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Modeling and Parameter Identification of an Unmanned Ground Vehicle

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Abstract: ***This paper deals with the modeling of an unmanned ground vehicle with two degrees of freedom 2DOF. Model parameter was identified experimentally using linear and yaw uniform motions.***

Keywords: unmanned ground vehicle, dynamic model, parameter identification

1. Introduction

A mathematical model is used to simulate the behaviour of a vehicle and in order to facilitate the design of controllers and system guidance.

In previous works such as [1] [2], non-linear control theory based on feedback linearization for single-input-single-output systems are designed for self-steering an unmanned wheeled vehicle.

In [3] [4], authors discuss feedback linearization for multi-input multi-output systems

In addition. Fossen [5] goes in depth on the theory of motion control systems.

In [6, 7, 8, 9], authors provide a complete description on vehicle dynamic models. More than that, papers [10,11] deal with the measurement and the estimation of a vehicle sideslip angle.

There are several ways to develop a dynamic model for a ground vehicle such as Newton Euler formula [12]:

"*The direction of the acceleration of an object is in the direction of the net external force acting on it. The acceleration of an object is proportional to the net external force* $\overrightarrow{F_{net}}$ *, in accordance with* $\overrightarrow{F_{net}} = m\,\vec{a}$ *. The net force acting on an object is the vector sum of all forces acting on it:* $\overrightarrow{F_{net}} = \sum \vec{F}$*. Thus,*

$$\sum \vec{F} = m\,\vec{a}$$ "

For rotational motion the same principals apply, but with inertia and momentsinstead of mass and force. Newton's second law for translational motion and rotational motion can be presented as :

$$\sum \vec{F} = \frac{d}{dt} m\vec{V}$$
$$\sum \vec{M} = \frac{d}{dt} J\vec{\Omega}$$

where $\vec{F}$ is the coordinate free force vector acting on the center of gravity and $\vec{V}$ isthe velocity of the center of gravity with respect to the inertial frame.

The remainder of the paper is structured as follows:

The second section deals with the development of a mathematical model with 2DOF to simulate the behavior of the vehicle. The modeling parameters were calculated experimentally during the third section. The proposed approach for damping model identifying represents a contribution in this paper.

Figure 1: The considered wheeled vehicle

2. Dynamic model of the vehicle

2.1. Reference systems

Let define two coordinate systems as follows: (O, X_0, Y_0, Z_0) the inertial reference frame $R_0.(G, X, Y, Z)$ the body frame R as shown in figure 2.

The origin of the body frame is chosen atthe vehicle center of gravity G. The (GX) axis is pointed towards the front of the vehicle, the (GY) one is pointed towards the left side of the vehicle and the (GZ) one is chosen oriented towards the high.

Let $(\vec{\imath}, \vec{\jmath}, \vec{k})$ be the unit vectors of the (GX), (GY) and (GZ) axis respectively.

Let $(\overrightarrow{\imath_0}, \overrightarrow{\jmath_0}, \overrightarrow{k_0})$ be the unit vectors of the (OX_0), (OY_0) and (OZ_0) axis respectively as shown in figure 2:

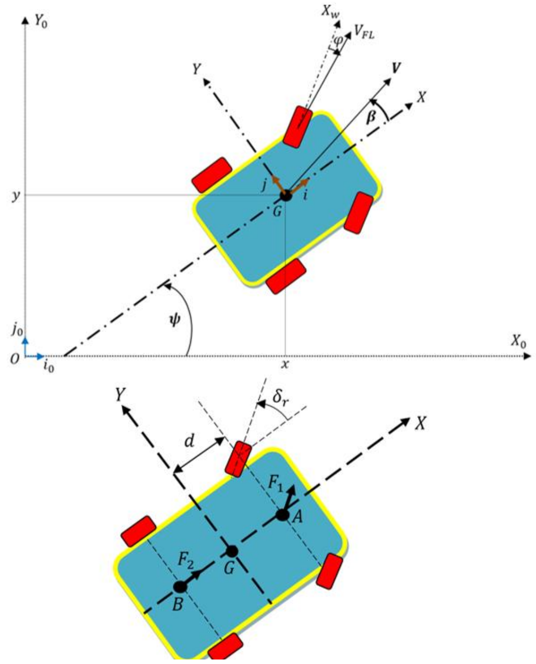

Figure 2: Geometry model of the considered vehicle

The considered vehicle is equipped with four identical wheels. They are driven by four identical electric motors. The two rear wheels create a pushing force $\vec{F_2}$ applied at point B. The two front wheels create a thrust force $\vec{F_1}$ applied at point A. The vehicle orientation ψ (heading)is controlled using the steering angle of the front wheels $\delta_r \in [-\delta_{r_{max}}, \delta_{r_{max}}]$.

