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A Comparative Study between Experimental & Numerical Value of Subharmonic Acoustic Resonances

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Abstract:

In a non linear system, a response at other then the driving frequency in often observed. We consider a closed tube in which the oscillations of a gas corum driven by the sinusoidal motion of a piston. For the investigated case (subharmonic resonances) the frequency of the piston displacement in the neighbourhood of half of the fundamental frequency. It can be show that near subharmonic resonances the acoustic theory breaks down for sufficiently small boundary–layer friction. The reason for this "breakdown" is that a second order nonlinear correction to the acoustic solution becomes resonant. we get a small boundary layer friction shocks are formed and the amplitude at subharmonic resonance is comparable with the non-resonant acoustic solution. Theoretical results are compared with the pressure signals measured at the closed end of a resonance tube. These fundamental experiments show the existence of a subharmonic resonance with shocks predicted by the nonlinear theory bv. J.J. Keller (1975) and consequently the breakdown of the acoustic solution in a case for which the acoustic solution does not become singular.

1. Introduction

In present paper we discuss the oscillations of a gas column contained in a tube driven by the sinusoidal motion of a piston (figure 1). The displacement of the piston is $l\,.\,\sin(\omega\, t)$, where l is much less then L. According to acoustic theory the particle velocity in the gas is given by

$$v,(x,t)=\frac{\omega l\sin(\omega x/a_0)\cos(\omega t)}{\sin(\omega L/a_0)} \tag{1}$$

where a_0 is the undisturbed speed of sound and L is the length of the tube. A question of interest now is what happens when the piston motion is within some neighbourhood of one–half of the fundamental resonance frequency of the gas column with

$$\omega=\frac{(2N+1)\pi a_0}{2L}+\Delta\omega \tag{2}$$

N = 0, 1, 2, 3, ...,,...

This report aimes at verifying the breakdown of the acoustic theory experimentally. Theoretical results, based on a paper written by J.J. Kellers are compared with the Pressure response measured at the closed end of the tube.

2. Numerical Simulations

For the particle velocity v & the soundspeed a the first order acountic equation with first approximate (v_1, a_1) for forward & backward system are given by equations :

$$v_1 + \frac{2}{k-1} a_1 = 2a_0 f\left(t - \frac{x}{a_0}\right) \tag{3}$$

$$v_1 - \frac{2}{k-1} a_1 = 2a_0 g\left(t + \frac{x}{a_0}\right)$$

The boundary condition v (x = 0, t) = 0 at the closed end requires

$$g = -f \tag{4}$$

In reality resonant solutions in closed tubes are strongly influenced by the boundary layer effects at the tube wall. Chester[2] takes into account the boundary layer effect by a convolution integral over Riemann's invariant f in the continuity equation. The boundary–laver term together with the quadratic inviscid terms in f lead to a non–linear integral equation. We now consider the case where v (L, t) given by:

$$v(x = L, t) = v_A \cos\left(\frac{\omega}{2} t\right) \tag{5}$$

The following approximations are valid if the frequency w lies near to $\pi\, a_0 /$ L:

$$f\left(t - \frac{L}{a_0}\right) = f\left(t - \frac{L_r}{a_0}\right) - \Delta t\, f'\left(t - \frac{L_r}{a_0}\right)$$

$$f\left(t + \frac{L}{a_0}\right) = f\left(t + \frac{L_r}{a_0}\right) + \Delta t\, f'\left(t + \frac{L_r}{a_0}\right) \tag{6}$$

with the abbreviations $L_r = \pi\, a_0/\omega$ and Δ t = (L–L)/a_0 Considering that the period of f is 4L_r / a, the following splitting into symmetric and asymmetric terms can be made:

$$f = f_A + f_S$$

$$f_A\left(t - \frac{L_r}{a_0}\right) = -f_A\left(t + \frac{L_r}{a_0}\right) \tag{7}$$

$$f_S\left(t - \frac{L_r}{a_0}\right) = f_S\left(t + \frac{L_r}{a_0}\right) \tag{8}$$

