we have

$$M^2(Pz,Qx_{2n+1},kt) \geq \begin{pmatrix} M^2(ABz,STx_{2n+1},t) \star M^2(Pz,ABz,t) \star \\ M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(Pz,STx_{2n+1},t) * M^2(ABz,ABz,t) \end{pmatrix}$$

Tak$n \to \infty$ and using equation 1.2.1 (i) we have

$$M^2(Pz,z,kt) \geq M^2(ABz,z,t) \star M^2(Pz,ABz,t) \star M^2(z,z,t) \star M^2(Pz,z,t) * M^2(ABz,z,t) \star M^2$$

That is $M^2(Pz,z,kt) \geq M^2(Pz,z,t)$
And hence $M(Pz,z,kt) \geq M(Pz,z,t)$
Therefore by using lemma 1.1.14, we get

$$Pz = z$$

So we have $ABz = Pz = z$.
Step- 4 Putting $x = Bz and y = x_{2n+1} in$ 2.2.1$(e), we get$

$$M^2(PBz,Qx_{2n+1},kt) \geq M^2(ABBz,STx_{2n+1},t) \star M^2(PBz,ABBz,t) \star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(PBz,STx_{2n+1},t) * M^2(ABBz,ABBz,t)$$

As $BP = PB and AB = BA$, so we have

$$P(Bz) = B(Pz) = Bz and (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.$$

Taking $n \to \infty$ and using 1.2.1(i) we get

$$M^2(Bz,z,kt) \geq M^2(Bz,z,t) \star M^2(Bz,Bz,t) \star M^2(z,z,t) \star M^2(Bz,z,t) * M^2(Bz,z,t)$$

$$M^2(Bz,z,kt) \geq M^2(Bz,z,t)$$

That is $M(Bz,z,kt) \geq M(Bz,z,t)$
Therefore by Lemma 1.1.14 we have $Bz = z$
And also we have $ABz = z implies Az = z$
Therefore $Az = Bz = Pz = z$. 1.2.1 (iv)
Step – 5 As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$$z = Pz = STu$$

Putting $x = x_{2n} and y = u$ in 1.2.1(e) we get

$$M^2(Px_{2n},Qu,kt) \geq M^2(ABx_{2n},STu,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(Qu,STu,t) \star M^2(Px_{2n},STu,t) * M^2(ABx_{2n},STu,t)$$

Taking $n \to \infty$ and using 1.2.1(i) and 1.2.1(ii) we get

$$M^2(z,Qu,kt) \geq M^2(z,STu,t) \star M^2(z,z,t) \star M^2(Qu,STu,t) \star M^2(z,STu,t)$$

$$M^2(z,Qu,kt) \geq M^2(z,Qu,t)$$

That is $M(z,Qu,kt) \geq M(z,Qu,t)$
TF by using Lemma 1.1.13 we have $Qu = z$
Hence $STu = z = Qu$.
Hence (Q,ST) is WC, therefore, we have

$$QSTu = STQu$$

Thus $Qz = STz$.
Step – 6 Putting $x = x_{2n} and y = z$ in 1.2.1(e) we get

$$M^2(Px_{2n},Qz,kt) \geq M^2(ABx_{2n},STz,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(Qz,STz,t) \star M^2(Px_{2n},STz,t) M^2(ABx_{2n},ABx_{2n},t)$$

Taking $n \to \infty$ and using 1.2.1(ii) and step 5 we get

$$M^2(z,Qz,kt) \geq M^2(z,STz,t) \star M^2(z,z,t) \star M^2(Qz,STz,t) \star M^2(z,STz,t) M^2(z,z,t)$$

$$M^2(z,Qz,kt) \geq M^2(z,Qz,t)$$

And hence $M(z,Qz,kt) \geq M(z,Qz,t)$

Therefore by using Lemma 1.1.13 we get $Qz = z$.

Step 7: Putting $x = x_{2n} and y = Tz$ in 2.2.1(e) we get

$$M^2(Px_{2n},QTz,kt) \geq M^2(ABx_{2n},STTz,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(QTz,STTz,t) \star M^2(Px_{2n},STTz,t) * M^2(ABx_{2n},ABx_{2n},t)$$

As $QT = TQ and ST = TS$ we have

$$QTz = TQz = Tz$$

And $ST(Tz) = T(STz) = TQz = Tz$.

Taking $n \to \infty$ we get

$$M^2(z,Tz,kt) \geq M^2(z,Tz,t) \star M^2(z,z,t) \star M^2(Tz,Tz,t) \star M^2(z,Tz,t) M^2(z,z,t)$$

$$M^2(z,Tz,kt) \geq M^2(z,Tz,t)$$

Therefore $M(z,Tz,kt) \geq M(z,Tz,t)$
Therefore by Lemma 1.1.13 we have $Tz = z$
Now $STz = Tz = z implies Sz = z$.
Hence
$Sz = Tz = Qz = z$ 1.2.1(v)
Combining 2.2.1(iv) and 2.2.1(v) we have

$$Az = Bz = Pz = Sz = Tz = Qz = z$$

Hence z is the CFP of A, B, S, T, P and Q.