2.2. Assumptions

Assumptions used in this paper are :

- The vehicle is considered as a rigid body moving on a flat surface.
- The vehicle body frame is defined with the origin located at the center of mass of the vehicle.
- The speed of the considered vehicle is very low (less than 1.6 m/s), therefore, the slip angles β and φ (see figure 2) are small. The sideways wheel forces acting in the contact point between the tire and the ground is also neglected. Only the longitudinal wheel forces are considered.
- The aerodynamic drag is defined along the longitudinal axis of the vehicle.
- The frictional force toward the longitudinal movement and the frictional force around the yaw axis are supposed decoupled.

2.3. System modeling

The notations used in this paper are described in Table 1.

Table 1: Used notation and description

Notation	Description
$\boldsymbol{x}, \boldsymbol{y}$	Horizontal coordinates of G in R_0
$\boldsymbol{\psi}$	Heading angle
$\boldsymbol{u}$	Velocity in surge motion
$\boldsymbol{v}$	Velocity in sway motion
$\boldsymbol{r}$	Yaw rate
$\boldsymbol{\delta_r}$	Steering angle
$\vec{F_1}$	Forward propulsion force
$\vec{F_2}$	Rear propulsion force
$\vec{N}$	Moment due to $\vec{F_1}$ around (GZ) axis
$\vec{f}$	Linear damping
$\vec{R}$	Turn damping
$\boldsymbol{m}$	Mass of the vehicle
$\boldsymbol{I_z}$	Moment of inertia around (GZ) axis
$\boldsymbol{J}$	The inertia matrix
$\boldsymbol{d}$	Distance between G and point A

The velocity of the vehicle $\vec{V}$ is given by:

$$\vec{V} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}_R = \begin{pmatrix} \dot{x} \\ \dot{y} \\ 0 \end{pmatrix}_{R_0} \quad (1)$$

The resultant force expressed in R and R_0 are:

$$\vec{F} = \begin{pmatrix} F_x \\ F_y \\ 0 \end{pmatrix}_R = \begin{pmatrix} F_{x_0} \\ F_{y_0} \\ 0 \end{pmatrix}_{R_0} \quad (2)$$

The resultant moment expressed in R_0 is:

$$\vec{M} = \begin{pmatrix} 0 \\ 0 \\ N \end{pmatrix}_{R_0} \quad (3)$$

By applying the Newton second law of motion, we obtain:

$$\begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix} = \begin{pmatrix} m.\vec{a} \\ \vec{\delta} \end{pmatrix} \quad (4)$$

With

$\vec{a} = \frac{d\vec{V}}{dt}$: Linear acceleration.

$\vec{\delta} = \frac{d\vec{\sigma}}{dt}$: Dynamic moment.

$\vec{\sigma} = J\vec{\Omega}$: Kinetic moment

J : inertia matrix of the vehicle

$\vec{\Omega}$: angular velocity of the center of gravity G in the inertial frame.

The linear acceleration $\vec{a}$ is thus written as follows:

$$\vec{a} = \ddot{x}.\vec{\iota_0} + \ddot{y}.\vec{J_0} \quad (5)$$

and the angular acceleration is noted $\ddot{\psi}$

Consequently, equation (4) becomes:

$$\begin{cases} F_{x_0} = m.\ddot{x} \\ F_{y_0} = m.\ddot{y} \\ N = I_z.\ddot{\psi} \end{cases} \quad (6)$$

With I_z the moment of inertia around the (GZ) axis.