A similar splitting can be made for the periodic integrals $F_A(t)$ and $F_S(t)$, where

$$\frac{dF_A(t)}{dt} = f_A(t), \qquad , \frac{dF_S(t)}{dt} = f_S(t)$$

If we make use of the boundary condition (5), equations (7), (8), (9) and neglecting terms which vanish at resonance or are of higher order, Chester's integral equation yields:

$$\pm v_A \cos\left(\frac{w}{2}t\right) = \pm 2a_0 f_A\left(t - \frac{L_r}{a_0}\right) - 2\Delta t a_0 f_S\left(t - \frac{L_r}{a_0}\right)$$

$$+\frac{k+1}{2} L_S \left[f_A^2\left(t - \frac{L_r}{a_0}\right) + f_S^2\left(t - \frac{L_r}{a_0}\right)\right]$$

$$\pm \frac{a_0(3-k)}{2}\left[f_A\left(t - \frac{L_r}{a_0}\right) F_S\left(t - \frac{L_r}{a_0}\right) - f_S\left(t - \frac{L_r}{a_0}\right) F_A\left(t - \frac{L_r}{a_0}\right)\right]' \tag{10}$$

$$\pm \frac{a_0(3-k)}{2}\left[f_A\left(t - \frac{L_r}{a_0}\right) F_S\left(t - \frac{L_r}{a_0}\right) - f_S\left(t - \frac{L_r}{a_0}\right) F_A\left(t - \frac{L_r}{a_0}\right)\right]'$$

$$-\beta L \int_0^\infty f_S'\left(t - \frac{L_r}{a_0} - \xi\right) \xi^{1/2} d\xi$$

$$\pm \frac{\beta a_0}{2} \int_0^\infty f_A'\left(t - \frac{L_r}{a_0} - \xi\right) \xi^{-1/2} d\xi$$

where $\beta = \left(\frac{v_0}{\pi}\right)^{1/2} \left\{1 + \frac{k-1}{Pr^{1/2}}\right\}$, R is the tube radius,

v, Pr and *K* are the kinematic viscosity, the Prandtl number and the ratio of specific heats respectively.

If the time t is replaced by $t + 2\pi/\omega$ the upper signs change to the lower ones. The difference of the two equations for t and $t + 2\pi/\omega$ leads to the linear solution

$$= u_A \sin\left(\frac{\omega}{2}t\right) = 2a_0 f_A(t) + \frac{\beta a_0}{2} \int_0^\infty f_A(t-\xi)\xi^{-1/2} d\xi, \tag{11}$$

which corresponds to the solution expected on the basis of acoustic theory. If the influence of the viscosity is small (which is usually true for typical experimental conditions) equation (11) reduces to

$$f_A(t) = -\frac{M}{2} \sin\left(\frac{\omega}{2}t\right) \tag{12}$$

where $M = v_A / a_0$ is the Mach number of the piston velocity.

The sum of the equations (10) can be integrated and gives:

$$c - f_A^2(t) = \left\{ f_S(t) - \frac{2r}{\pi} \varepsilon^{1/2} \right\}^2 - s\left(\frac{\omega\varepsilon}{\pi}\right)^{1/2} \int_0^\infty f_S(t-\xi)\xi^{1/2} d\xi \tag{13}$$

where

$$\varepsilon = \frac{M^2}{4},$$

$$r = \frac{\pi}{(k+1)\varepsilon^{1/2}} \frac{\Delta\omega}{\omega}$$

$$S = \frac{2\beta}{(k+1)} \left(\frac{\pi}{\omega\varepsilon} \right)^{1/2}$$

with the driving non–linear term $f_A^2(t)$ on the left–hand side;

s is a friction parameter which is essentially the ratio of the boundary layer thickness and the radius of the tube; *r* is a frequency parameter which sub the frequency departure from the resonant frequency.