Case - II suppose P is nonstop
As P is continuous

$$P^2x_{2n} \to Pz and P(AB)x_{2n} \to Pz$$

As (P,AB) is compatible pair of type (β),

$$M(PPx_{2n},(AB)(AB)x_{2n},t) = 1 \; for all t > 0$$

Or $M(Pz,(AB)(AB)x_{2n},t) = 1$
Therefore $(AB)^2x_{2n} \to Pz$.
Step -8 Putting $x = Px_{2n} and y = x_{2n+1}$ in 2.2.1(e) then we get

$$M^2(PPx_{2n},Qx_{2n+1},kt) \geq M^2(ABPx_{2n},STx_{2n+1},t) \star M^2(PPx_{2n},ABPx_{2n},t) \star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(PPx_{2n},STx_{2n+1},t) * M^2(ABPx_{2n},ABPx_{2n},t)$$

Taking $n \to \infty, we get$

$$M^2(Pz,z,kt) \geq M^2(Pz,z,t) \star M^2(Pz,Pz,t) \star M^2(z,z,t) \star M^2(Pz,z,t) * M^2(Pz,Pz,t)$$

$$M^2(Pz,z,kt) \geq M^2(Pz,z,t)$$

Hence $M(Pz,z,kt) \geq M(Pz,z,t)$
Therefore by Lemma 1.1.13 we get $Pz = z$
Step- 9 Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in 2.2.1(e) then we get

$$M^2(PABx_{2n},Qx_{2n+1},kt) \geq M^2(ABABx_{2n},STx_{2n+1},t) \star M^2(PABx_{2n},ABABx_{2n},t) \star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(PABx_{2n},PABx_{2n},t) * M^2(ABABx_{2n},STx_{2n+1},t)$$

Taking $n \to \infty$ we get

$$M^2(ABz,z,kt) \geq M^2(ABz,z,t) \star M^2(ABz,z,t) \star M^2(z,z,t) \star M^2(ABz,z,t)M^2(ABz,ABzz,t)$$

Therefore $M^2(ABz,z,kt) \geq M^2(ABz,z,t)$

And hence

$$M(ABz,z,kt) \geq M(ABz,z,t)$$

By Lemma 1.1.13 we get $ABz = z$

By applying step 4 ,5,6,7,8 we get

$$Az = Bz = Sz = Tz = Pz = Qz = z.$$

That is z is a common fixed point of A,B,S,T,P,Q in X.

Uniqueness Let u be another common fixed point of A,B,S,T,P and Q. Then

$$Au = Bu = Su = Tu = Pu = Qu = u$$

Putting $x = u and y = z$ in 1.2.1(e) then we get

$$M^2(Pu,Qz,kt) \geq M^2(ABu,STz,t) \star M^2(Pu,ABu,t) \star M^2(Qz,STz,t) \star M^2(Pu,STz,t) * M^2(ABu,ABu,t)$$

Taking limit both side then we get

$$M^2(u,z,kt) \geq M^2(u,z,t) \star M^2(u,u,t) \star M^2(z,z,t) \star M^2(u,z,t)M^2(u,u,t)$$

$$M^2(u,z,kt) \geq M^2(u,z,t)$$

And hence $M(u,z,kt) \geq M(u,z,t)$

By lemma 1.1.13 we get $z = u$.

That is z is a unique common fixed point of A,B, S, T, P and Q in X.

Remark 1.2.2: If we take $B = T = I$ identity map on X in Theorem 2.2.1 then condition 1.2.1(b) is satisfy trivially and we get following Corollary

Corollary 1.2.3 suppose $(X,M,\star)$ be a FMS and let $A,B,S,T,P and Q$ be mappings from X into itself s.t the following conditions are satisfied:

1.2.3(a) $P \subset S and Q(X) \subset A$, are in universal space

1.2.3(b)$AB = BA, ST = TS, PB = BP, QT = TQ$,

1.2.3(c) either P or AB is continuous,

1.2.3(d) (P,AB) is compatible of type (β) and (Q,ST) is weak compatible,

1.2.3(e) $\exists k \in (0,1)$ such that for every $x,y \in X and t > 0$

$$M^2(Px,Qy,kt) \geq M^2(Ax,STy,t) \star M^2(Px,Ax,t) \star M^2(Qy,Sy,t) \star M^2(Px,Sy,t) * M^2(Ax,STy,t)M^2(Ax,Ax,t)$$

To show A, S, P and Q have a UCFP in X.

Remark 1.2.4 If we take the pair (P,AB) is weakly compatible in place of compatible type of (β) in Theorem 2.2.1 then we get the following result.

Corollary 1.2.5 suppose $(X,M,\star)$ be a CFMS and let $A,B,S,T,P and Q$ be mappings from X into itself s.t the following conditions are satisfied:

1.2.5(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,

1.2.5(b)$AB = BA, ST = TS, PB = BP, QT = TQ$,

1.2.5(c) either P or AB is continuous,

1.2.5(d) (P,AB) is compatible of type (β) and (Q,ST) is weak compatible,

1.2.5(e) $\exists k \in$ (zero, one)s.t for each point $x,y \in X and t > 0$

$$M^2(Px,Qy,kt) \geq M^2(ABx,STy,t) \star M^2(Px,ABx,t) \star M^2(Qy,STy,t) \star M^2(Px,STy,t) * M^2(ABx,ABx,t)$$

To show A, B,S,T, P and Q have a UCFP in X.

