International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

A Mathematical Model for Estimating Varied Irrigation Efficiency

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Abstract: ***This study proposed a mathematical model for estimating irrigation efficiency. The proposed model considered the previous situation, and current situation and introduces up-to-date concepts in assessing irrigation efficiencies to manage water resources more efficiently and productively. The proposed model not only determines overall irrigation efficiency but covers all aspects of improvement factors such as the capacity of stakeholders, product costs, etc. The approaching model was applied to determine of irrigation efficiency of the Nam Houm Irrigation Scheme, (in the Central region of Lao PDR). The results obtained that the mathematical models can be used to estimate varying irrigation efficiencies, given the total available water resources, farmer participation, and previous irrigated area. The model provided the optimal condition of the irrigation efficiencies which are close to the actual irrigation efficiency.***

Keywords: Mathematical models, varied irrigation efficiency, farmer participation, water resources management

1. Introduction

Irrigation Efficiency is an obvious issue for agricultural development. Improvement irrigation efficiency not only can improve equity in water distribution but also can minimize the gap between crop water requirements and actual water use. In consequence, it will lead to determining the effectiveness of water use and lead to improving the livelihood of people. Generally, irrigation efficiency is the overall system efficiency which affecting by conveyance, distribution, and paddy field application. Most previous studies mainly analyzed field-level efficiencies and considered irrigation efficiency as a constant value for all seasons [1],[2]. However, there are not enough reliable to estimate irrigation efficiencies and actual water use.

However, it is likely that the efficiencies tend to vary due to the uncertainty of water resources. The participation of stakeholders in water resources management is the main effect of irrigation efficiencies [3]. Hence, the participation of stakeholders in water resources management is an important factor for estimating irrigation efficiency.

Multiple regression is a mathematical relationship that describes a system by a set of variables and a set of equations that establish relationships between the variables. The values of the variables can be practically anything such as mathematical relationships between the advance time and distance from the stream end using a regression function for analysis [4]. The multiple regressions can be a useful tool to determine the irrigation timing as a function of environmental and operational conditions (e.g. working pressure, air temperature, relative humidity, etc.) in order to minimize evaporation and drift loss [5]. A mathematical model is search and optimization techniques based on linear and non-linear multiple equations. The mathematical model is a method for searching the memberships of functions and it was applied to solve the optimal solution of water resources and varied irrigation efficiencies problems [6]. In addition, this mathematical model is relatively easy to explain and understand.

This study thus proposes the mathematical model for finding the varied efficiency corresponding to seasonal inflow, farmer participation in water resources management, land area cultivation in the previous season, and cost of crop production. The relationship between input and output variables is defined from a mathematical formula, according to human processes in thinking and decision

2. Model Formulation

Multiple regression (or, more generally, "regression") allows researchers to examine the effect of many different factors on some outcomes at the same time. The general purpose of multiple regressions is to describe the relationship between several independent or predictor variables and a dependent variable. For some kinds of research questions, a regression can be used to examine how much a particular set of predictors explains differences in some outcomes.

Thus, multiple regressions are usually composed by variables, which are abstractions of quantities of interest in the described systems, and operators that act on these variables, which can be algebraic operators, functions, differential operators, etc. If all the operators in a mathematical model present linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise.

In order to account for any uncertainty on seasonal inflow, the land area of cultivation, product cost, and farmer participation in water resources management, the mathematical model by using linear and non-linear multiple equations for estimating varied irrigation efficiency. System inputs include the seasonal inflow, seasonal requested area, participation of stakeholders, and product cost. Output is the seasonal irrigation efficiency from various seasonal operation factors.

