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Application of Linear Programming (Transportation Problem)

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Abstract: ***The aim of this paper is to explain the theoretical aspects of the transportation problem and provide a suitable model that reduces the cost of transportation of goods from the different sources to the various destinations which lead to increase profits as well as enhancing productivity and satisfying the desired goals and objective of profit maximization through cost minimization, the Quantity System Business software (WIN QSB) is used for the analysis.***

Keywords: Source, Destination, Supply, Demand, Cost, Least cost method, North West corner method, Vogel approximation method.

1. Introduction

Transportation problem is one of the most important application of the linear programming problems which deals with the distribution of goods from the different supply points (sources) to the various points of demand (destinations) with minimum transportation cost.

This paper has been arranged as follows:

Introduction in Section one, Formulation of transportation problem will be discussed in section two,

2. Formulation of transportation Problem

During the construction of the transportation problem the following assumptions should be considered

2.1 Assumptions of transportation problem:

1) A single product is to be shipped from the source to destination.
2) Each source has specific level of supply.
3) Each destination has specific level of demand.
4) Unit transportation cost between source and destination should be known.
5) Costs are assumed to be linear.

2.2 Transportation algorithm consists of the following stages:

1) Developing initial solution.
2) Test the optimality of the solution
3) Improving solution.

2.3 General Formulation of transportation problem:

$$Min\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

S.t $\sum_{j=1}^{n} x_{ij} \le s_i$ (i =1,2, 3,…,m) (Supply constraints)

$\sum_{i=1}^{m} x_{ij} \ge d_j$ (j=1,2, 3,…,n) (Demand constraints)

$x_{ij} \ge 0$ (i=1,2, 3,…, m, j=1,2,3,…,n) **(**Non-negative constrain)

2.4 Supply and demand assumptions:

Each source has a fixed supply of units, where this entire supply must be distributed to the various destinations. (let S_i denotes the number

of units being supplied by source i, for i = 1, 2, . . ., m.) Similarly, each destination has a fixed demand for units, where this entire demand must be received from the different sources. (let D_j denotes the number of units being received by destination j, for j = 1, 2, . . ., n.) This assumption State that there is no leeway in the amounts to be sent or received means that there needs to be a balance between the total supply from all sources and the total demand at all destinations.
For feasible solution to exist, its necessary that total supply should equal total demand i.e.

$$\sum_{i=1}^{n} s_i = \sum_{j=1}^{m} d_j$$

Otherwise it is called unbalanced Transportation problem.

		Demand			
		D_1	D_2	…	D_n
Supply	S_1	X_{11} C_{11}	X_{12} C_{12}	…	X_{1n} C_{1n}
	S_2	X_{21} C_{21}	X_{22} C_{22}	…	X_{2n} C_{2n}
	. .	. .	. .	…	. .
	S_m	X_{m1} C_{m1}	X_{m2} C_{m2}	…	X_{mn} C_{mn}

The above problem is actually a linear programming problem of the transportation problem type. To formulate the model, let Z denote total shipping cost, and let (x_{ij})be the number of truckloads to be shipped from source i to destination j where (i = 1, 2, 3..., m; j = 1, 2, 3, ..., n). Thus, the objective is to determine the values of the decision variables (x_{ij}) so as to reduce cost of shipping (Z)

Minimize $Z = C_{11}X_{11} + C_{12}X_{12} + \ldots. + C_{1n}X_{1n} + C_{14}X_{14} + C_{21}X_{21} + C_{22}X_{22} + \ldots. + C_{2n}X_{2n} + C_{m1}X_{m1} + C_{m2}X_{m2} + \ldots. + C_{mn}X_{mn}$ **subject to the constraints**

$$\left.\begin{array}{l} x_{11}x_{12} \ldots. x_{1n} = S_1 \\ x_{21}x_{22} \ldots. x_{2n} = S_2 \\ . \quad . \quad . \\ . \quad . \quad . \\ . \quad . \quad . \\ x_{m1}x_{m2} \ldots. x_{mn} = S_m \end{array}\right\} \text{Supply Constraints}$$

$$\left.\begin{array}{l} x_{11}x_{21} \ldots. \; x_{m1} = D_1 \\ x_{12} \quad x_{22} \quad \ldots. \; x_{m2} = D_2 \\ . \, . \, . . \\ . . \, . . \\ . . \, . \qquad . \\ x_{1n}x_{2n} \; \ldots. \; x_{mn} = D_n \end{array}\right\} \text{Demand constraints}$$

and

$x_{ij} \geq 0$ where (i = 1, 2, 3...m; j = 1, 2, 3, ..., n).