Remark 1.2.6 If we take $B = T = I$ identity map on X in Corollay 1.2.5 then condition 1.2.1(b) is satisfy trivially and we get following Corollary

Corollary 1.2.7

suppose $(X,M,\star)$ be a CFMS and let $A,B,S,T,P and Q$ be mappings from X into itself s.t the following conditions are satisfied:

1.2.7(a) $P \subset S and Q(X) \subset A$, are in universal space

1.2.7(b)$AB = BA, ST = TS, PB = BP, QT = TQ$,

1.2.7(c) either P or AB is continuous,

1.2.7(d) (P,AB) is compatible of type (β) and (Q,ST) is weak compatible,

1.2.7(e) $\exists k \in (0,1)$ such that for every $x,y \in X and t > 0$

$$M^2(Px,Qy,kt) \geq M^2(Ax,STy,t) \star M^2(Px,Ax,t) \star M^2(Qy,Sy,t) \star M^2(Px,Sy,t) * M^2(Ax,STy,t)M^2(Ax,Ax,t)$$

To show A, ,S,, P and Q have a UCFP in X.

Definition 1.2.8SM A and S of a FMS$(X,M,\star)$are said to be compatible of type $(\alpha) \Leftrightarrow$if $M(ASx_n,SSx_n,t) \to 1$ and $M(AAx_n,ASx_n,t) \to 1 \forall t > 0$, where $\{x_n\}$ is a in X s.t$Sx_n, Ax_n \to p$ for some $p \in X as n \to \infty$.

It is easy to see that compatible map of type (α) is equivalent to the compatible map of type (β).

Now following results are equivalent to Theorem 1.2.1

Theorem 1.2.9 Let $(X,M,\star)$ be a complete fuzzy metric space and let $A,B,S,T,P and Q$ be mappings from X into itself such that the following conditions are satisfied:

1.2.9(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,

1.2.9(b)$AB = BA, ST = TS, PB = BP, QT = TQ$,

1.2.9(c) moreover P or AB is nonstop,

1.2.9(d) (P,AB) is compatible of type (α) and (Q,ST) is WC,

1.2.9(e) there exists $k \in (0,1)$ such that for every $x,y \in X and t > 0$

$$M^2(Px,Qy,kt) \geq M^2(ABx,STy,t) \star M^2(Px,ABx,t) \star M^2(Qy,STy,t) \star M^2(Px,STy,t) * M^2(ABx,ABx,t)$$

Then A, B,S,T, P and Q have a UCFP in X.

Proof : Form the definition 1.2.8 and proof of the Theorem 1.2.1, we get the result.

Remark 1.2.10 If we take $B = T = I$IM on X in Theorem 1.2.9 then condition 1.2.9(b) is satisfy trivially and we get following Corollary

Corollary 1.2..10 if $(X,M,\star)$ be a CFMS and let $A,S,P and Q$ be mappings from X into itself s.t :

1.2.10(a) $P(X) \subset S(X) and Q(X) \subset A(X)$,

1.2.10(b) either P or A is continuous,

1.2.10(c) (P,A) is compatible of type (α) and (Q,S) is weak compatible,

1.2.10(d) $\exists k \in$ (zero, one)s.t for every $x,y \in X and t > 0$

$$M^2(Px,Qy,kt) \geq M^2(Ax,Sy,t) \star M^2(Px,Ax,t) \star M^2(Qy,Sy,t) \star M^2(Px,Sy,t) * M^2(Ax,Ax,t)$$

To show A, S,P and Q have a UCFP unique common fixed point in X.

1.3 Common Fixed Point Theorem for Integer Type Mapping in Fuzzy Metric Space.

On the way of generalization of Banach contraction principle [21] one of the most famous generalization is introduced by Branciari [21] in general setting of lebgesgue integrable function and proved following fixed point theorems in metric spaces.

Theorem 1.3.1: Suppose (X,d) be a CMS, $\alpha \in (0,1)$ and let $T: X \to X$, be a mapping s.t for each $x, y \in X$,

$$\int_0^{d(Tx,Ty)} \xi(v)\,dv \le \int_0^{d(x,y)} \xi(v)\,dv$$

i.e ξ : [zero, +infinite] → [zero, +infinite] is a LIM which is summable on ECSS of [zero, +infinite] , NN, and such that, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\,dv > 0$to show , T has UFP$z \in X$ such that for each $x \in X, T^n x \to z$ as $n \to \infty$.

It should be noted that if $\xi(v) = 1$to show BCP is obtained.

Inspired from the result of Branciari [] we prove following CFPT in FMS.

Theorem 1.3.2 if $(X, M, \star)$ be a CMFS and let $A, B, S, T, P and Q$ be mappings from X into s.t:

1.3.2(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,

1.3.2 (b)$AB = BA, ST = TS, PB = BP, QT = TQ$,

1.3.2 (c) moreover P or AB is nonstop,

1.3.2 (d) (P, AB) is compatible of type (β) and (Q, ST) is w.c,

1.3.2 (e) $\exists k \in (0,1)$s.t for every $x, y \in X and t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\,dv \ge \int_0^{W(x,y,t)} \xi(v)\,dv$$

$$W(x,y,t) = M^2(ABx, STy, t) \star M^2(Px, ABx, t) \star M^2(Qy, STy, t) \star M^2(Px, STy, t) * M^2(ABx, ABx, t)$$

i.eξ : $[0, +\infty] \to [0, +\infty]$ is a LIM which is summable on each CSS of $[0, +\infty]$, NN, and s.t, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\,dv > 0$. to show A,B,S,T, P and Q have a UCFP in X.

