Stochastic Analysis of a System Having Two Dissimilar Components and Two Service Facilities

Praveen Gupta, Ruchi Yadav

School of Studies in Statistics Vikram University, Ujjain, Madhya Pradesh, India

Abstract: ***In order to improve reliability, system is considered. The system has two dissimilar components working in parallel. The failure time of the components are assumed to be exponentially distributed with different parameters. Failure of one component puts the work pressure on the second component, causing its changed (increased) failure rates. There are two repair facilities to repair the components. The repair time distribution of each server is exponential. We obtain the expressions for reliability, the mean time to system failure (MTSF) and steady state availability for both the systems.***

Keywords: Availability, Exponential Distribution, Mean time to system failure, Reliability

1. Introduction

Two-unit standby system models have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industry. Recently, Mokaddis and Matta (2010), Khaled (2010) and Sharma et. al (2010) have studied two unit standby systems. They have considered a single repair facility to repair both the units. When both the units are failed, one failed unit waits for repair. Researchers in reliability have shown keen interest in the analysis of two (or more) component parallel systems. Owing to the practical utility in modern industrial and technological set-ups of these systems, we come across with the systems in which the failure in one component affects the failure rate of the other component.

2. System Description

1) The system consists of a single unit having two dissimilar components, say A and B arranged in parallel.
2) Failure of one component affects the failure rate of the other component due to increase in working stresses.
3) The system remains operative even if a single component operates.
4) There are two repair facilities to repair the components. When both the components are failed, they work independently on each component.
5) The repair rates are different, when both the repair facilities work on same component and when both work on different components.
6) After repair, each component is as good as new.

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3. Notations and States of the System

E = Set of regenerative States

α = Constant failure rate of component A when B is also operating

β = Constant failure rate of component B when A is also operating

α' = failure rate of component A when B has already failed

β' = failure rate of component B when A has already failed

γ = repair rate of component A when B is operating

δ = repair rate of component B when A is operating

θ = repair rate of component B when A is also under repair

η = repair rate of component A when B is also under repair

μ = rate of conducting preventive maintenance

λ = rate with which system goes for preventive maintenance.

$A_{N:}$ component A is in normal mode and operative

$B_{N:}$ component B is in normal mode and operative

$A_{R:}$ component A is under repair

$B_{R:}$ component B is under repair

$A_{f:}$ component A is in failure mode needs repair

$B_{f:}$ component B is in failure mode needs repair.

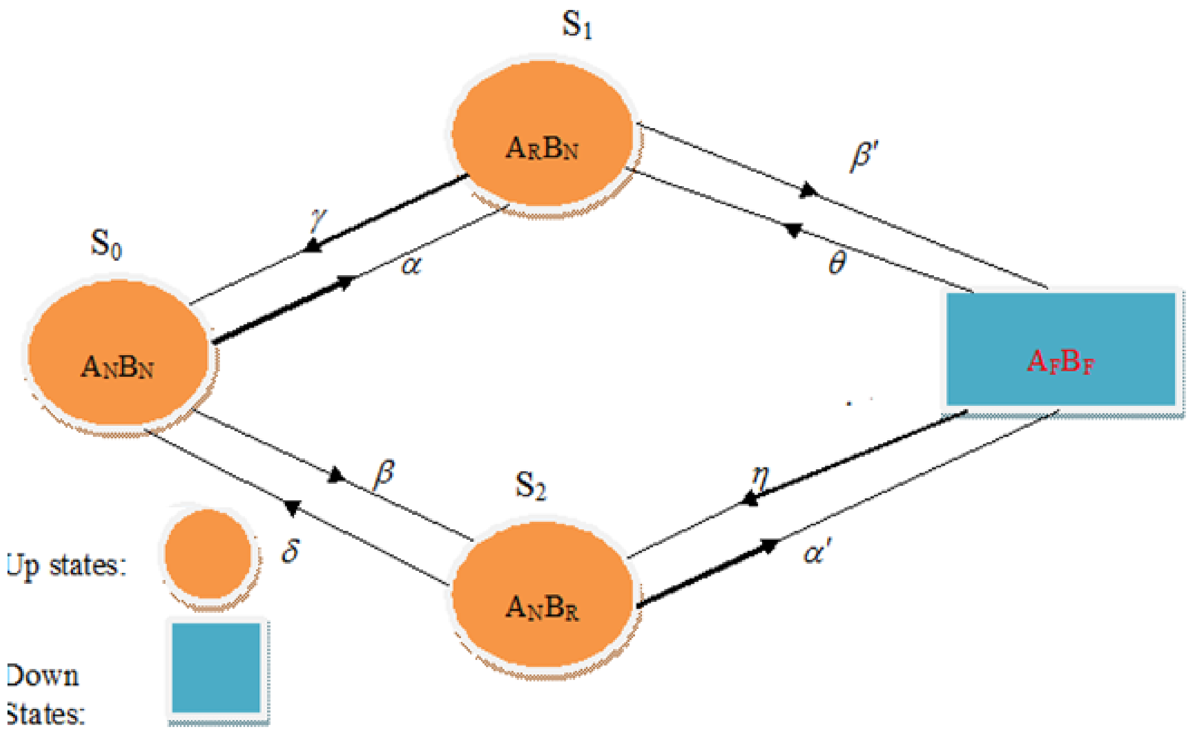
$A_{NP:}$ component A is under preventive maintenance.

