A Short Review on Poisson Distribution with Some Applications and Properties

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Abstract: ***There are three important Discrete Distributions that use integer as random variables are Binomial, Poisson and Hyper Geometric Distribution. In this paper, we thoroughly discuss about Poisson Distribution. It was named after French Mathematician Simeon Denis Poisson. The applications of Poisson Distribution are also described in this paper, and how it related to our daily life problems. Here we shall find out PMF, CDF, Mean, Variance etc. of Poisson distribution. In this paper we shall learn about the relation between Poisson and other Distributions.***

Keywords: Poisson Distribution, Probability, Mean, Variance.

1. Introduction

In real world we notice various wonders in which occasions happens pretty much at arbitrary. Occasions which happen perfectly at arbitrary are occasions which are not influenced by the event of different occasions; they are not correlated [1], So Poisson Distributionshows how often an occasion is probably going to happen inside a predetermined timeframe.It helps to predict the probability of certain events that will happen in future. It is denoted by Pois (λ). In Poisson Distribution the probability of an occurrence is constant over time and occurrences are independent. Poisson distribution is used where numbers of trials are large and probability of successive is small, where trials are independent. Poisson Distribution was initially derived as estimation to the Binomial Distribution. Poisson Distribution also called probability of rare events. The Poisson Distribution is a tool which is used in probability theory statistics. It is also a Discrete Probability Distribution and it is limiting form of Binomial Distribution. Poisson Distribution is positively skewed distribution (skewed means lack of symmetry) [5].

Poisson Distribution has many applications in different fields. For example, the quantity of clients who show up in a shop is free of the quantity of clients that show up in some other hour.Another example is, consider the lighting of cigarettes by a crowd of people viewing a film. From the start it seems, by all accounts, to be very irregular with individuals illuminating at their individual impulse. However, is it truly irregular? Does some individual choose to have a cigarette since he sees another light up? Will a strained scene cause various individuals to illuminate? On the off chance that such factors are available, at that point the occasions have some relationships thus don't happen clearly at arbitrary, a way to test for randomness is to setup a model for events occurring and from it to develop a theoretical distribution for a series of identical time intervals, then to observe whether or not the phenomena obey such a distribution over a large number of time intervals. Such a model leads to Poisson distribution [1].

2. Properties

2.1 PMF (Probability Mass Function)

A discrete irregular variable is said to have a Poisson Distribution with boundary λ>0, in the event that for x=0, 1, 2, …. at that point the likelihood mass capacity is given by

$$P(X) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad (1)$$

Where e is Euler's number (e=2.178…) and x is number of occurrences and x! is factorial of x[4].

Graphical representation of of pmf is shown in fig no. 1.1

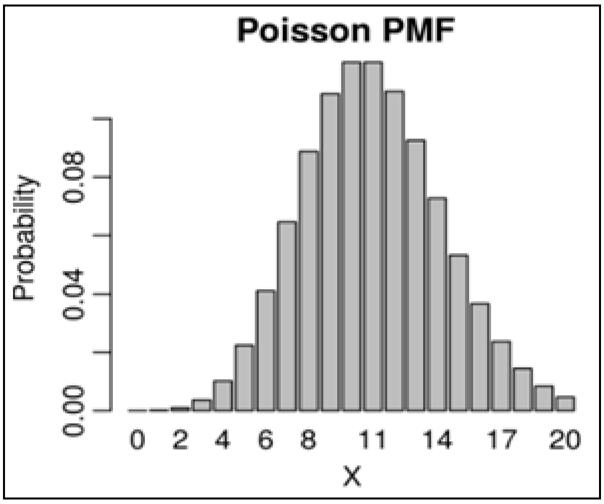

Figure 1.1

And the coding for this program is written given below

```
Barplot(height=dpois(0:20, lambda=11),
name.arg=0:20,
main="Poisson PMF",
xlab='X',
ylab='Probability')
```