Proof: if $x_0 \in X$, then from 2.3.2(a) we have $x_1, x_2 \in X$s.t

$$Px_0 = STx_1 and Qx_1 = ABx_2$$

Inductively, we CS$\{x_n\} and \{y_n\}$ in X s.t for $n \in N$

$$Px_{2n-2} = STx_{2n-1} = y_{2n-1} and Qx_{2n-1} = ABx_{2n} = y_{2n}$$

Step 1 Put $x = x_{2n} and y = x_{2n+1}$ in 2.3.2 (e) i.e

$$\int_0^{M^2(Px_{2n},Qx_{2n+1},kt)} \xi(v)\,dv \ge \int_0^{W(x_{2n},x_{2n+1},t)} \xi(v)\,dv$$

$$W(x_{2n}, x_{2n+1}, t) = M^2(ABx_{2n}, STx_{2n+1}, t) \star M^2(Px_{2n}, ABx_{2n}, t) \star M^2(Qx_{2n+1}, STx_{2n+1}, t) \star M^2(Px_{2n}, STx_{2n+1}, t) * M^2(ABx_{2n}, ABx_{2n}, t)$$

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\,dv \ge \int_0^{W(y_{2n+1},y_{2n+2},t)} \xi(v)\,dv$$

$$W(y_{2n+1}, y_{2n+2}, t) = M^2(y_{2n}, y_{2n+1}, t) \star M^2(y_{2n+1}, y_{2n}, t) \star M^2(y_{2n+2}, y_{2n+1}, t) \star M^2(y_{2n+1}, y_{2n+1}, t) * M^2(y_{2n}, y_{2n}, t)$$

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\,dv \ge \int_0^{M^2(y_{2n},y_{2n+1},t)\star M^2(y_{2n+2},y_{2n+1},t)} \xi(v)\,dv$$

From Lemma 1.1.13 and 1.1.14 we have

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\,dv \ge \int_0^{M^2(y_{2n},y_{2n+1},t)} \xi(v)\,dv$$

Since $\xi(v)$ is LIFs.t

$$M^2(y_{2n+1}, y_{2n+2}, kt) \ge M^2(y_{2n}, y_{2n+1}, t)$$

That is

$$M(y_{2n+1}, y_{2n+2}, kt) \ge M(y_{2n}, y_{2n+1}, t)$$

likewise

$$M(y_{2n+2}, y_{2n+3}, kt) \ge M(y_{2n+1}, y_{2n+2}, t)$$

Thus we have

$$M(y_{n+1}, y_{n+2}, kt) \ge M(y_n, y_{n+1}, t)$$

$$M(y_{n+1}, y_{n+2}, t) \ge M\left(y_n, y_{n+1}, \frac{t}{k}\right)$$

$$M(y_n, y_{n+1}, t) \ge M\left(y_0, y_1, \frac{t}{k^n}\right) \to 1\ as n \to \infty,$$

in addition to hence $M(y_n, y_{n+1}, t) \to 1\ as n \to \infty\ for all t > 0$.

Every point$\epsilon > 0\ and t > 0$, we can choose $n_0 \in N$s.t $M(y_n, y_{n+1}, t) > 1 - \epsilon for all n > n_0$.

Every point$m, n \in N$ we suppose that $\ge n$. consider

$$M(y_n, y_m, t) \ge M\left(y_n, y_{n+1}, \frac{t}{m-n}\right) \star M\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right) \star \dots \star M\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$$

$$M(y_n, y_m, t) \ge (1-\epsilon) \star (1-\epsilon) \star \dots \dots \star (1-\epsilon)(m-n) times$$

$$M(y_n, y_m, t) \ge (1-\epsilon)$$

And hance $\{y_n\}$ is a CS in X.

Since $(X, M, \star)$ is complete, $\{y_n\}$ cons to SP$z \in X$. Also its SSC to the SP$\in X$.

i.e.

$\{Px_{2n+2}\} \to z and \{STx_{2n+1}\} \to z$ 1.3.2 (i)

$\{Qx_{2n+1}\} \to z and \{ABx_{2n}\} \to y_{2n}$ 1.3.2(ii)

Case 1 Suppose AB is nonstop

Since AB is nonstop, we have

$$(AB)^2 x_{2n} \to ABz and ABPx_{2n} \to ABz$$

As (P, AB) is CP of type (β), we have

$$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = 1, for all t > 0$$

Or $M(PPx_{2n}, ABz, t) = 1$

Therefore, $PPx_{2n} \to ABz$.

Step 2 Put $x = (AB)x_{2n} and y = x_{2n+1}$ in 1.3.2(e) we have

$$\int_0^{M^2(P(AB)x_{2n},Qy,kt)} \xi(v)\,dv \ge \int_0^{W(P(AB)x_{2n},Qy,kt)} \xi(v)\,dv$$

$$W(P(AB)x_{2n}, Qy, t) = M^2(AB(AB)x_{2n}, STx_{2n+1}, t) \star M^2(P(AB)x_{2n}, AB(AB)x_{2n}, t) \star M^2(Qx_{2n+1}, STx_{2n+1}, t) \star M^2(P(AB)x_{2n}, STx_{2n+1}, t) * M^2(AB(AB)x_{2n}, AB(AB)x_{2n}, t)$$

