International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

Upper and Lower Weakly Quasi Continuous Fuzzy Multifunctions

Saurabh Saxena

Department of Mathematics, GOVT.S.S.A.PG College Sihora Jabalpur M.P., India

Abstract: ***The aim of this paper is to initiate the study and to find different characterizing properties of upper and lower weakly continuous fuzzy multifunction's, where the domain of these functions is a topological space with these values as arbitrary fuzzy sets in fuzzy topological spaces. The study of different classes of non-continuous functions between topological spaces and there genralizations to multi valued cases has long been of interest to topologist. Some of these functions have been extended to fuzzy topological space by many authors. A further step ahead in this process of generalization is the introduction of fuzzy multifunction's. Malakar introduced fuzzy θ-continuous multifunctions. Again Mukherjee and Malakar have introduced fuzzy almost continuous multifunctions. In this paper we introduced a mutual relationships among these fuzzy multifunctions.***

According to Papageorgious (1985), If (X, τ) is a topological space in the sense Chang (1968) ,then a fuzzy multifunction F: X → Y is a function which maps a point of X to a set in Y. We shell adhere to this terminology and simply by X and Y, we shell denote the topological space (X ,τ) and the fuzzy topological space (Y,σ) respectively.

Definition:

A fuzzy multifunction F : X → Y is said to be

(a) Fuzzy upper weakly quasi continuous at a point x ϵ X if for reach open set U in X containing x and each fuzzy open set V in Y containing F(x) ,there exists a non empty open set τ in X such that τ ⊂ U and F(u) ≤ cl (V).
(b) Fuzzy lower weakly quasi continuous at a point x ϵ X if for reach open set U in X containing x and each fuzzy open set V in Y with F(x)qV, there exists a non empty open set τ in X such that τ ⊂ U and F(g)qclV for each g ϵ τ.
(c) Fuzzy upper weakly quasi continuous resp. fuzzy lower weakly quasi continuous on X .If F has corresponding property at each point x of X.

Definition:

A fuzzy multifunction F : X → Y is said to be

(a) Fuzzy upper almost quasi continuous at a point x ϵ X if for reach open set U in X containing x and each fuzzy open set V in Y containing F(x) ,there exists a non empty open set G in X such that G ⊂ X and F(τ) ≤scl (V).
(b) Fuzzy lower almost quasi continuous at a point x ϵ X if for reach open set U in X containing x and each fuzzy open set V in Y with F(x)qV ,there exists a non empty open set G in X such that G⊂ X and F(g)qsclV for all g ϵ G.
(c) Fuzzy upper almost quasi continuous resp. fuzzy lower almost quasi continuous on X .If F has corresponding property at each point x of X.

Theorem: A fuzzy multifunction F : X → Y is Fuzzy upper almost quasi continuous iff $F^+(V)$ ϵ SO(X), for every fuzzy regular open set V in Y.

Theorem: A fuzzy multifunction F: X → Y is Fuzzy lower almost quasi continuous iff $F^-(V)$ ϵ SO(X), for every fuzzy regular open set V in Y.

Theorem: A fuzzy multifunction F : X → Y is Fuzzy lower almost quasi continuous at a point x of X iff for any fuzzy open set V in Y containing F (x),there exist Uϵ SO(X) with x ϵ U such that F(U) ≤ scl(V).

Definition: An fuzzy topological space X is said to be fuzzy almost regular if for each fuzzy point X_α in X and for each fuzzy regular open q-neighbourhood U of X_α ,there exist a fuzzy regular open q-neighbourhood V of X_α such that clV ≤ U.

Theorem:

IF A fuzzy multifunction F : X → Y is Fuzzy lower weakly quasi continuous and Y is fuzzy almost regular then F is Fuzzy lower almost quasi continuous.

Proof:

Let V be a fuzzy regular open set in Y and x ϵ F^- (V) .Then F(x)qV. Since Y is fuzzy almost regular , there exist a fuzzy regular open set W in Y such that F (x)qW ≤ cl W ≤ V.Since F is fuzzy lower weakly quasi continuous thereexists a semiopen set U_x in X containing x such that F(u)qclW,for all u ϵ U_x→ F(u)qV , for all u ϵ U_x. Therefore we have x ϵ U_x⊂ F^-(V) this implies that F^-(V) ϵ SO(X) and hence F is Fuzzy lower almost quasi continuous.

Definition:

A fuzzy multifunction F : X → Y is said to be fuzzy almost preopen if F(U) ≤ int(cl(F(U))) for every U ϵ SO(X).

Theorem

If F: X → Y is fuzzy upper weakly quasi continuous and fuzzy almost preopen ,then it is fuzzy upper almost quasi continuous.