It discontinuities of compression are admitted only the inviscid form of (13) at exact resonance ($r = 0$) yields the solution

$$t_S(t) = \pm \left(\frac{M}{2} \cos\left(\frac{\omega}{2} t \right) \right) \tag{14}$$

Correspondingly the combined solution *f* takes the form

$$f(t) = \frac{M}{2} \left(-\sin\left(\frac{\omega}{2} t \right) \pm \cos\left(\frac{\omega}{2} t \right) \right) \tag{15}$$

in figure 2 the functions f_A, f_S and the pressure response p ($x = 0$, t) at th closed end of the tube are plottea versus the time.

3. Experimental Apparatus

The resonance tube used in the experiment has an inner diameter of 20 mm and the wall thickness is 2 mm. The essential part of the present experimental apparatus is the arrangement to generate a sinusoidal motion of the piston crosshead with a horizontal slit transfers the rotation to a sinusoidal oscillation of the piston.

The voltage output from a piezoelectric pressure transducer is digitized and processed by a data aquisition system. The data are synchronously sampled with the piston motion, 60 or 72 samples per cycle controlled by photocells and a 360 tooth gear mounted on the flywheel. The wall temperature was measured at the positions $x = 0$, $x = L/2$ and 96 mm from the piston.

A further critical effect is the second harmonic of the piston motion. It is superimposed on the subharmonic resonance driven by the non–linear effect (equation (13)) and falsifies the result.

The critical amplitude e which corresponds to the driving term $f_A^2(t)$ can be calculated.

$$I_C = \frac{\pi(k+1)w\ell^2}{64a_0} = \frac{\pi(k+1)M\ell^2}{32} \tag{16}$$

A second harmonic part was determined with an accelerometer mounted on the moving piston. The error due to a second harmonic in the present experiments lies between 2 and 3%.

4. Results and Discussions

Figure 3 represents the shock domain in the r–s–plane. The points on the dotted lines are experimental results. Figures 4–6 illustrate a set of experiments for different tube lengths (L= 0.396 – 0.486 m) and frequencies. The tube was the with different heavy gases (to reduce the boundary–layer effect s $\rightarrow$ 0) includ Xenon, Sulfurhexafluoride SF and refrigerant RC 318 (Octafluorcyclobutan Excellent agreement between theory and experiments can be shown in the shock domain. The relative difference between the theoretical and exper results corresponds to the expected order of magnitude 0(M). We get much small variation between theoretical & experimental results.

These fundamental experiments are of significant theoretical interest the existence of a subharmonic resonance with shocks predicted by the me theory by J.J. Keller and consequently the breakdown of the acoustic a case for which the acoustic solution does not become singular. The striking result is that the correction of the acoustic solution (0(M) due to a resonant second–order term in the governing equation is of the same order as the acoustic solution itself.

Subharmonic resonance effects are often not observed in experiments because the attenuation (due to boundary–layer friction) of the resonant components is by one order of magnitude stronger than that of the acoustic components.

Another reason is that subharmonic resonance occurs in a relatively small frequency band $\left(\frac{\Delta w}{w} = 0\left(\frac{\ell}{L}\right)\right)$ compared with the fundamental resonat frequency $\left(\frac{\Delta w}{w} = 0\left(\sqrt{\frac{\ell}{L}}\right)\right)$.

A similar consideration can be made if the frequency of the piston lies in the hbourhood of one third (or two thirds of the fundamental frequency. However, third-order subharmonic resonance causes relatively small corrections $\left(0\left(\sqrt{M}\right)\right)$ only.

A experiment with refrigerant R 114 shows the third–order subharmonic reson (figure 7). The three shocks per period are strongly attenuated by the boundary–layer effect.