Taking $n \to \infty$ we get

$$\int_0^{M^2(P(AB)x_{2n},Qy,kt)} \xi(v)\,dv \ge \int_0^{W(P(AB)x_{2n},Qy,t)} \xi(v)\,dv$$

$$M^2((AB)z, z, kt) \ge M^2((AB)z, z, t) \star M^2((AB)z, (AB)z, t) \star M^2((AB)z, z, t) \star M^2((AB)z, z, t) * M^2((AB)z(AB), z, t)$$

$$\int_0^{M^2((AB)z,z,kt)} \xi(v)\,dv \geq \int_0^{M^2((AB)z,z,t)\star M^2((AB)z,z,t)} \xi(v)\,dv$$

i.e of $\xi(v)$ we have

$$M((AB)z,z,kt) \geq M((AB)z,z,t)$$

i.e by lemma 1.1.14 we have

$ABz = z$. 1.3.2(iii)

Step -3 Put $x = z and y = x_{2n+1}$ in 2.3.2(e) we have

$$\int_0^{M^2(Pz,Qx_{2n+1},kt)} \xi(v)\,dv \geq \int_0^{W(Pz,Qx_{2n+1},t)} \xi(v)\,dv$$

$$W(Pz,Qx_{2n+1},t) = M^2(ABz,STx_{2n+1},t) \star M^2(Pz,ABz,t) \star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(Pz,STx_{2n+1},t)$$

Taking $n \to \infty$ and using equation 1.3.2 (i) we have

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\,dv \geq \int_0^{W(Pz,z,t)} \xi(v)\,dv$$

$$W(Pz,z,t) \geq M^2(ABz,z,t) \star M^2(Pz,ABz,t) \star M^2(z,z,t) \star M^2(Pz,z,t) * M^2(ABz,ABz,t)$$

So that $M^2(Pz,z,kt) \geq M^2(Pz,z,t)$

And hence $M(Pz,z,kt) \geq M(Pz,z,t)$

i.e by using lemma 1.1.14, we get $Pz = z$

i.e $ABz = Pz = z$.

Step- 4 Putting $x = Bz and y = x_{2n+1} in$ 2.3.2$(e), we get$

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(v)\,dv \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(v)\,dv$$

$$W(PBz,Qx_{2n+1},t) = M^2(ABBz,STx_{2n+1},t) \star M^2(PBz,ABBz,t) \star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(PBz,STx_{2n+1},t) * M^2(ABBz,ABBz,t)$$

As $BP = PB and AB = BA$, so we have

$$P(Bz) = B(Pz) = Bz and (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.$$

Taking $n \to \infty$ and using 1.3.2(i) we get

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(v)\,dv \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(v)\,dv$$

$$\int_0^{M^2(Bz,z,kt)} \xi(v)\,dv \geq \int_0^{W(Bz,z,t)} \xi(v)\,dv$$

$$W(Bz,z,t) = M^2(Bz,z,t) \star M^2(Bz,Bz,t) \star M^2(z,z,t) \star M^2(Bz,z,t) * M^2(Bz,z,t)$$

So we have $M^2(Bz,z,kt) \geq M^2(Bz,z,t)$

That is $M(Bz,z,kt) \geq M(Bz,z,t)$

Consequently by Lemma 1.1.14 we have $Bz = z$

And also we have $ABz = z implies Az = z$

Therefore

$$Az = Bz = Pz = z \text{ . 1.3.2 (iv)}$$

Step – 5 As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$$z = Pz = STu$$

Putting $x = x_{2n} and y = u$ in 2.3.2(e) we get

$$\int_0^{M^2(Px_{2n},Qu,kt)} \xi(v)\,dv \geq \int_0^{W(Px_{2n},Qu,t)} \xi(v)\,dv$$

$$W(Px_{2n},Qu,t) = M^2(ABx_{2n},STu,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(Qu,STu,t) \star M^2(Px_{2n},STu,t) * M^2(ABx_{2n},ABx_{2n},t)$$

Taking $n \to \infty$ and using 1.3.2(i) and 1.3.2(ii) we get

$$\int_0^{M^2(z,Qu,kt)} \xi(v)\,dv \geq \int_0^{W(z,Qu,t)} \xi(v)\,dv$$

$$W(z,Qu,t) = M^2(z,STu,t) \star M^2(z,z,t) \star M^2(Qu,STu,t) \star M^2(z,STu,t) * M^2(z,Z,t)$$

So we have $M^2(z,Qu,kt) \geq M^2(z,Qu,t)$

i.e $M(z,Qu,kt) \geq M(z,Qu,t)$

Consequently by using Lemma 1.1.13 we have $Qu = z$

Hence $STu = z = Qu$.

Hence (Q,ST) is weak compatible, therefore, we have

$$QSTu = STQu$$

Thus $Qz = STz$.

Step – 6 Putting $x = x_{2n} and y = z$ in 1.3.2(e) we get

$$\int_0^{M^2(Px_{2n},Qz,kt)} \xi(v)\,dv \geq \int_0^{W(Px_{2n},Qz,t)} \xi(v)\,dv$$

$$W(Px_{2n},Qz,t) = M^2(ABx_{2n},STz,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(Qz,STz,t) \star M^2(Px_{2n},STz,t)$$

Taking $n \to \infty$ and using 1.3.2(ii) and step 5 we get

$$\int_0^{M^2(z,Qz,kt)} \xi(v)\,dv \geq \int_0^{W(z,Qz,t)} \xi(v)\,dv$$

$$W(z,Qz,t) = M^2(z,STz,t) \star M^2(z,z,t) \star M^2(Qz,STz,t) \star M^2(z,STz,t) * M^2(STz,STz,t)$$

That is $M^2(z,Qz,kt) \geq M^2(z,Qz,t)$

And therefore $M(z,Qz,kt) \geq M(z,Qz,t)$

consequently by using Lemma 1.1.13 we get $Qz = z$.