The damping force $\vec{f}$ in surge motion and the angular damping force $\vec{N_f}$ are given by:

$$\vec{f} = -g(u)\vec{\imath} \quad (7)$$

$$\vec{N_f} = -h(r)\vec{k} \quad (8)$$

With

$g(u)$: damping model in surge motion

$h(r)$: damping model in yaw motion

We obtain thus:

$$\begin{cases} m\ddot{x} = f_{\vec{\iota_0}} + F_{1_{\vec{\iota_0}}} + F_{2_{\vec{\iota_0}}} \\ m\ddot{y} = f_{\vec{J_0}} + F_{1_{\vec{J_0}}} + F_{2_{\vec{J_0}}} \\ I_z\ddot{\psi} = N_{f_{\vec{k_0}}} + N_{1_{\vec{k_0}}} \end{cases} \quad (9)$$

Where

- $F_{1_{\vec{\iota_0}}}, F_{2_{\vec{\iota_0}}}$ are the components of $\vec{F_1}$ and $\vec{F_2}$ respectively along (OX_0) axis.
- $F_{1_{\vec{j_0}}}, F_{2_{\vec{j_0}}}$ are the components of $\vec{F_1}$ and $\vec{F_2}$ respectively along (OY_0) axis.
- $f_{\vec{\iota_0}}, f_{\vec{j_0}}$ are the components of the damping force $\vec{f}$ along (OX_0) and (OY_0) axis respectively.
- $N_{1_{\vec{k_0}}}$ is the moment created by the force $\vec{F_1}$ around the (OZ_0)axis.
- $N_{f_{\vec{k_0}}}$is the component of the moment $\vec{N_f}$around the (OZ_0) axis.

In the body frame we have:

$$\vec{F_1} = \begin{pmatrix} F\cos(\delta_r) \\ F\sin(\delta_r) \\ 0 \end{pmatrix}_R \quad (10)$$

$$\vec{F_2} = \begin{pmatrix} F \\ 0 \\ 0 \end{pmatrix}_R \quad (11)$$

$$\vec{f} = \begin{pmatrix} -g(u) \\ 0 \\ 0 \end{pmatrix}_R \quad (12)$$

By using the following transformation matrix

$$P_{R_0 \to R} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (13)$$

We get

$$\vec{F_1} = \begin{pmatrix} F\cos(\delta_r + \psi) \\ F\sin(\delta_r + \psi) \\ 0 \end{pmatrix}_{R_0} \quad (14)$$

$$\vec{F_2} = \begin{pmatrix} F\cos(\psi) \\ F\sin(\psi) \\ 0 \end{pmatrix}_{R_0} \quad (15)$$

$$\vec{f} = \begin{pmatrix} -g(u)\cos(\psi) \\ -g(u)\sin(\psi) \\ 0 \end{pmatrix}_{R_0} \quad (16)$$

Moment $\vec{N_1}$ is calculated as follows:

$$\vec{N_1} = \vec{F_1} \wedge \vec{AG} \quad (17)$$

With

$$\vec{AG} = (-d \quad 0 \quad 0\,)_R = (-d\cos(\psi) \; -d\sin(\psi) \quad 0\,)_{R_0} \quad (18)$$

Moment $\vec{N_1}$ becomes:

$$\vec{N_1} = \begin{pmatrix} F\cos(\delta_r + \psi) \\ F\sin(\delta_r + \psi) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -d\cos(\psi) \\ -d\sin(\psi) \\ 0 \end{pmatrix} = dF\sin(\delta_r)\,\vec{k_0} \quad (19)$$

Consequently we get :

$$\begin{cases} m\ddot{x} = F\cos(\delta_r + \psi) + F\cos(\psi) - g(u)\cos(\psi) \\ m\ddot{y} = F\sin(\delta_r + \psi) + F\sin(\psi) - g(u)\sin(\psi) \\ I_z.\ddot{\psi} = dF\sin(\delta_r) - h(r) \end{cases} \quad (20)$$