There are three steps in developing a mathematical model as the following:

The first step of creating a mathematical model is to test the variable's input. If all variables in the mathematical model present linearity, the resulting mathematical model is defined as linear and if otherwise will be considered as non-linear. The mathematical formula for estimating variables is:

$$y_{ij} = a * X_1^{b_1} * X_2^{b_2} * X_3^{b_3} * ..... * X_m^{b_m} \quad (1)$$

where y_{ij} is estimated irrigation efficiency of scenario *i* during the season *j; a* and b_i are parameters of multiple functions during the scenario *i (i=1,2,3…m); X_1* is seasonal variable of total irrigated areas; X_2 is seasonal variables of farmer participation in water resources management; X_3 is seasonal variable of inflow; X_4 is seasonal product cost; and X_m is seasonal variables input for estimating parameters of the mathematical model of scenario *m.*

The secondly, the parameters of multiple functions were used to estimate irrigation efficiencies on each historical data. The actual historical data of irrigation efficiency will be presented in the next section.

Finally, the mathematical model was evaluated by considering the coefficient of correlation (r) and coefficient of determination (R^2) which define base on the actual irrigation efficiency and estimated irrigation efficiency from the model as:

$$R^2 = \frac{\left(\sum_{j=1}^{n} e_j \hat{e}_j - n\bar{e}_j \bar{\hat{e}}_j\right)^2}{\left(\sum_{j=1}^{n} e_j^2 - n\bar{e}_j^2\right)\left(\sum_{j=1}^{n} \hat{e}_j^2 - n\bar{\hat{e}}_j^2\right)} \quad (2)$$

where e_j is the estimated irrigation efficiency of the scenario during season *j* which is calculated using a mathematical model, $\hat{e}_j$ is the actual irrigation efficiency of the scenario during season *j* which is calculated from irrigated area, participation of stakeholders, product cost and value of available water, $\bar{e}$ and $\bar{\hat{e}}$ are respectively the average of above mentions and *n* is the number of annual historic data.

Generally, irrigation efficiency is the overall system efficiency which affecting by conveyance, delivery and field application efficiencies [7],[8]. The actual irrigation efficiency of the system can be computed by the following equation:

$$\hat{e}_j = 100\left(\frac{V_{net}}{V_{gross}}\right) \quad (3)$$

where: V_{net} is the net volume of crop water requirement (m^3), V_{gross} is the gross water diverted from the source to the conveyance system (m^3).

The net value of crop water requirement is computed as:

$$V_{net} = \sum_{j=1}^{j}\sum_{k=1}^{K} CWR_{jk} X_{jk} \quad (4)$$

$$CWR = WR_{vu} \text{ x Stage of crop development} \quad (5)$$

where: CWR_{jk} is crop water requirement rate of crop *k* during season *j* (mm/ha), X_{jk} is cropped area of crop *k* during season *j*(ha), WR_{vu} is crop water requirement (mm/dayx10^{-3}) of crop type *u* at the day *v*.

3. Illustrative Application

The 18-year (1989-2007) of seasonal flow, irrigated area, irrigation efficiency, crop water requirement, related evaporation and effective rainfall of the Nam Houm