Proof:

For any x ϵ X and any fuzzy open set V of Y containing F(X) ,there exist U ϵ SO(X) with x ϵ U such that F (U) ≤clV. Since F is fuzzy almost preopen ,F(U) ≤ int(cl(F(U))) ≤ int(cl(V) = sclV. Hence F is fuzzy upper almost quasi continuous.

Paper ID: SR22424183316 DOI: 10.21275/SR22424183316

Theorem:
Let F:X → Y be a fuzzy multifunction such that F(x) is fuzzy open in Y for each x ϵ X. Then the following are equivalent:
(a) F is fuzzy lower quasi continuous.
(b) F is fuzzy lower almost quasi continuous.
(c) F is fuzzy lower weakly quasi continuous.

Proof:
(a) →(b)→(c) areobvious. We only show that (c)→(a)
(c)→ (a) :Let x ϵ X and V be a fuzzy open set in Y such that F (x)qV. Since F is fuzzy lower weakly quasi continuous, there exist U ϵ SO(X) containing x such that F(u)qclV for every u ϵ U. Since F(u) is fuzzy open in Y by the given condition, F(u)qV for every u ϵ U and hence F is fuzzy lower quasi continuous.

Example:
Let X = {a ,b ,c} , Y= [0,1], τ = { φ , X }, τY = {0Y ,1Y ,A ,B} where A(Y) = 0.35 , B(y) = .04 for all y ϵ Y.Let F: (X,τ) → (y,τY) be a fuzzy multifunction defined by F(a) = A ,F(b) = B, F(c) = C where C(y) = 0.6 , for all y ϵ Y. Now 1Y and B are only non null fuzzy regular open sets in (Y ,τY) and F^+ (B) = {x ϵ X :F(x) ≤ B}= {a,b} f ϵ SO(X). And so F is not fuzzy upper almost quasi continuous.

Example:
Let X = {a, b, c}, Y= [0,1], τ = { φ , X }, τY = {0Y ,1Y ,A ,B} where A(Y) = 0.35 , B(y) = .04 for all y ϵ Y.Let F: (X,τ) → (y,τY) be a fuzzy multifunction defined by F(a) = A ,F(b) = B, F(c) = C where A(y) = 0.45,C(y) = 0.61 , for all y ϵ Y. Now 1Y and B are only non null fuzzy regular open sets in (Y ,τY) now F^-(B) = {x ϵ X :F(x)qB}= {c} f ϵ SO(X). And so F is not fuzzy lower almost quasi continuous.

Definition:
A fuzzy multifunction F: X→Y is called :
(a) Fuzzy upper θ-continuous at some point x_0 ϵ X iff for every fuzzy open set V in Y with x_0 ϵ F^+ (V) ,there exist an open nbd U of x_0 such that cl U ⊂ F^+(cl V).
(b) Fuzzy lower θ-continuous at some point x_0 ϵ X iff for every fuzzy open set V in Y with x_0 ϵ F^- (V) ,there exist an open nbd U of x_0 such that cl U ⊂ F^-(cl V).

Example:
Let X = {a ,b ,c} , Y= [0,1],τ = { φ , X }, τY = {0Y ,1Y ,A ,B} where A(Y) =
0.35 , B(y) = .04 for all y ϵ Y.Let F: (X,τ) → (y,τY) be a fuzzy multifunction defined by F(a) = A ,F(b) = B, F(c) = C where C(y) = 0.61 , for all y ϵ Y. Here F is not fuzzy upper θ-continuous But F is fuzzy upper quasi continuous.

References

[1] Ajmal N.and Sharma R,D. Various Fuzzy topological notions and fuzzy almost continuity.Fuzzy sets and Systems , 50 (2)(1992),209-223.
[2] Azad K.K. On Fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity. J. Math. Anal. Appl., 82,(1981), 14-32.
[3] BinShahnaA.S.Mapping in fuzzy topological spaces. Fuzzy sets and systems, 61(1994), 209-213.
[4] Chang C.L. Fuzzy topological spaces. J. Math. Anal. Appl. 24 (1968), 182-190.
[5] EwartJ.Fuzzy valued maps. Math.Nachr,137 (1988), 79-87.
[6] Hu Chang-Ming.Fuzzy topological spaces. J. Math, Anal., 110 (1985), 114-178.
[7] Malakar S. On fuzzy semi irresolute and fuzzy strongly irresolute functions. Fuzzy sets and systems 45(2) (1992), 239-244.
[8] Anda S. On Fuzzy topological spaces , Fuzzy sets and systems 19(2) (1986)193-197.
[9] Ppapageorgiov N. S. Fuzzy topology and fuzzy multi functions. J. Math.Anal.Appl.109(1985) ,397-425.
[10] Zadah L. A. Fuzzysets. In form and control, 8 (1965), 338-358.