

# An Interesting Number Cycle that takes Two Consecutive Integers (a, b) and One Third Integer (c), and with any c, it Always Returns a or b

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**Abstract:** In this paper, I introduce an interesting number cycle that takes two consecutive integers and one third integers and after repeating multiple operations on them including adding and dividing operations, returns one of the two consecutive integers. This cycle always is true for all the integers and no matter which integers are chosen for this number cycle.

**Keywords:** number theory, integer, numerical, collatz

## 1. Introduction

Collatz's conjecture is one of the most famous unsolved problems in mathematics. It is also known as  $3n + 1$  problem. It was introduced by Lothar Collatz in 1937. I was trying to find a solution for the collatz's conjecture and during one of these efforts, I found that there is another number cycle that after several simple math operation on three different integers, it will return one of these integers. In this cycle, at first consider two consecutive integers (any consecutive integers) and then choose another integer (any integer), next add the third integer to one of the two consecutive integers (odd to odd and even to even) and then divide the results by 2. Then put the result as third integer. Repeating this operation will get you to one of the consecutive integers and this is true for all the integers.

## 2. Explanation of this Cycle

First we choose two consecutive integers: "a" and "b" so that  $b = a + 1$ . These integers could be chosen from  $-\square$  to  $+\square$ , then another integer ("c") is chosen. This integer also could be chosen from  $-\square$  to  $+\square$ . In next step, we should add one of the two consecutive integers ("a" or "b") to third integer ("c"). If the third integer ("c") is odd, we should select the odd integer from two consecutive integers: a or b and if the third integer ("c") is even, we should select even integer from two consecutive integers: a or b (zero is considered as even). After adding a or b to c, the resulting integer will be an even integer (because we add odd to odd and even to even). Now we divide this integer by 2 and put it as the third integer ("c") and repeat the previous operations (adding and dividing). These operations will result to one of the consecutive integers (a or b): If  $c \geq b$ , the resultant integer will be "b", and if  $c \leq a$ , the resultant integer will be "a". We always will reach to these results, no matter which integer is selected.

### Examples

Here I will present some examples from different range of integers.

### Example 1

$a > 0, b > 0, c > 0$  and  $c = b$ :  
 $a = 7, b = 8$  and  $c = 8 \rightarrow 8 + 8 = 16, 16 : 2 = 8, 8 + 8 = 16, 16 : 2 = 8, \dots$

### Example 2

$a > 0, b > 0, c > 0$  and  $c > b$  and c is odd:  
 $a = 7, b = 8$  and  $c = 37 \rightarrow 37 + 7 = 44, 44 : 2 = 22, 22 + 8 = 30, 30 : 2 = 15, 15 + 7 = 22, 22 : 2 = 11, 11 + 7 = 18, 18 : 2 = 9, 9 + 7 = 16, 16 : 2 = 8, 8 + 8 = 16, 16 : 2 = 8, \dots$

### Example 3

$a > 0, b > 0, c > 0$  and c is even:  
 $a = 7, b = 8, c = 38 \rightarrow 38 + 8 = 46, 46 : 2 = 23, 23 + 7 = 30, 30 : 2 = 15, 15 + 7 = 22, 22 : 2 = 11, 11 + 7 = 18, 18 : 2 = 9, 9 + 7 = 16, 16 : 2 = 8, 8 + 8 = 16, 16 : 2 = 8, \dots$

### Example 4

$a < 0, b < 0$  and  $c > 0$ :  
 $a = -8, b = -7, c = 29 \rightarrow 29 + (-7) = 22, 22 : 2 = 11, 11 + (-7) = 4, 4 : 2 = 2, 2 + (-8) = -6, -6 : 2 = -3, (-3) + (-7) = -10, -10 : 2 = -5, (-5) + (-7) = -12, -12 : 2 = -6, (-6) + (-8) = -14, -14 : 2 = -7, (-7) + (-7) = -14, -14 : 2 = -7, \dots$