$B_{NP:}$ component B is under preventive maintenance.

The system can be in one of the following states:

Up states: S_0 (A_NB_N), S_1 (A_RB_N), S_2 (A_NB_R)

Down states: S_3 (A_FB_F)



State Transition Diagram for the first system

Transition probabilities and sojourn times

Let T_0 (=0), T_1, T_2, be the epochs at which the system enters the state $S_i \in E$, and let X_n denotes the state entered at epoch T_{n+1}. i.e. just after the transition of T_n. Then $\{X_n, T_n\}$ constitute a Markov-renewal process with the state space E, and $Q_{ij}(t) = \Pr[X_{n+1} = S_j, T_{n+1} - T_n \le t \mid x_n = S_i]$

Then the transition probability matrix of the embedded Markov chain is:

$$P = (P_{ij}) = \lim_{t\to\infty} Q_{ij}(t) = Q(\infty)$$

By simple probabilistic considerations, the non-zero elements of $Q_{ij}(t)$ are:

$$Q_{01}(t) = \int_0^t \alpha e^{-(\alpha+\beta)u} du, \quad Q_{02}(t) = \int_0^t \beta e^{-(\alpha+\beta)^u} du$$

$$Q_{10}(t) = \int_0^t \gamma e^{-(\gamma+\beta)^u} du, \quad Q_{13}(t) = \int_0^t \beta^1 e^{-(\gamma+\beta)^4} du$$

$$Q_{20}(t) = \int_0^t \delta e^{-(\alpha+\delta)u} du, \quad Q_{23}(t) = \int_0^t \alpha^1 e^{-(\alpha+\delta)^u} du$$

$$Q_{31}(t) = \int_0^t \theta e^{-(\theta+\eta)u} du, \quad Q_{32}(t) = \int_0^t \eta e^{-(\theta+\eta)u} du \qquad (1)$$

Taking limit as $t \to \infty$, the steady state transition probabilities p_{ij} can be obtained from (1). Thus

$$P_{ij} = \lim_{t\to\infty} Q_{ij}(t)$$

$$p_{01} = \alpha/(\alpha+\beta) \quad p_{02} = \alpha/(\alpha+\beta) \quad p_{10} = \gamma/(\gamma+\beta^1) \quad p_{13} = \beta^1/(\gamma+\beta^1)$$

$$p_{20} = \delta/(\alpha^1+\delta) \quad p_{23} = \alpha^1/(\alpha^1+\delta) \quad p_{31} = \theta/(\theta+\eta) \quad p_{32} = \eta/(\theta+\eta)$$

From the above probabilities the following relations can be easily verified as:

$$p_{01} + p_{02} = p_{02} + p_{23} = p_{10} + p_{13} = p_{31} + p_{32} = 1.$$

Mean Sojourn Times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu = \int_0^\infty P[T > t]\, dt$$

Where T is the time of stay in state S_i by the system. s

To calculate mean sojourn time □$_i$ in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore,

$$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)t}\, dt \ = 1/(\alpha+\beta),$$
$$\mu_1 \ = 1/(\gamma+\beta^1),$$
$$\mu_2 = 1/(\alpha^1+\beta),$$
$$\mu_3 = 1/(\theta+\eta). \ (2)$$