2.2 CDF (Cumulative. Distribution Function)

The cdf of a genuine esteemed irregular variable X is the likelihood that X will take a worth not exactly or equivalent to x. At that point the cdf is given by [4]

$$F.(X) = e^{-\lambda} \sum_{i=0}^{\lfloor x \rfloor} \frac{\lambda^i}{i!}$$

Graphical representation of cdf is shown in given fig 1.2

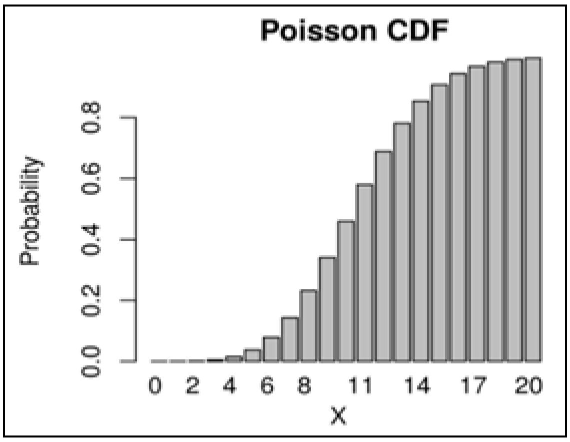

Figure 1.2

And the coding for this program is written given below

```
Barplot(height=ppois(0:20, lambda=11),
  name.arg=0:20,
  main="Poisson CDF",
 xlab='X',
ylab='Probability')
```

2.3 Mean

It is the normal estimation of Poisson distribution and is signified by E(X).

Mean= E(X) =. $\sum_x xP(X)$

$$= .\sum_x x \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{(From equation (1))}$$

$$= \sum_{x.} x \frac{\lambda^x e^{-\lambda}}{x(x-1)!}$$

$$= .e^{-\lambda} \sum_x \lambda \frac{\lambda^{x-1}}{(x-1)!}$$

$$= .e^{-\lambda}\, \lambda \sum_x \frac{\lambda^{x-1}}{(x-1)!}$$

$$= .e^{-\lambda}\, \lambda e^{\lambda}$$

$$E(X) = \lambda \quad \text{....... (2)}$$

2.4 Variance

The change of an arbitrary variable X is the normal estimation of the squared deviation from the mean X. It is denoted by Var(X) or σ^2

E.[X(X-1)] = $.\sum_x x(x-1)$P(X)

$$= .\sum_x x(x-1) \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{(From equation (1))}$$

$$= \sum_{x.} x(x-1) \frac{\lambda^2 \lambda^{x-2} e^{-\lambda}}{x(x-1)(x-2)!}$$

$$= \lambda^2 e^{-\lambda} \sum_x \frac{\lambda^{x-2}}{(x-2)!}$$

$$= \lambda^2 e^{-\lambda} e^{\lambda}$$

$$E.[X(X-1)] = \lambda^2 \quad \text{.... (3)}$$

Then

$$E[X^2] = E[X(X-1)] + E[X]$$

$$E[X^2] = \lambda^2 + \lambda \quad \text{....... (4)}$$

(From equation (2) and (3))

Var(X). = $E[X^2]$ - $(E[X])^2$(From equation (2) and (4))

$$= (\lambda^2 + \lambda) - \lambda^2$$

Var(X) = σ^2 = λ

$$\sigma = \sqrt{\lambda}$$

In Poisson Distribution the mean (which is equal to the variance) performs a function similar to that performed by the mean and standard deviation together in the Normal Distribution, for knowledge of the mean enables one to determine the distribution and all its moments [2].

2.5 Moment Generating Function (MGF)

In statistics the mgf of a genuine esteemed irregular variable is an elective determination of its probability distribution. It is generally denoted by M_X(t).

M_X(t) = E(e^{xt}) = $\sum_x e^{xt} P(X)$

$$= \sum_x e^{xt} \frac{\lambda^x . e^{-\lambda}}{x!}$$

$$= .e^{-\lambda} \sum_x \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e t}$$