With :$r = \dot{\psi}$

By dividing by the maximum applied force F_{max} we get :

$$\begin{cases} \frac{m\ddot{x}}{F_{max}} = \frac{F\cos(\delta_r+\psi) + F\cos(\psi) - g(u)\cos(\psi)}{F_{max}} \\ \frac{m\ddot{y}}{F_{max}} = \frac{F\sin(\delta_r+\psi) + F\sin(\psi) - g(u)\sin(\psi)}{F_{max}} \\ \frac{I_z.\ddot{\psi}}{F_{max}} = \frac{dF\sin(\delta_r) - h(r)}{F_{max}} \end{cases} \quad (21)$$

Let us pose:

- ✓ $m' = \frac{m}{F_{max}}$
- ✓ $I_z' = \frac{I_z}{F_{max}}$
- ✓ $F' = \frac{F}{F_{max}}$

We get

$$\begin{cases} m'\ddot{x} = F'\cos(\delta_r + \psi) + F'\cos(\psi) - g(u)\cos(\psi) \\ m'\ddot{y} = F'\sin(\delta_r + \psi) + F'\sin(\psi) - g(u)\sin(\psi) \\ I_z'\ddot{\psi} = dF\sin(\delta_r) - h(r) \end{cases} \quad (22)$$

and since we have

$$\vec{V} = \dot{x}.\vec{\iota_0} + \dot{y}.\vec{j_0} = u.\vec{\iota} + v.\vec{j} \quad (23)$$

and

$$\begin{cases} \vec{\iota_0} = \cos\psi.\vec{\iota} - \sin\psi.\vec{j} \\ \vec{j_0} = \sin\psi.\vec{\iota} + \cos\psi.\vec{j} \end{cases} \quad (24)$$

We get

$$\begin{cases} u = \dot{x}\cos\psi + \dot{y}\sin\psi \\ v = -\dot{x}\sin\psi + \dot{y}\cos\psi \end{cases} \quad (25)$$

Time derivative of equations (25) gives:

$$\begin{cases} \dot{u} = \ddot{x}\cos\psi - \dot{x}.\dot{\psi}.\sin\psi + \ddot{y}.\sin\psi + \dot{\psi}.\dot{y}.\cos\psi \\ \dot{v} = -\ddot{x}\sin\psi - \dot{x}.\dot{\psi}.\cos\psi + \ddot{y}.\cos\psi - \dot{\psi}.\dot{y}.\sin\psi \end{cases} \quad (26)$$

It is equivalent to

$$\begin{cases} m'.\dot{u} = m'\ddot{x}\cos\psi - m'\dot{x}.\dot{\psi}.\sin\psi + m'\ddot{y}.\sin\psi + m'\dot{\psi}.\dot{y}.\cos\psi \\ m'.\dot{v} = -m'.\ddot{x}\sin\psi - m'.\dot{x}.\dot{\psi}.\cos\psi + m'.\ddot{y}.\cos\psi - m'\dot{\psi}.\dot{y}.\sin\psi \end{cases} \quad (27)$$

So, we get:

$$\dot{u} = \frac{F'(1+\cos(\delta_r)) + m'.r.v - g(u)}{m'} \quad (28)$$

The dynamic model of the vehicle is thus given by the following differential equations:

$$\begin{cases} \dot{x} = u\cos\psi \\ \dot{y} = u\sin\psi \\ \dot{\psi} = r \\ \dot{u} = \frac{\tau_1 - g(u)}{m'} \\ \dot{r} = \frac{\tau_2 - h(r)}{I_z'} \end{cases} \quad (29)$$

With

$$\begin{cases} \tau_1 = F'(1 + \cos(\delta_r)) \\ \tau_2 = dF'\sin(\delta_r) \end{cases} \quad (30)$$

The developed model is nonlinear. It has the form:$\dot{X} = f(X, U)$.

Where $X = (xy\psi ur)^T$ is the state vector and $U = (F' \quad \delta_r)^T$ is the input vector.

In the following section, we propose an approach for damping identification $g(u)$ and $h(r)$.

3. Parameter identification

3.1 Determination of the damping model $g(u)$

The damping model $g(u)$ in surge motion can be determined using linear uniform motion: This motion is obtained by applying each time a thrust force while keeping the steering angle $\delta_r = 0$. It leads to a steady state characterized by a constant speed in surge $\dot{u} = 0$.