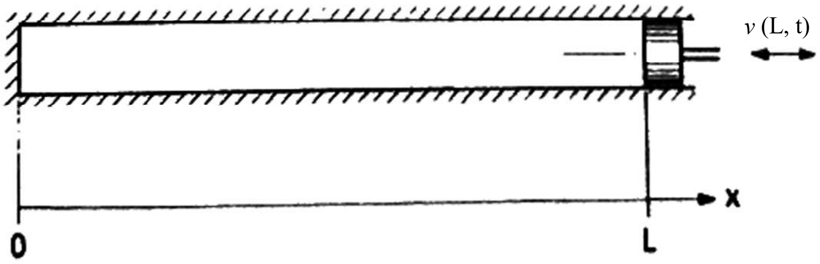

Figure 1: Gas–filled resonance tube

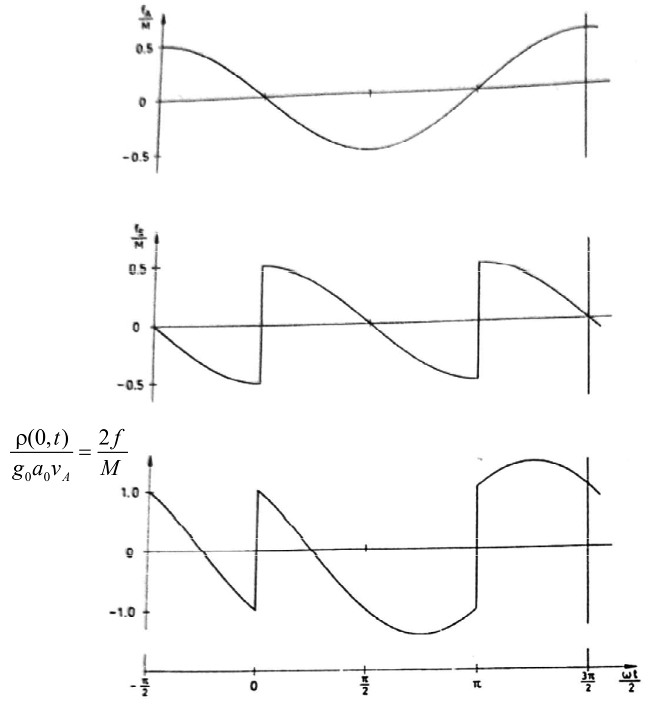

Figure 2. The function f_A, f_S and p versus the time for $s = 0$ and $r = 0$

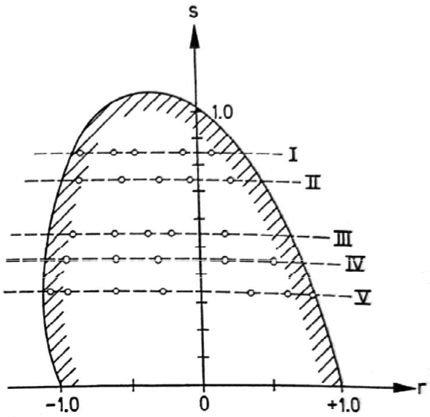

Figure 3. r– s – plane

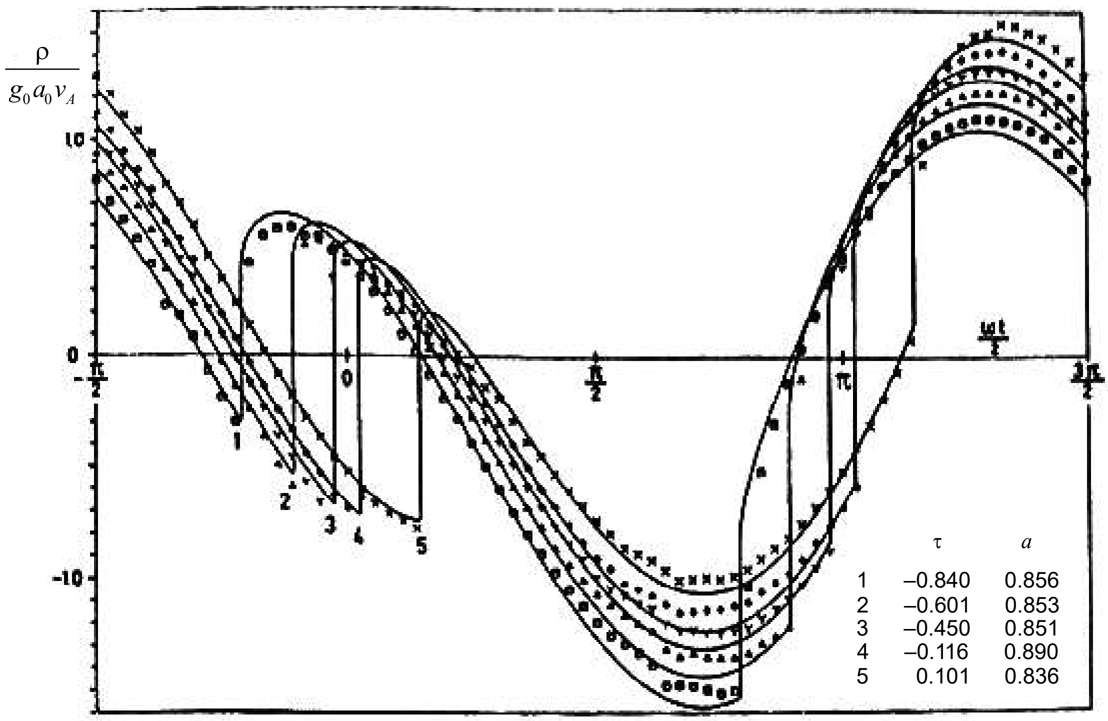