$$= e^{-\lambda + \lambda e t}$$

M_X(t) = $e^{\lambda(e^{t-1})}$

3. Some Real-Life Applications of Poisson Distribution

1) A book store rent an average of 150 books every Sunday night. By the help of this data, we can predict the probability that the more books will sell (perhaps 250 0r 350) on following Sunday nights. Poisson Distribution is also used by businessmen. In business overstocking means losses if the goods are not sold also if the stock is less that means you are not able to maximize your stock during the sale. So, Poisson Distribution helps to predict that when the interest is peculiarly higher so they can buy more stock [6].
2) Every week, on average, 20 people clap for my blog post. I would like to predict the # of people who would clap next week because I get paid weekly by those numbers. What is the probability that exactly 20 people will clap for the blog post next week? [3].
3) Consider a simple emergency room example where 2 patients show up on normal each 10 minutes (this is comparable to 0.2 patients per one minute). Successive appearances are measurably free. This implies that each given appearance has no effect on the likelihood of next appearances. Letting N(t) speak to the quantity of such appearance in t minutes, one can assess the accompanying probabilities, utilizing the Poisson Distribution [7].
4) Customers Purchasing a Commodity. Suppose that customers arrive at an amusement park according to a Poisson process of intensity λ; each customer must pay $1 to enter the park. If the value of price is disconnected back to time t=0 according to an Exponential Distribution with rate β, we can determine the total expected value of money collected [8].

4. Relation between Poisson Distribution and Binomial Distribution

The Binomial Distribution inclines toward the Poisson distribution as n→∞. p→0 and λ= np says steady.

Now the formula for Binomial is

$$P(X=x) = \binom{n}{x}.p^x(1-p)^{n-x} \qquad \text{..... (5)}$$

Where the mean of Binomial Distribution is **np**

Now we know np =λ

$$p = \frac{\lambda}{n}$$

Put the value of p in equation (5), then we get

$$= .\binom{n}{x}\left(\frac{\lambda}{n}\right)^x.\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{.n!}{x!.(n-x)!}.\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda^x}{x!}.\frac{n!}{(n-x)!}.\frac{1}{n^x}\left(.1-\frac{\lambda}{n}\right)^{n-x}$$

$$= .\frac{\lambda^x}{x!}\left(.1-\frac{1}{.n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\ldots\left(1-.\frac{x+1}{n}\right)\left(1-\frac{\lambda}{n}\right)^{.n-x}$$

$$.= .\frac{\lambda^x}{x!}\left(.1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\ldots\left(1-.\frac{x+1}{n}\right)\left(1-.\frac{\lambda}{n}\right)^n\left(1-.\frac{\lambda}{n}\right)^{-x}$$

Now take limit as n→ ∞, we get

$$= .\frac{\lambda^x}{x!}*1.*e^{-\lambda}*1$$

$$= .\frac{\lambda^x e^{-\lambda}}{x!}$$

This is the Poisson Distribution.

5. Literature Survey

In 1972 M.S Lafleur et.al [1], they studied about how they can challenge the students to use the distribution function. So, they got a distribution that was a sensible fit to the Poisson Distribution, and the reaction to this test was in a way that is better than to insights analyze which included tossing dice or flipping coins. Nonetheless, it was clear to the students that this examination was picked on the grounds that the appropriation would be Poisson and thusly they had minimal motivation to play out the dreary assignment of information gathering, particularly when they realized that numerous different students had gathered the very same information.

Jerzy Letkowski [7] brings issues to light of a few application openings and to give more finish case examination of the Poisson Distribution. He learned about how to execute regular use cases, utilizing Google spread sheet, a cloud computing, information examination apparatus. He wants to create Poisson Probability Distribution applications in a cloud. He examined in his paper about academic issues, with respect to difficulties in showing Statistics and using the accounting page innovation.

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In 2013 Phillip Minogla [8], the motivation behind his paper is to make an investigation so as to more readily comprehend the Poisson and related cycles and utilizations of these stochastic cycles. He would accumulate a few outcomes that he accepts would be helpful and open to undergraduate math students. His paper contains primer data about binomial and Poisson arbitrary factors, and a consequence of examination. He likewise referenced a few properties of Poisson Distribution.

6. Conclusion

In this paper we have studied about Poisson Distribution with parameters, it discloses to us that how frequently an occasion is happen in a given timeframe. The paper mainly focuses on the properties of the Poisson Distribution with coding and graphical representation of some of the properties with the help of R programming language. We illustrate the practical applications of Poisson Distribution in the areas like natural and in business area. In this paper we learnt about the relation of Poisson and Binomial Distribution.

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