

Mathematics of Finance: Internal Rate of Return (IRR)

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Abstract: This article abstract discusses the concept of the Internal Rate of Return in finance and provides a detailed explanation of how to calculate it, using practical examples and mathematical analysis. It highlights the importance of IRR in evaluating the profitability of investments and presents step-by-step instructions for determining the IRR. The article also includes two illustrative examples with calculations, demonstrating the application of IRR in real-world scenarios. In the first example, the IRR is calculated for a project with given cash flows, while the second example deals with a different investment scenario. The abstract concludes by emphasizing the importance of understanding IRR and its practical utility in financial decision-making.

Keywords: Internal rate of return, net present value, yield rate

1. Introduction

Analysis of finance through mathematical way is very intelligible. Here we discuss what is the rate of return of an investment and explain how to calculate it with some illustrations.

Net Present Value (NPV)

Net present value is the sum of the present values of the net cash flows over n periods. The value of NPV(i) can be +ve, -ve or zero depending on how large or small value of 'i' is.

Internal Rate of Return (IRR)

When NPV(i)=0, yield rates are same from either the borrowers or lenders perspective. Yield rates are often used to measure how favourable or unfavourable a transaction might be. From the lender's perspective, a higher yield rate makes transaction more favourable. From the borrower's perspective, a lower yield rate makes transaction more favourable. Based on financial factors projects with positive NPV are considered acceptable.

Determination of Internal Rate of Return (IRR)

Steps to follow:

- 1) Prepare the cash flow table using an arbitrary assumed discount rate, to discount the net cash flow to the present value.
- 2) Find out net present value, NPV.
- 3) If NPV is positive, apply higher rate of discount.
- 4) If the higher discount rate still gives a positive net present value, increase the discount rate until it gets negative.
- 5) If NPV is negative at this higher rate, IRR lies between these two rates.

Problem: Calculate Internal Rate of Return, if

Initial investment Rs.60,000/-

Life of the asset 4 years

Estimated net annual cash flow:

Year	Year 1	Year 2	Year 3	Year 4
Amount (Rs)	15,000	20,000	30,000	20,000

Solution:

Table 1

Initial investment: 60,000, Discount rate: 10%

Year	Annual cash flow	P.V. F	P. V
0	60,000	1	-60,000
1	15,000	0.9091	13636.5
2	20,000	0.8264	16528
3	30,000	0.7513	22539
4	20,000	0.6830	13660
			6363.5

Table 2

Discount rate 12%		Discount rate :14%		Discount rate: 15%	
P.V. F	P. V	P.V. F	P. V	P.V. F	P. V
0.8929	13393.5	0.8772	13158	0.8696	13044
0.7972	15944	0.7695	15390	0.7561	15122
0.7118	21354	0.6750	20250	0.6575	19725
0.6355	12710	0.5921	11842	0.5718	11436
	63401.5		60640		59327

From the table, NPV at 14% = 60640-60000= 640

NPV@15% = 59327- 60000= -673

Therefore, the IRR Value lies between 14% and 15%

$$\begin{aligned} \text{IRR} &= 14\% + 640 \times (15-14) / (640+673) \\ &= 14 + 640/1313 \\ &= 14 + 0.4874 \\ &= 14.4874\% \end{aligned}$$

Problem 2:

An investment of Rs.10,000 returns Rs.3000 at the end of years 1 and 2 and Rs.3500 at the end of years 3 and 4. Calculate IRR.

Solution: Initial investment Rs.10,000

Table 1

Year	A.C. F	Amount
1	3000	3000
2	3000	6000
3	3500	10500
4	3500	14000
	13000	33500

Assumed rate of interest = (13000-10000)/33500= 8.9%

Table 2

Year	A.C. F	Amount	Discount rate @9%		Discount rate @10%		Discount rate @12%	
			P.V. F	P. V	P.V. F	P. V	P.V. F	P. V
1	3000	3000	0.9174	2752.2	0.9091	2727.3	0.8929	2678.7
2	3000	6000	0.8417	2525.1	0.8265	2479.5	0.7972	2391.6
3	3500	10500	0.7722	2702.7	0.7513	2629.55	0.7118	2491.3
4	3500	14000	0.7084	2479.4	0.6830	2390.5	0.6355	2224.25
				10459.4		10226.85		9785.85

From the table,

Net Present Value, NPV @ 9% = 10459.4 – 10000 = 459.4, +ve

Net Present Value, NPV @ 10% = 10226.85 – 10000 = 226.85, +ve

Net Present Value, NPV @ 12% = 9785 – 10000 = -214.15, -ve

Therefore, IRR value lies between 10% & 12%.

$$\begin{aligned}
 \text{IRR} &= 10 + \left\{ \frac{[226.85 \times (12 - 10)]}{226.85 - (-214.15)} \right\} \\
 &= 10 + \left\{ \frac{453.7}{441} \right\} \\
 &= 10 + 1.028 \\
 &= 11.028\%
 \end{aligned}$$

2. Conclusion

United information from reference books and online contents to comprehend a simple, lucid way to understand the concept 'internal rate of return'. Some illustrations are also provided to calculate the same.

References

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