Figure 4. Pressure signals at the closed end for Xenon (1)

(L = 0.486 m, 1 = 12.58 mm)

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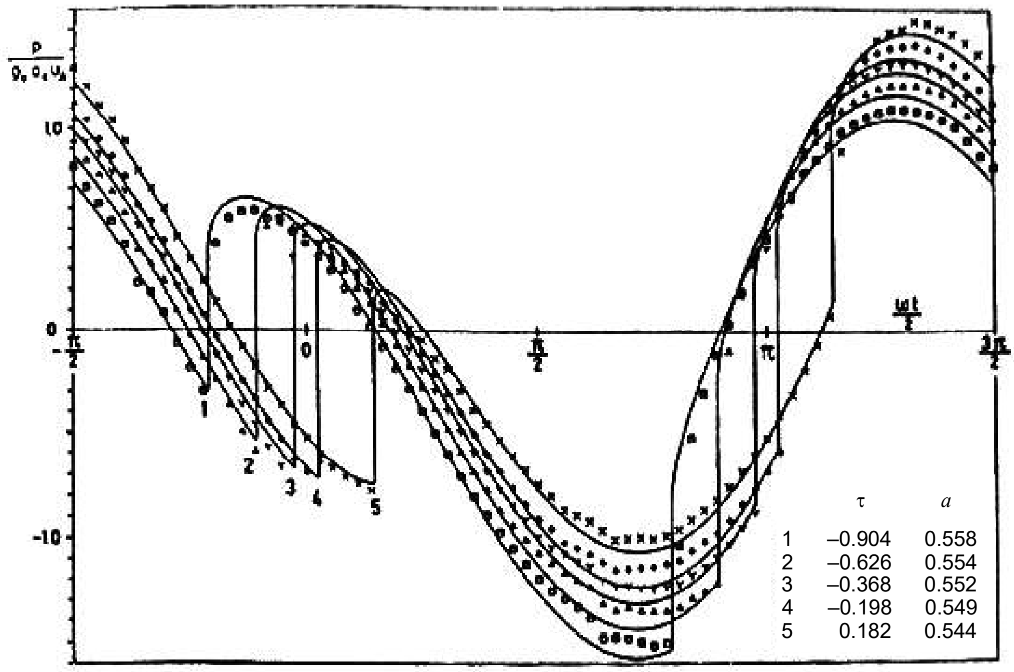

Figure 5. Pressure signals at the closed end for SF_6, (III)

(L = 0.476 m, I = 12.58 mm)