In this trial, the elapsed time Δt, taking by the vehicle to travel a known distance D during the steady state, is determined.
So we have:

$$\dot{u} = \frac{\tau_1 - g(u)}{m'} = 0 \tag{31}$$

Meaning that

$$\tau_1 = g(u) \tag{32}$$

All data corresponding to this trial is presented in Table 2.

Table 2: Experimental data corresponding to turning trial

	$D = 10\ m$										
	Trial 1	Trial 2	Trial3	Trial4	Trial5	Trial6	Trial7	Trial8	Trial9	Trial 10	Trial11
τ_1	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$\Delta t\ (s)$	0	79.16	35.3	25.5	18	13.9	11.3	9.26	7.9	6.7	6
$u\ (m.s^{-1})$	0	0.12	0.28	0.39	0.55	0.72	0.88	1.03	1.26	1.49	$u_{max} = 1.66$

By plotting $\tau_1 = g(u)$,we get the red points shown on figure 4.

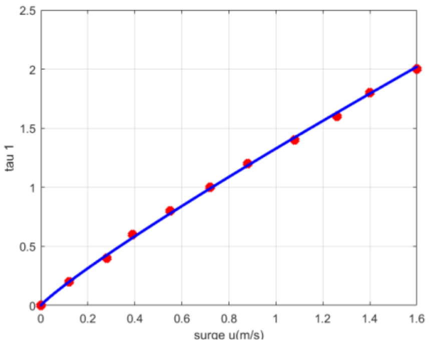

Figure 3: Damping model of the linear motion

The damping model of the linear motion can be thus approximated by $g(u) = \alpha_1 u^{\beta_1}$ (the curve presented with blue color on figure 4). With $\alpha_1 > 0$ and $\beta_1 \in [0,1]$ that were calculated using the Least mean squares (LMS) method [13][14].

3.3 Determination of the yaw damping model $h(r)$

This trial consists on making the vehicle to turn with a uniform yaw motion (the thrust force is chosen to be constant $F' = 0.7$ during this trial). The turn motion is obtained by setting a constant steering angle δ_r letting the vehicle turn with a constant yaw rate r. The yaw rate r is measured by counting the elapsed time needed to accomplish a complete turn (360°).

The angular acceleration $\dot{r}$ is given by :

$$\dot{r} = \frac{dF' \sin(\delta_r) - h(r)}{I_z'} \tag{33}$$

However, when the vehicle is turning, its yaw rate r is constant meaning that $\dot{r} = 0$.
So,

$$\tau_2 = dF' \sin(\delta_r) = h(r) \tag{34}$$

All data corresponding to this trial is presented in Table 3.

Table 3 : Experimental data corresponding to turning trial

	$F' = 0.7$				
δ_r (°)	5	10	15	20	25
Elapsed time Δt (s)	83.34	71	41.54	29.37	23.9
r (rad/s)	0.038	0.088	0.15	0.22	0.265
τ_2	0.02	0.042	0.064	0.086	0.1

The curve shown on figure 4, represents the yaw moment τ_2 as a function of the angular speed r. Points in green color are the result of the experimental tests presented in Table 3. The curve in pink color represents an approximation of the yaw damping model.

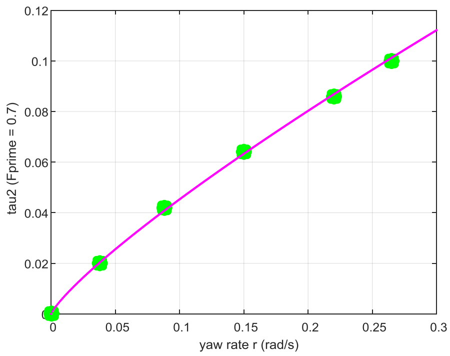

Figure 4: Turndamping at 70 % of the total thrust forse

$h(r)$ can be therefore approximated by :

$$h(r) = \alpha_2 r^{\beta_2} \tag{35}$$

Where α_2 and β_2 are calculated using the same method as the previous section.