Reliability and Mean Time to System Failure (MTSF)

To determine R_i (t), the reliability of the system when it starts initially from regenerative state

S_i (i= 1, 2), we assume the failed state S3 as absorbing. Using simple probabilistic arguments in regenerative point technique, we have

$$R_0(t) = Z_0(t) + q_{01}(t) \copyright\ R_1(t) + q_{02}(t) \copyright\ R_2(t)$$
$$R_1(t) = Z_1(t) + q_{10} \copyright\ R_0(t)$$
$$R_2(t) = Z_2(t) + q_{20} \copyright\ R_0(t), \ (3)$$

Where we define Z_i (t) as the probability that starting from state S_i the system remains up till epoch t without passing through any regenerative state.

$$Z_0(t) = e^{-(\alpha+\beta)t},$$
$$Z_1(t) = e^{-(\gamma+\beta')t},$$
$$Z_2(t) = e^{-(\delta+\alpha')t}$$

Taking Laplace transform of relations and solving, we get

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*} \quad (4)$$

Here for brevity the argument s is omitted. Now by taking the limit as $s \to 0$ in equation (4), the mean time to system failure when the initial state S_0, is

$$E(T) = \frac{\mu_0 + p_{01}\mu_1 + p_{02}\mu_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \quad (5)$$

Availability Analysis

Let A_i (t) be the probability that starting from state S_i the system is available at epoch t without passing through any regenerative state,

Now, obtaining A_i (t) by using elementary probability arguments:

$$A_0(t) = Z_0(t) + q_{01}(t) \copyright\ A_1(t) + q_{02}(t) \copyright\ A_2(t)$$
$$A_1(t) = Z_1(t) + q_{10}(t) \copyright\ A_0(t) + q_{13}(t) \copyright\ A_3(t)$$
$$A_2(t) = Z_2(t) + q_{20}(t) \copyright\ A_0(t) + q_{23}(t) \copyright\ A_3(t)$$
$$A_3(t) = q_{31} \copyright\ A_1(t) + q_{32}(t) \copyright\ A_2(t)$$

Taking Laplace transform of above equations and solving for $A_0^*(s)$, by omitting the argument 's' for brevity, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Where

$$N_1(s) = [Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*]\,[1 - q_{13}^* \ q_{32}^* \ q_{23}^*]$$
$$+ [q_{01}^* \ q_{13}^* + q_{02}^* \ q_{23}^*]\,[q_{31}^* Z_1^* + q_{32}^* Z_2^*]$$
$$D_1(s) = [1 - q_{13}^* q_{31}^* - q_{23}^* q_{32}^*] \ [1 - q_{01}^* \ q_{10}^* \ q_{20}^* q_{02}^*]$$
$$- [q_{01}^* \ q_{13}^* + q_{02}^* \ q_{23}^*]\,[q_{32}^* q_{20}^* + q_{31}^* q_{10}^*]$$

Therefore, the steady state availability of the system when its starts operation from S_0 is

$$A_0(\infty) = \lim_{t\to\theta} A_0(t)$$
$$= \lim_{s\to\theta} S.A_0^*(s) = N_2(0) \,|\, D_2^1(0) = N_2 \,|\, D_2$$

Where N_1 and D_1 are as

$N_1 = N_1(0) = (\mu_0 + P_{01}\mu_1 + P_{02}\ \mu_2)(1 - P_{13}P_{31} - P_{32}P_{23}) + (P_{01}P_{13} + P_{02}P_{23})(P_{31}\ \mu_1 + P_{32})$ (6)

$D_1 = D_1^1$ (0) =

$$(P_{20}P_{32} + P_{01}P_{31})\mu_1 + (P_{32}\ P_{13} + P_{02}\ P_{23})\ \mu_2$$

(7)

4. Conclusion

This paper describes an improvement over the Khaled (2010) and Sharma et. al (2010) have studied two unit standby systems. They have considered a single repair facility to repair both the units. Using regenerative point technique reliability analysis, availability analysis, busy period analysis which shows that the proposed model is better than Khaled and Sharma (2010).

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