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Modification of Feynman Technique of Differentiation

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Abstract: ***Sir Richard P. Feynman mentioned a technique to differentiate a type of functions in a book named "Feynman's Tips on Physics: Reflections, Advice, Insights, Practice" which was written by Michael A. Gottlieb, Ralph Leyton and Richard P. Feynman himself. This method made differentiation easier. This paper is to reduce the constraints of the method by introducing some modifications to make it applicable for more type of functions.***

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1. Introduction

There are conventional methods to differentiate functions $f(x)$ in the form 1.$f(x) = u(x)v(x)$ or 2.$f(x) = \frac{u(x)}{v(x)}$. The methods are:

$$\frac{df}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} (for form 1) \quad (1)$$

$$\frac{df}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} (for form 2) \quad (2)$$

For functions of the form $f(x) = u(x)v(x)w(x)$:

$$\frac{df}{dx} = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx} \quad (3)$$

Feynman's differentiation method gives a short and easy process where we can differentiate functions of the general form :

$$f(x) = \frac{[a_1(x)]^{c_1}[a_2(x)]^{c_2}[a_3(x)]^{c_3}......}{[b_1(x)]^{d_1}[b_2(x)]^{d_2}[b_3(x)]^{d_3}......} \quad (4)$$

Where, $(c_1, c_2, c_3, ...)$ and $(d_1, d_2, d_3, ...)$ are constants.
The modified version helps to differentiate functions of the same form but with variable exponents, i.e. $(c_1, c_2, c_3, ...)$ and $(d_1, d_2, d_3, ...)$ are also functions of x.Therefore, the general form becomes :

$$f(x) = \frac{[a_1(x)]^{c_1(x)}[a_2(x)]^{c_2(x)}[a_3(x)]^{c_3(x)}......}{[b_1(x)]^{d_1(x)}[b_2(x)]^{d_2(x)}[b_3(x)]^{d_3(x)}......} \quad (5)$$

2. Feynman's Method

We can write Eq.(4) like this,

$$f(x) = [a_1(x)]^{c_1}[a_2(x)]^{c_2}[a_3(x)]^{c_3}.....[b_1(x)]^{-d_1}[b_2(x)]^{-d_2}[b_3(x)]^{-d_3}.....$$

Therefore, $\frac{df}{dx}$ will be,

$$\frac{df}{dx} = f(x)[\sum_{i=1}^{n_1} \frac{c_i}{a_i}\frac{da_i}{dx} - \sum_{j=1}^{n_2} \frac{d_i}{b_i}\frac{db_i}{dx}]$$

Where, n_1 and n_2 are the number of functions in numerator and denominator of Eq.(4) respectively.

By generalizing, we get, for a function $f(x) = \prod_{i=1}^{n} [a_i(x)]^{b_i}$ [Where, b_i is constant and n is the number of functions],

$$\frac{df}{dx} = f(x)\sum_{i=1}^{n} \frac{b_i}{a_i}\frac{da_i}{dx} \quad (6)$$

(Proof is in a latter section)

3. Modified Method

If there is a differentiable function $f(x) = \prod_{i=1}^{n} [a_i(x)]^{b_i(x)}$, where a and b are differentiable functions of x, then $\frac{df}{dx}$ will be,

$$\frac{df}{dx} = f(x)\sum_{i=1}^{n} (\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]) \quad (7)$$

(Proof is in the next section)

In Eq.(7) we can see that $f(x)$ is differentiated by multiplying $\sum_{i=1}^{n} (\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i])$ with it.This can be called **Differentiating Factor ($\phi(x)$)**. So, we can say,

$$\phi(x) = \frac{1}{f(x)}\frac{df}{dx} = \sum_{i=1}^{n} (\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]) \quad (8)$$

This is a more generalized version of Feynman's Differentiating Technique.

By putting b as a constant in Eq.(8), we get,

$$\phi(x) = \sum_{i=1}^{n} \frac{b_i}{a_i}\frac{da_i}{dx} \quad (9)$$

Which gives us Eq.(6), the Feynman Differentiating Technique.

Also, by putting a as a constant in Eq.(8), we get,

$$\phi(x) = \sum_{i=1}^{n} \frac{db_i}{dx}\log_e[a_i] \quad (10)$$

4. Proof

To prove Eq.(7), we take,

$$f(x) = \prod_{i=1}^{n} [a_i(x)]^{b_i(x)}$$

Where $f(x)$, $a_i(x)$ and $b_i(x)$ are differentiable functions.

By taking log with base e on both sides and then differentiating with respect to x, we get,

$$\log_e f(x) = \sum_{i=1}^{n} b_i(x)\log_e a_i(x)$$

$$\frac{1}{f(x)}\frac{df}{dx} = \sum_{i=1}^{n} (\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]) = \phi(x)$$

This gives us,

$$\frac{df}{dx} = f(x)\sum_{i=1}^{n} (\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]) = f(x)\phi(x)$$

Which is Eq.(7). By putting $b = constant$, Eq.(6) can also be proved.

5. Example

Using this method, one can save time while differentiating functions of the form $f(x) = \prod_{i=1}^{n} [a_i(x)]^{b_i}$. Here is an example,

Differentiating $f(x) = x^x e^{(x^2+2x+2)}$ by calculating the differentiating factor $\phi(x)$ instead of the conventional method. $\phi(x)$ will be,

$$\phi(x) = \frac{x}{x}\frac{dx}{dx} + \frac{dx}{dx}\log_e x + [\frac{d}{dx}(x^2 + 2x + 2)]\log_e e$$

This gives us,

$$\phi(x) = 3 + 2x + \log_e x$$

Therefore,

$$\frac{df}{dx} = f(x)\phi(x) = x^x e^{(x^2+2x+2)}(3 + 2x + \log_e x)$$

6. Conclusion

Modified Feynman's Method states that there always is a differentiating factor $\phi(x)$ for any differentiable function $f(x)$ such that,

$$\frac{df}{dx} = f(x)\phi(x)$$

If the function $f(x)$ is of the form $f(x) = \prod_{i=1}^{n} [a_i(x)^{b_i(x)}]$, then $\phi(x) = \sum_{i=1}^{n} [\frac{b_i(x)}{a_i(x)}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e(a_i)]$. Where $a_i(x)$ and $b_i(x)$ are differentiable.

Also, we can conclude that if $\phi(x)$ is the differentiating factor of $f(x)$, then,

$$f(x) = \int f(x)\phi(x)dx + c$$

Where, $c = Constant$

7. Summary

Every differentiable function has a differentiating factor such that the product of a function and its differentiating factor gives the first order differentiation of the function with respect to the independent variable.

If the function is of the form of product of multiple differentiable functions with constant powers then the value of its differentiating factor is given by the Feynman's technique of differentiation.

If the function is of the form of product of multiple differentiable functions with variable powers which are also differentiable functions then the differentiating factor is given by the Modified Feynman's technique of differentiation.

Also, if a function $f(x)$ is of the form of another function $g(x)$ multiplied with its differentiating factor, then integrating $f(x)$ will give $g(x)$ with an integrating constant c.

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