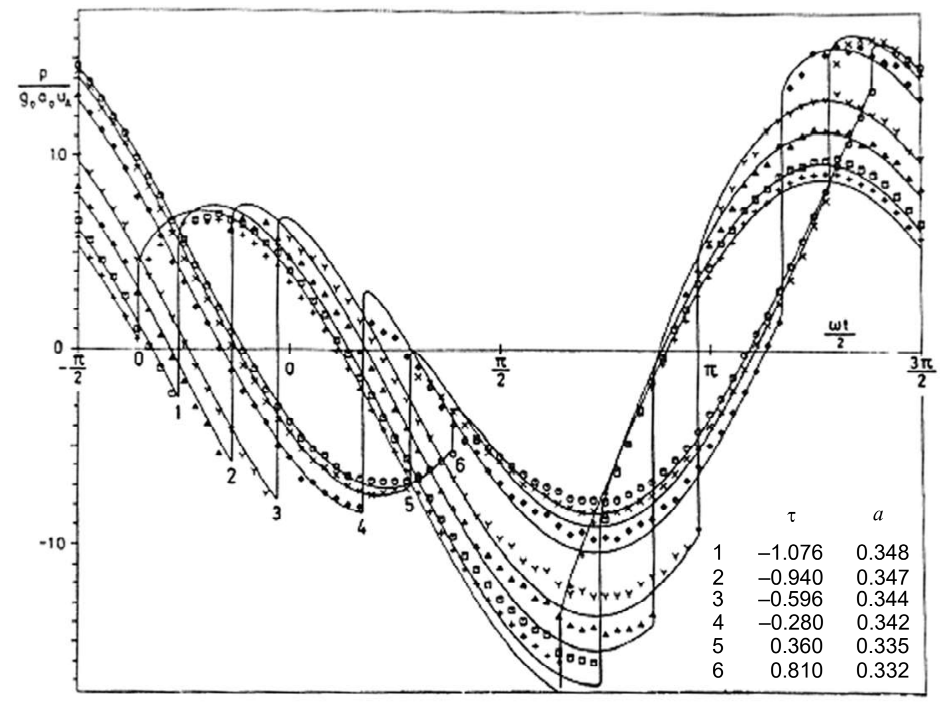

Figure 6. Pressure signals at the closed end for refrigerant RC 318 (V)

(Freon RC 318, L = 0.396 m, l = 12.58 mm)

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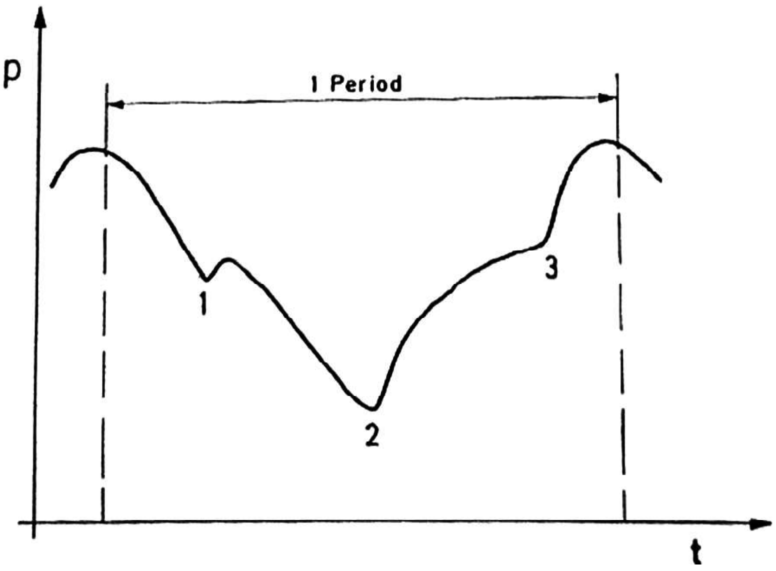

Figure 7. Secondo rubatormomis, resonance (Refrigerant R 114, L = 0, 407 m, 1 = 12.58 mm)

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