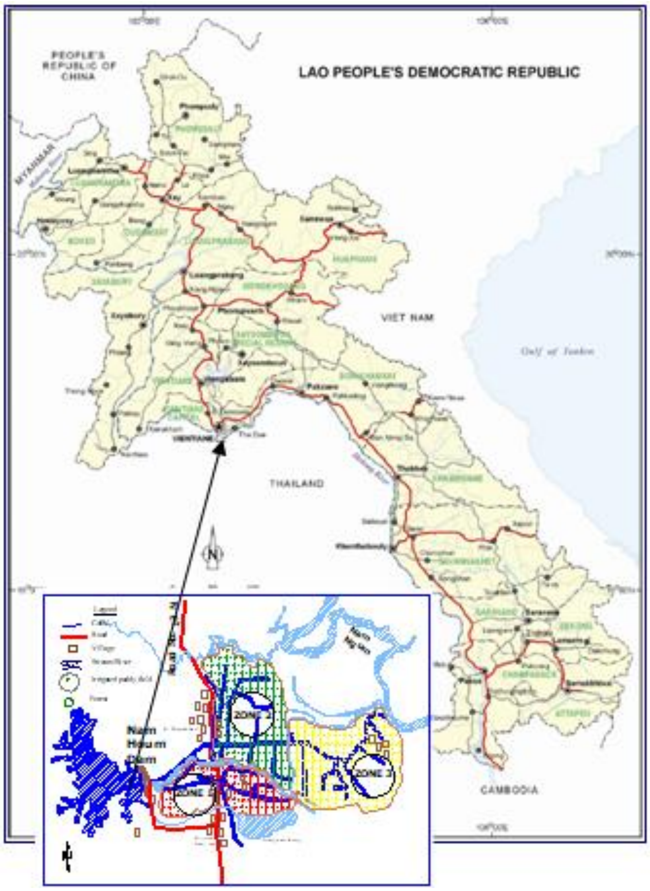

Figure 1: Location Map of Nam Houm Irrigation Scheme

The irrigation Scheme during the dry season (November-April) was considered for illustrative application of the proposed approach. Figure 1 presents the location of the Nam Houm Irrigation scheme in the Central of Lao PDR.

The Nam Houm Irrigation System (NISO) has an estimated irrigation service area of 4,200 hectares during the wet season and 2,400 hectares during the dry season.

The historical record of irrigated areas in Nam Houm from 1989 to 2007 is used for the application. The average irrigated area during the wet season is 1,986 hectares and 1,441.79 during the dry season (rice, non-rice crops, and fish ponds). It must be noted that the reported wet season irrigation area does not represent the total area planted during the wet season as the 4,200 hectares. Most areas are planted to rice during the wet season. Most of these areas depend on rain for water supply.

The reservoir has a catchments area of 108 km^2 with an annual inflow of 149.5 Mm^3. The maximum storage is 60 Mm^3 (active of 54 Mm^3, and dead of 6 Mm^3.). The estimation of the maximum flood level is 190.1 MSL (mean sea level) and the crest of the dam of 192.2 MSL, the spillway at 189.1 MSL, and the intake at 178.8 MSL. The reservoir was not filled-up to maximum capacity in the past wet season 2006.

For water distribution practices, water balance is considered before every cultivation season. If the storage is kept as high as 189.1 MSL (in other words 60 Mm^3) at the end of the rainy season, a constant amount of water will be supplied continuously with no limitation. However, a rotation method is applied when water in the reservoir is insufficient. When available water levels are under normal level, cultivated areas are estimated according to water storage e.g., in 2003, a water level of 186.41 MSL (36 Mm^3) could irrigate only 1,500 ha.

Due to the cultivated areas being limited according to available water. Water delivery is limited to one zone at the same time with regular rotation from zone 1 to zone 3. Some areas are abandoned, especially the areas which are located far away from the canals. The overall irrigation efficiency of the Nam Houm Irrigation Scheme is 70.52 % in 2006 [2].

Table 1: Historical data of requested irrigation area, participation of stakeholders in water resources management, available inflow, product cost, and actual irrigation efficiency

Year	Total irrigated area last yrs	Farmer Partici-pation	Inflow	Unit Cost Rice	Actual irrigation efficiency
	(ha)	(%)	($x10^3m^3$)	(Kip/kg)	(%)
1989	473.16	20.55	7,836.52	180	79.27
1990	493.16	23.55	8,613.05	212	82.76
1991	565.20	29.72	11,398.13	230	79.19
1992	713.20	26.50	10,079.26	255	79.22
1993	635.90	22.27	8,366.85	283	79.26
1994	534.50	29.80	11,164.54	334	78.83
1995	715.26	41.60	15,701.09	633	78.37
1996	998.51	45.10	16,856.32	803	77.98
1997	1,082.35	53.04	22,158.92	1,305	77.45
1998	1,405.35	54.33	22,604.72	1,708	77.87
1999	1,440.70	60.00	25,098.74	1,871	77.33
2000	1,581.81	91.67	38,331.21	2,000	76.77
2001	2,376.05	99.63	40,781.55	2,200	77.05
2002	2,467.05	102.38	42,098.40	2,648	78.34
2003	2,672.17	58.33	30,596.23	2,919	62.44
2004	1,575.17	100.00	46,293.38	2,897	69.85
2005	2,635.17	100.00	45,298.95	3,750	71.56
2006	2,643.96	68.75	31,642.78	4,069	70.45
2007	2,644.96	69.55	34,642.78	4,070	70.40

The total area of the scenario for several years of cultivation is 2,400 ha (1 ha = 10,000m^2) which is divided into 3 zones, including zone No. 1 of 550 ha, zone No. 2 of 900 ha and zone No. 3 of 950 ha.