Step – 7 Putting $x = x_{2n} and y = Tz$ in 2.3.2(e) we get

$$\int_0^{M^2(Px_{2n},QTz,kt)} \xi(v)\,dv \geq \int_0^{W(Px_{2n},QTz,t)} \xi(v)\,dv$$

$$W(Px_{2n},QTz,t) = M^2(ABx_{2n},STTz,t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(QTz,STTz,t) \star M^2(Px_{2n},STTz,t) * M^2(ABx_{2n},ABx_{2n},t)$$

As $QT = TQ and ST = TS$ we have $QTz = TQz = Tz$

And $ST(Tz) = T(STz) = TQz = Tz$.

Taking $n \to \infty$ we get

$$\int_0^{M^2(z,Tz,kt)} \xi(v)\,dv \geq \int_0^{W(z,Tz,t)} \xi(v)\,dv$$

$$W(z,Tz,t) = M^2(z,Tz,t) \star M^2(z,z,t) \star M^2(Tz,Tz,t) \star M^2(z,Tz,t) * M^2(Tz,Tz,t)$$

And hence $M^2(z,Tz,kt) \geq M^2(z,Tz,t)$

Consequently $M(z,Tz,kt) \geq M(z,Tz,t)$

Consequently by Lemma 1.1.13 we have $Tz = z$

Now $STz = Tz = z implies Sz = z$.

Hence

$Sz = Tz = Qz = z$ 1.3.2(v)

Combining 1.3.2(iv) and 1.3.2(v) we have

$$Az = Bz = Pz = Sz = Tz = Qz \ = z$$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case - II suppose P is continuous

As P is continuous

$$P^2x_{2n} \to Pz and P(AB)x_{2n} \to Pz$$

As (P, AB) is compatible pair of type (β),

$$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = \ 1\, for all t > 0$$

Or $M(Pz, (AB)(AB)x_{2n}, t) = \ 1$

Therefore $(AB)^2x_{2n} \to Pz$.

Step -8 Putting $x \ = \ Px_{2n} and y = x_{2n+1}$ in 1.3.2(e) then we get

$$\int_0^{M^2(PPx_{2n},Qx_{2n+1},kt)} \xi(v)\,dv \geq \int_0^{W(PPx_{2n},Qx_{2n+1},t)} \xi(v)\,dv$$

$$W(PPx_{2n}, Qx_{2n+1}, t) = M^2(ABPx_{2n}, STx_{2n+1}, t) \star M^2(PPx_{2n}, ABPx_{2n}, t)$$

$$\star M^2(Qx_{2n+1}, STx_{2n+1}, t) \ \star M^2(PPx_{2n}, STx_{2n+1}, t)M^2(ABPx_{2n}, ABPx_{2n}, t)$$

Taking $n \to \infty, we get$

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\,dv \geq \int_0^{W(Pz,z,t)} \xi(v)\,dv$$

$$W(Pz, z, t) = M^2(Pz, z, t) \star M^2(Pz, Pz, t) \star M^2(z, z, t) \star M^2(Pz, z, t)M^2(Pz, Pz, t)$$

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\,dv \geq \int_0^{M^2(Pz,z,t)} \xi(v)\,dv$$

consequently we have

$$M^2(Pz, z, kt) \geq \ M^2(Pz, z, t)$$

Hence $M(Pz, z, kt) \geq \ M(Pz, z, t)$ consequently by Lemma 1.1.13 we get $Pz \ = z$

Step- 9 Put $x \ = \ ABx_{2n}$ and $y = x_{2n+1}$ in 1.3.2(e) then we get

$$\int_0^{M^2(PABx_{2n},Qx_{2n+1},kt)} \xi(v)\,dv \geq \int_0^{W(PABx_{2n},Qx_{2n+1},t)} \xi(v)\,dv$$

$$W(PABx_{2n}, Qx_{2n+1}, t) = M^2(ABABx_{2n}, STx_{2n+1}, t) \star M^2(PABx_{2n}, ABABx_{2n}, t)$$

$$\star M^2(Qx_{2n+1}, STx_{2n+1}, t) \star M^2(PABx_{2n}, STx_{2n+1}, t)M^2(ABABx_{2n}, ABABx_{2n}, t)$$

Taking $n \to \ \infty$ we get

$$\int_0^{M^2(ABz,z,kt)} \xi(v)\,dv \geq \ \int_0^{W(ABz,z,t)} \xi(v)\,dv$$

$$W(ABz, z, t) = M^2(ABz, z, t) \star M^2(ABz, z, t) \star M^2(z, z, t) \ \star M^2(\ ABz, z, t)$$

$$\int_0^{M^2(ABz,z,kt)} \xi(v)\,dv \geq \ \int_0^{M^2(ABz,z,t)} \xi(v)$$

Therefore $M^2(ABz, z, kt) \geq \ M^2(ABz, z, t)$

And hence $M(ABz, z, kt) \geq \ M(ABz, z, t)$

By lemma 2.1.13 we get $ABz \ = z$

By applying step 4 ,5,6,7,8 we get

$$Az \ = Bz = Sz = Tz \ = Pz = Qz \ = z.$$

That is z is a CFP of A,B,S,T,P,Q in X.

