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Binomial Theorem

Understanding the Definition

B. Tabsum

Assistant Professor, Department of Mathematics, Government College of Arts, Science and Commerce Quepem Goa, India

Abstract: ***This study has been undertaken to understand the Binomial theorem and the pattern of its terms. Here the role and the significance of Pascal's triangle is studied.***

Keywords: factorial

1. Introduction

The binomial theorem is the most commonly used theorem in Mathematics. It is a technique for expanding a binomial expression raised to any finite power. It is used to solve problems in combinatorics, algebra, calculus, probability etc. It is also used in proving many important equations in Physics and Mathematics, in weather forecast services, Ranking up candidates, to estimate the cost of projects in Architecture etc.

Definition

Consider the following expansions
$(x+y)^0 = 1$---1 term
$(x+y)^1 = x+y = x^1+y^1$ ------------------------------------2 terms
$(x+y)^2 = x^2+2xy+y^2 = x^2+2x^1y^1+y^2$----------------------------------3 terms
$(x+y)^3 = x^3+3x^2y+3xy^2+y^3 = x^3+3x^2y^1+3x^1y^2+y^3$------------------------------------4 terms
$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4 = x^4+4x^3y^1+6x^2y^2+4x^1y^3+y^4$---------------5 terms

In general, for $(x+y)^n$, we get $n+1$ terms

Here we note that, in all the above expansions, the power of x is that of $(x+y)$ in the beginning and keeps decreasing till 0. Whereas the power of y is 0 in the beginning an keeps increasing till the power is that of $(x+y)$.

Let's observe the sum of the powers of x and y in each term of each expression mentioned above. We rewrite the expansions as follows
$(x+y)^1 = x^1+y^1 = x^1y^0+y^1x^0$
$(x+y)^2 = x^2+2x^1y^1+y^2 = x^2y^0+2x^1y^1+y^2x^0$
$(x+y)^3 = x^3+3x^2y^1+3x^1y^2+y^3 = x^3y^0+3x^2y^1+3x^1y^2+y^3x^0$
$(x+y)^4 = x^4+4x^3y^1+6x^2y^2+4x^1y^3+y^4$
$= x^4y^0+4x^3y^1+6x^2y^2+4x^1y^3+y^4x^0$

and so on. Here each term is expressed as a product of some power of x and that of y, and sum of these powers is the power of $x+y$ in respective expansion.

Lastly we observe the coefficients in each term of the expansions. And on observation we get

Expression	Coefficients	Coefficients in terms of combinations
$(x+y)^0$	1	$\binom{0}{0}$
$(x+y)^1$	1,1	$\binom{1}{0},\binom{1}{1}$
$(x+y)^2$	1,2,1	$\binom{2}{0},\binom{2}{1},\binom{2}{2}$
$(x+y)^3$	1,3,3,1	$\binom{3}{0},\binom{3}{1},\binom{3}{2},\binom{3}{3}$
$(x+y)^4$	1,4,6,4,1	$\binom{4}{0},\binom{4}{1},\binom{4}{2},\binom{4}{3},\binom{4}{4}$
$(x+y)^5$	1,5,10,10,5,1	$\binom{5}{0},\binom{5}{1},\binom{5}{2},\binom{5}{3},\binom{5}{4},\binom{5}{5}$
$(x+y)^6$	1,6,15,20,15,6,1	$\binom{6}{0},\binom{6}{1},\binom{6}{2},\binom{6}{3},\binom{6}{4},\binom{6}{5},\binom{6}{6}$
$(x+y)^7$	1,7,21,35,35,21,7,1	$\binom{7}{0},\binom{7}{1},\binom{7}{2},\binom{7}{3},\binom{7}{4},\binom{7}{5},\binom{7}{6},\binom{7}{7}$
$\vdots$	$\vdots$	$\vdots$
$(x+y)^n$	$1, n, \cdots, n, 1$	$\binom{n}{0},\binom{n}{1},\cdots,\binom{n}{n-1},\binom{n}{n}$

The above triangle in the second or third column is called as Pascal's Triangle

Hence we have

$$(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^0 + \cdots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} x^0 y^n$$

Or

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Acknowledgment

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression, "One of us (R.B.G.) thanks..." Instead, try "R. B. G. thanks". Put applicable sponsor acknowledgments here; DONOT place them on the first page of your paper or as a foot note.

References

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