4. Results and Discussions

Table 2 shows the parameters of multiple regression functions. The results have shown that the parameters of determination of multiple regressions functions provided the optimal condition for calibration the varied efficiencies.

Table 2: Parameters of multiple regression function for estimating irrigation efficiency.

Data input	Multiple coefficient	Total Area (X_1)	Farmer Partici-pation (X_2)	Inflow (X_3)	Unit Cost Rice (X_4)
	a	b_1	b_2	b_3	b_4
X_1-X_2	4.962	-0.129	0.071		
X_1-X_3	4.720	-0.107		0.037	
X_1-X_4	4.801	-0.057			-0.010
X_2-X_3	8.696		0.768	-0.742	
X_3-X_4	4.196			0.071	-0.079
X_4-X_2	4.529		-0.093		0.115
X_1-X_2-X_3	8.733	0.033	0.774	-0.750	
X_1-X_2-X_4	4.711	-0.055	0.114		-0.063
X_1-X_3-X_4	4.351	-0.061		0.074	-0.047
X_2-X_3-X_4	8.847		0.786	-0.769	0.006
X_1-X_2-X_3-X_4	8.842	-0.003	0.783	-0.766	0.007

The mathematical model was evaluated by considering the coefficient of correlation (r) and coefficient of determination (R^2) which are defined based on the actual irrigation efficiency and the estimated irrigation efficiency of all variables input as shown in table 3. The coefficient of correlation (r) of the relationship between irrigated area, participation of stakeholders in water resources management, available inflow, and product cost (X_1-X_2-X_3-X_4) highest values is 0.940 of all variables input and also the coefficient of determination (R^2) of all variables is highest values of 88.3% respectively. Therefore, these variables input with X_1-X_2-X_3-X_4 is suitable for estimating irrigation efficiency given the optimal condition of calibration. The least value of the coefficient of correlation (r) is 0.729 of the total irrigated area and inflow relationship(X_1-X_4) and also the coefficient of determination is 52.3%.

The power regression equation of the mathematical model for the relationship between irrigated area, participation of stakeholders in water resources management, available inflow, and product cost (X_1-X_2-X_3-X_4) is near non-linearity if the comparison between other relationships with the values of exponent of (Xi) and the coefficients of determination (R^2) which it showed in table 2, the multiple regression function as follow:

$$y = 6918.816 * X_1^{-0.003} * X_2^{0.783} * X_3^{-0.766} * X_4^{0.007} \quad (6)$$

Table 3: Parameters of multiple regression function for estimating irrigation efficiency.

Variables input	Average of Actual irrigation efficiency	Average of Estimated efficiency from model	Coefficient of correlation	Coefficient of determination
	(%)	(%)	r	R^2 (%)
X_1-X_2	76.02	75.62	0.759	57.6
X_1-X_3	76.02	75.89	0.729	53.2
X_1-X_4	76.02	75.78	0.723	52.3
X_2-X_3	76.02	75.91	0.939	88.1
X_3-X_4	76.02	75.65	0.733	53.8
X_4-X_2	76.02	143.81	0.784	61.4
X_1-X_2-X_3	76.02	76.08	0.939	88.2
X_1-X_2-X_4	76.02	75.95	0.798	63.7
X_1-X_3-X_4	76.02	75.78	0.748	56.0
X_2-X_3-X_4	76.02	75.52	0.939	88.2
X_1-X_2-X_3-X_4	76.02	75.42	0.940	88.3

Figure 2 shows the relationship between actual irrigation efficiency and estimated irrigation efficiency for all variables input with the highest value of the coefficient of correlation, r. The graphs have shown that the estimated irrigation efficiency from the model was very close to the actual irrigation efficiency for all variables of the multiple regression function, with consideration of the irrigated area, participation of stakeholders in water resources management, seasonal inflow, and product cost.