Exclusivity Let u be another CFP of A,B,S,T,P and Q. Then

$$Au = Bu = Su = Tu = Pu = Qu \ = u$$

Putting $x \ = u and y = z$ in 1.2.1(e) then we get

$$\int_0^{M^2(Pu,Qz,kt)} \xi(v)\,dv \geq \ \int_0^{W(Pu,Qz,t)} \xi(v)\,dv$$

$$W(Pu, Qz, t) = M^2(ABu, STz, t) \star M^2(Pu, ABu, t) \star M^2(Qz, STz, t) \ \star M^2(Pu, STz, t)M^2(ABu, ABu, t)$$

Taking limit both side then we get

$$\int_0^{M^2(u,z,kt)} \xi(v)\,dv \geq \int_0^{W(u,z,t)} \xi(v)\,dv$$

$$W(u, z, t) = M^2(u, z, t) \star M^2(u, u, t) \ \star M^2(z, z, t) \star M^2(u, z, t) * M^2(u, u, t)$$

i.e. $M^2(u, z, kt) \geq \ M^2(u, z, t)$

And hence $M(u, z, kt) \geq \ M(u, z, t)$

By lemma 1.1.13 we get $z \ = \ u$.
i.e. z is a UCFP of A,B, S, T, P and Q in X.

Remark 1.3.3 Theorem 1.2.1 is a special case of the Theorem 1.3.2. It is sufficient if we take $\xi(v) \ = \ 1$ in Theorem 1.3.2.

Remark 1.3.4 If we take $B = T = I$IM on X in Theorem 1.2.3.2 to show condition 1.3.2(b) is satisfy trivially and we get following Corollary

Corollary 1.3.5 suppose $(X, M, \star)$ be a CFMS and suppose$A, S, P and Q$ be mappings from X into itself s.t:
1.3.5(a) $P(X) \subset S(X) and Q(X) \subset A(X)$,
1.3.5 (b) moreover P or AB is nonstop,
1.3.5 (c) (P, AB) is c of t (β) and (Q, ST) is WC,
1.3.5 (d) $\exists k \in$ (zero, one)s.t for every $x, y \in X and t > \ 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\,dv \geq \ \int_0^{W(x,y,t)} \xi(v)\,dv$$

$$W(x, y, t) \ = M^2(Ax, Sy, t) \star M^2(Px, Ax, t) \star M^2(Qy, Sy, t) \ \star M^2(Px, Sy, t) * M^2(Ax, Ax, t)$$

anywhere $\xi : \ [0\,, +\infty] \to \ [0\,, +\infty]$ is a LIM which is summable on each CSS of $[0\,, +\infty]$, NN, and s.t, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\,dv > 0$. to show A,B,S,T, P and Q have a UCFP in X.

Remark 1.3.6 If we take the pair (P, AB) is WC in place of CT of (β) in Theorem 1.3.2 then we get the following result.

Corollary 1.3.7 suppose$(X, M,\star)$ be a CFMC and let $A, B, S, T, P and Q$ be mappings from X into itself s.t the subsequent situation are fulfilled:

1.3.7(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,
1.3.7 (b)$AB = BA, ST = TS, PB = BP, QT = TQ$,
1.3.7(c) moreover P or AB is nonstop,
1.3.7 (d) (P, AB) and (Q, ST) are WC,
1.3.7 (e) $\exists f(x) = k \in (0,1)$s.tevery point$x, y \in X and t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\, dv \geq \int_0^{W(x,y,t)} \xi(v)\, dv$$
$$W(x,y,t) = M^2(ABx, STy, t) \star M^2(Px, ABx, t) \star M^2(Qy, STy, t) \star M^2(Px, STy, t) * M^2(ABx, ABx, t)$$

i.e ξ : [zero, +infinite] → [zero, +infinite] is a LIM which is summable on ECSS of [zero, +infinite] , NN, and ST, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\, dv > 0$. To show A,B,S,T, P and Q have a UCFT in X.

Remark 1.3.8 If we take $B = T = I$IP on X in Corollay 1.3.7 then condition 1.3.7(b) is satisfy insignificantly and we get following Corollary

Corollary 1.3.9 Let $(X, M, \star)$ be a CFMS and let $A, S, P and Q$ be mappings from X into itself such that the following conditions are satisfied:
1.3.9(a) $P(X) \subset S(X) and Q(X) \subset A(X)$,
1.3.9(b) either P or AB is continuous,
1.3.9 (c) (P, A) and (Q, S) are weak compatible,
1.3.9 (d) $\exists f(x) = k \in (0,1)$s.tevery point$x, y \in X and t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\, dv \geq \int_0^{W(x,y,t)} \xi(v)\, dv$$
$$W(x,y,t) = M^2(Ax, Sy, t) \star M^2(Px, Ax, t) \star M^2(Qy, Sy, t) \star M^2(Px, Sy, t)$$

i.e. ξ : [zero, +infinite] → [zero, +infinite] is a LIM which is summable on ECSS of [zero, +infinite] , NN, and ST, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\, dv > 0$. To show A,S, P and Q have a UCFT in X.
Now following results are also equivalent to Theorem 2.3.2