### Example 5

$a < 0, b < 0$  and  $c = 0$ :  
 $a = -8, b = -7, c = 0 \rightarrow 0 + (-8) = -8, -8 : 2 = -4, (-4) + (-8) = -12, -12 : 2 = -6, (-6) + (-8) = -14, -14 : 2 = -7, (-7) + (-7) = -14, -14 : 2 = -7, \dots$

### Example 6

$a < 0, b < 0$  and  $b \leq c < 0$ :  
 $a = -21, b = -20, c = -4 \rightarrow (-4) + (-20) = -24, -24 : 2 = -12, (-12) + (-20) = -32, -32 : 2 = -16, (-16) + (-20) = -36, -36 : 2 = -18, (-18) + (-20) = -38, -38 : 2 = -19, (-19) + (-21) = -40, -40 : 2 = -20, (-20) + (-20) = -40, -40 : 2 = -20, \dots$

### Example 7

$a < 0, b < 0$  and  $c \leq a$ :  
 $a = -22, b = -21, c = -67: (-67) + (-21) = -88, -88 : 2 = -44, (-44) + (-22) = -66, -66 : 2 = -33, (-33) + (-21) = -54, -54 : 2 = -27, (-27) + (-21) = -48,$

-48:  $2 = -24, (-24) + (-22) = -46, -46: 2 = -23, (-23) + (-21) = -44, -44: 2 = -22, (-22) + (-22) = -44, -44: 2 = -22, \dots$

### 3. Discussion

There are many number theories and conjectures in mathematic that suggest many number patterns and structures in numerical field. Number theory has always fascinated amateurs as well as professional mathematicians. In contrast to other branches of mathematics, many of the problems and theorems of number theory can be understood by laypersons [2]. For example there are seven millennium problems that only one of them has been solved and other six problems are unsolved and \$1 million prize is for solving each of them and some of them are type of number theory.

Fibonacci numbers series is also a very famous numerical series in math. The *Fibonacci numbers*  $F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$  are a numerical sequence given by the following recurrence relation:  $F_n = F_{n-1} + F_{n-2}$  with the initial terms  $F_1 = F_2 = 1$  [1].

One of the interesting conjectures in math is Collatz's conjecture. In this conjecture, Collatz presents a formula in which you take a positive integer and if it is even, divide it by 2 and if it is odd, multiply it by 3 and add 1 to it and then divide it by 2. Continuing by repeating these operations will get you to 1, no matter which positive integer you select. The Collatz's conjecture also named  $3n+1$  conjecture [3].

Collatz's conjecture is unsolved and I did some try on it to find a solution for it. I couldn't solve it but during these efforts, I find out that there is another number cycle (cycle of integers) that could be used on all integers. In this cycle at first you take two consecutive integers (any two consecutive integers from  $-\square$  to  $+\square$ ): "a" and "b" so that  $b = a + 1$ , then choose a third integer: "c" (any integer from  $-\square$  to  $+\square$ ). After choosing these integers, first add the third integer to one of the two consecutive integers (odd on odd and even on even), next divide the result by 2 and then put the result as "c" and repeat the operations until it returns "a", or "b" (as shown in the above examples).

### 4. Conclusion

This number cycle is a cycle of operations on integers so that after choosing two consecutive integers and a third integers, it will return one of the two consecutive integers, and you can repeat these operations with the same two consecutive integers and any other third integers i.e repeat it infinite times for the same two consecutive integers and return a fixed result.

This is different from Collatz's integer in which only positive integers are acceptable but in number cycle, any integer from  $-\square$  to  $+\square$  could be selected.

### References

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### Author Profile

**Mohammadreza Barghi**, lives in Calgary, Alberta, Canada. I am interested in numerical math and I was trying to solve Collatz's Conjecture that I found out the above mentioned numerical cycle. It seems to me that it might be interesting for other peoples. Email address: mreza7@yahoo.com. Post address: 284, Hidden Ranch Circle, NW, Calgary, Alberta, Canada. Postal Code: T3A 5R2