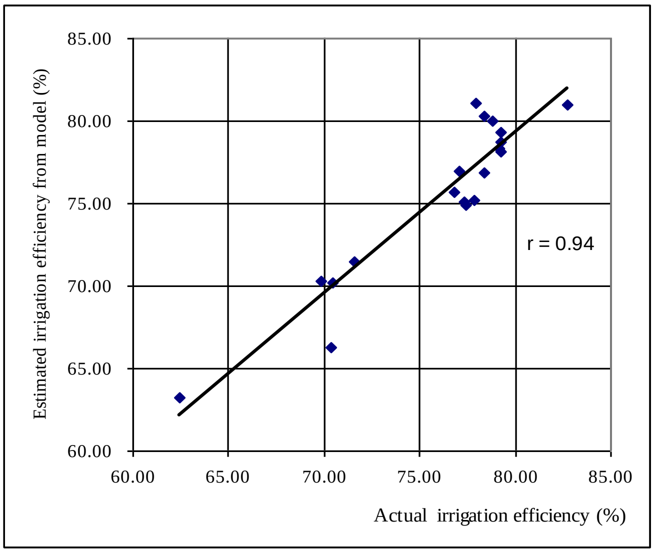

Figure 2: Actual irrigation efficiency and estimate irrigation efficiency from the mathematical model relationship for inputting variables of irrigated area, participation of stakeholders in water resources management, the value of available inflow, and cost of the product.

Table 4: Deviation between the estimated irrigation efficiency and the actual efficiency which is calculated from the irrigated area, participation of stakeholders, product cost, and value of available inflow

Year	Actual irrigation efficiency (%) $\hat{e}_j$	Estimated irrigation efficiency (%) e_j	Deviation $\left\|\hat{e}_j - e_j\right\|$
1989	79.27	78.12	1.15
1990	82.76	80.94	1.82
1991	79.19	78.37	0.83
1992	79.22	78.71	0.51
1993	79.26	79.32	0.06
1994	78.83	80.02	1.19
1995	78.37	80.31	1.95
1996	77.98	81.07	3.09
1997	77.45	74.89	2.56
1998	77.87	75.24	2.63
1999	77.33	75.10	2.23
2000	76.77	75.67	1.10
2001	77.05	76.99	0.07
2002	78.34	76.84	1.49
2003	62.44	63.20	0.76
2004	69.85	70.29	0.44
2005	71.56	71.49	0.07
2006	70.45	70.21	0.24
2007	70.40	66.25	4.15

Table 4 shows the deviation between the estimated irrigation efficiency and the actual efficiency which is calculated from the irrigated area, participation of stakeholders, product cost, and value of available inflow. The results have shown that the estimated irrigation efficiencies are closed to the actual irrigation efficiencies-all series. Except in 2007, the deviation is quite high with the number of 4.15. It indicates that the varied irrigation efficiency by using a mathematical model is more precise than the constant efficiency. Consequently, the parameters of the multiple regression function with all variables input (X_1-X_2-X_3-X_4) of the illustrative model are appropriate to find the varied irrigation efficiency for irrigation planning.

5. Conclusion

This paper is developed a mathematical model for finding the varied irrigation efficiency. The improvement process used multiple regression techniques to search for the optimal condition of parameters of multiple regression functions. The developed model was applied to determine the irrigation efficiency of the Nam Houm Irrigation Scheme (in Central Laos). The theory of mathematical functions applied in this study can be used to estimate irrigation efficiencies, when the previous irrigated area, participation of stakeholders in water resources management, available inflow, and product cost is given.

The results also show that the estimated irrigation efficiencies are closed to the actual irrigation efficiencies-all series of variables input. It indicates that the varied irrigation efficiency using a mathematical model is more precise. Consequently, the parameters of the multiple regression function with all variables input of the model are appropriate to find the varied irrigation efficiency for irrigation planning.

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