Theorem 1.3.10: Suppose $(X, M, \star)$ be a CFMS and suppose $A, B, S, T, P and Q$ be mappings from X into itself s.t the following conditions are satisfied:
1.3.10(a) $P(X) \subset ST(X) and Q(X) \subset AB(X)$,
1.3.10 (b)$AB = BA, ST = TS, PB = BP, QT = TQ$,
1.3.10(c) moreover P or AB is nonstop,
1.3.10(d) (P, AB) is CofT(α) and (Q, ST) is WC,
1.3.10 (e) there exists $k \in (0,1)$ such that for every $x, y \in X and t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\, dv \geq \int_0^{W(x,y,t)} \xi(v)\, dv$$
$$W(x,y,t) = M^2(ABx, STy, t) \star M^2(Px, ABx, t) \star M^2(Qy, STy, t) \star M^2(Px, STy, t)$$

Where ξ : $[0, +\infty] \to [0, +\infty]$ is a LIM which is summable on ECSSof $[0, +\infty]$NN, and such that, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\, dv > 0$. to show A,S, P and Q have a UNCFP in X.
Proof : Form the explanation 1.2.8 and proof of the Theorem 1.3.2, we get the result.

Remark 1.3.11: If we take $B = T = I$ identity map on X in Theorem 1.3.10 then condition 1.3.10(b) is ST and we get following Corollary

Theorem 1.3.12 suppose$(X, M, \star)$ be a CFMS and let $A, S, P and Q$ be mappings from X into itself i.e. the subsequent situation are fulfilled:
1.3.12(a) $P(X) \subset S(X) and Q(X) \subset A(X)$,
1.3.12 (b) moreover P or AB is nonstop,
1.3.12 (c) (P, AB) is CofT(α)&(Q, ST) is WC,
1.3.12 (d) $\exists k \in$ (zero, one)s.tevery point $x, y \in X and t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\, dv \geq \int_0^{W(x,y,t)} \xi(v)\, dv$$
$$W(x,y,t) = M^2(Ax, Sy, t) \star M^2(Px, Ax, t) \star M^2(Qy, Sy, t) \star M^2(Px, Sy, t) * M^2(Ax, Ax, t)$$

i.e ξ : [zero, +infinite] → [zero, +infinite] is a LIM which is summable on each ECSS of [zero, +infinite] , NN and S.t, $\forall\, \varepsilon > 0, \int_0^{\varepsilon} \xi(v)\, dv > 0$ .to show A,S, P and Q have a UNCFP in X.

2. Conclusion

Here we proved Common Fixed Point Theorem for Compatible Maps of Type (β) and Type (α) in Fuzzy Metric Space and Common Fixed Point Theorem for Integer Type Mapping in Fuzzy Metric Space important corollaries.

References

[1] Aage, C. T. and Salunke, J. N., "On Fixed Point Theorems in Fuzzy Metric Spaces." Int. J. Open Problems Compt. Math., 3 (2), **(2010)**, 123-131.
[2] Aage C.T., Salunke J.N., "On Fixed point Theorems in Fuzzy Metric Spaces Using a Control Function." International Journal of Nonlinear Analysis and Applications, 6(2), **(2010)**, 50-57.
[3] Aage, C.T., Salunke, J.N., "The Result on Fixed point Theorems in Dislocated Quasi- Metric Spaces." Applied Mathematical Sciences, 2(59), **(2008)**, 2941-2948.
[4] Aamri, M., El Moutawakil, D., "Some New Common Fixed Point Theorems underStrict Contractive Conditions." Journal of Mathematical Analysis and Applications, 226, **(1988)**, 251-258.
[5] Abbas, M., Ali Khan M., "Common Fixed Point Theorem of Two Mappings Satisfying a Generalized Weak Contractive Condition." Int J Math. Sci, 9, **(2009)**, Article ID 131068.
[6] Alaca, C., Turkoglu D. and Yildiz, C., "Fixed Point in Intuitionistic Fuzzy Metric Spaces." Chaos, Solitons & Fractals, 29, **(2006)**, 1073-1078.
[7] Alaca, C., Altun, I. and Turkoglu, D., "On Compatible Mapping of Type (I) and (II) in Intuitionistic Fuzzy Metric Spaces." Korean Mathematical Society, 23(3), **(1986)**, 771-779.
[8] Alber, Ya I, Guerre-Delabriere S., "Principles of Weakly Contractive Maps in Hilbert Spaces." Oper. Theory Adv. Appl., 98, **(1997)**, 7-22.
[9] Aliprantis, C.D. and TourkyR. , "Cones and Duality in Graduate Studies in Mathematics." Amer. Math. Soc., 84, **(2007)**, 215-240.

[10] Al-Thagafi, A. and Shahzad, N.,"Generalized I-NonexpansiveSelfmaps and Invariant Approximations." Acta Mathematica Sinica, English Series Vol. 24, No. 5, (**2008)**, 867-876.
[11] Atanssove, K., "Intuitionstic Fuzzy Sets." Fuzzy Sets and Systems, 2(1), **(1986)**, 87-96.
[12] Altun, I. and Turkoglu, D., "Some Fixed Point Theorems on Fuzzy Metric Spaces with Implicit Relations." Comm. Korean Math. Soc., 23(1), **(2008)**, 111-124.
[13] Banach, S. "Surles Operation Dansles Ensembles AbstraitesEtleur Application Integrals." Fund. Math., 3, **(1922)**, 133-181.
[14] Balasubramaniam, P., Muralisankar S., Pant R.P., "Common Fixed Points of Four Mappings in a Fuzzy Metric Space." J. Fuzzy Math., 10 (2), **(2002)**, 379-384.