3.4 Calculation of the inertia moment I_z'

The vehicle is assumed to be assimilated to a homogeneous rectangular parallelepiped as shown in figure 6 with the following dimensions:
Length $b = 1.32\ m$
Width $a = 0.78\ m$
Height $c = 0.53\ m$
Masse $m = 37\ Kg$

The inertia moment I_z of the vehicle around the the vertical axis is calculated using a numerical simulator. It is equal to :

$$I_z = 6.06\ \text{Kg}\,.m^3 \tag{37}$$

3.5 Model validation

3.5.1 First, trial : Linear motion

This Trial consists of applying the maximum thrust force $\tau_1 = 2$ without making a steering angle $\delta_r = 0$. When the linear uniform motion is obtained, the speed of the vehicle stabilizes at the maximum value u_{max} , as shown in figure 5.

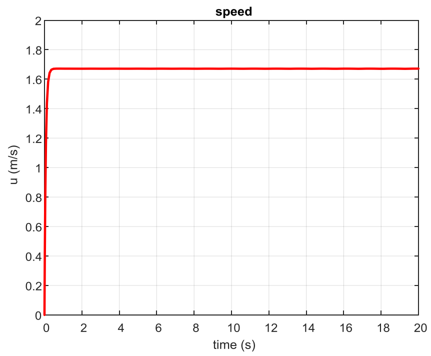

Figure 5: Time evolution of the vehicle Speed

3.5.2 Second trial: turning motion

This test consists on applying to the vehicle a constant thrust force $F' = 0{,}7$and a steering angle $\delta_r = 10°$. The vehicle will have to turn, so its trajectory will be a circle (figure 6), its speed will stabilize at a limit speed called turning speed (figure 8) and its yaw rate r (figure 7) will converge towards a constant value.

The simulation results obtained are presented below, they confirm the real behavior of the vehicle:

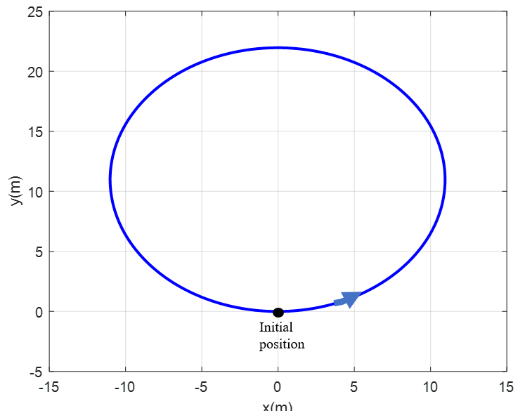

Figure 6: Trajectory traveled by the vehicle

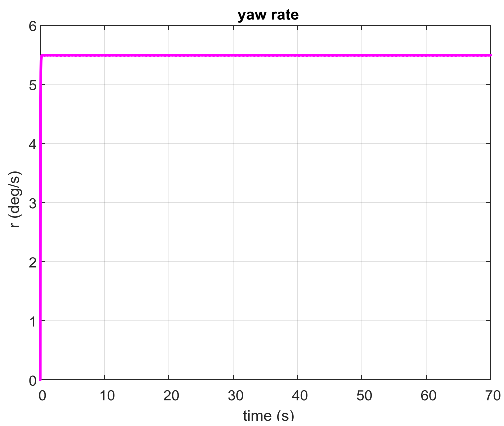

Figure 7: Time evolution of the yaw rate

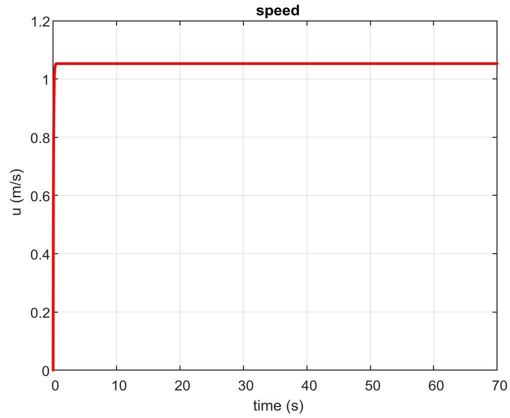

Figure 8: Time evolution of the vehicle speed

4. Conclusion

In this paper, we have identified the damping model for linear and turning motions of the wheeled vehicle designed model. Then a simulator was developed to validate the mathematical model of the vehicle. As afuture work,control and guidance systems will be designed for self-steering the considered vehicle

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