3. Methods of Finding Transportation Problem Initial Solution

3.1 Northwest corner Rule:

The idea is to find an initial basic feasible solution i.e., a set of allocations that satisfied the row and column totals. This method simply consists of making allocations to each row in turn, apportioning as much as possible to its first cell and proceeding in this manner to its following cells until the row total in exhausted. The algorithm involved under north-west corner rule consists for the following steps:

Before allocation ensures that the total of availability and requirement is equal. If not then make them equal, the first allocation is made in the cell occupying the upper left-hand corner of the matrix, the assignment is made in such a way that either the resource availability is exhausted or the demand at the first destination is satisfied, if the resource availability of the row one is exhausted first, we move down the second row and first column to make another allocation which either exhausts the resource availability of row two or satisfies the remaining destination demand of column one. If the first allocation completely satisfies the destination demand of column one, we move to column two in row one, and make a second allocation which either exhausts the remaining resource availability of row one or satisfies the destination requirement under column two, such procedure is repeated until all the row availability and column requirements are satisfied. Consider, for example, the following sample problem. This method does not use transportation costs which we shall bring in later in the other method.

3.2 The Least cost method:

Before starting the process of allocation ensure that the total of supply and demand are equal. The least cost method starts by making the first allocation in the cell whose shipping cost (or transportation cost) per unit is the lowest. This lowest cost cell is loaded or filled as much as possible in view of the origin capacity of its row and the destination requirements of its column, then we move to the next lowest cost cell and make an allocation in view of the remaining capacity and requirement of its row and column. In case there is a tie for the lowest cost cell during any allocation, we can arbitrarily choose cell for allocation, such procedure is repeated till all row requirements are satisfied.

3.3 Vogel's Approximation Method (VAM)

The Vogel's Approximation Method (VAM) is considered to be superior to the northwest corner rule in that it usually provides an initial solution that is optimal or near to optimal.

For each row of the transportation table identify the smallest and next smallest costs. Find the difference between the two costs and display it to the right of that row as "Difference" (Diff.). Likewise, find such a difference for each column and display it below that column. In case two cells contain the same least cost then the difference will be taken as zero.

From amongst these row and column differences, select the one with the largest difference. Allocate the maximum possible to the least cost cell in the selected column or row. If there occurs a tie amongst the largest differences, the choice may be made for a row or column which has least cost. In case there is a tie in cost cell also, choice may be made for a row or column by which maximum requirement is exhausted. Hatch that column or row containing this cell whose totals have been exhausted so that this column or row is ignored in further consideration. Recompute the column and row differences for the reduced transportation table and repeat the procedure until all the column and row totals are exhausted.

4. The problem

Let us consider the below transportation problem

From\To	Qaryat	Khobar	Deba	Mecca	Supply
Jeddah	150	130	140	50	200
Damam	100	45	120	100	120
Tabuk	50	100	45	120	80
Demand	45	65	140	150	

As it is mentioned before there are three different methods to find the optimal solution for the transportation problem, The Quantity System Business software (WIN QSB) is utilized to obtain the initial solution as well as the optimal solution.

Iteration One:

From\ To	Qaryat	Khobar	Deba	Mecca
Jeddah	150	130 50*	140 Cij**	50 150
Damam	100 45	45 15	120 60	100
Tabuk	50	100	45 80	120

Objective Value = 29975 (minimization)
****Entering: Jeddah to Deba**
***Leaving: Jeddah to Khobar**

Iteration Two:

From\ To	Qaryat	Khobar	Deba	Mecca
Jeddah	150	130	140 50	50 150
Damam	100 45	45 65	120 10	100
Tabuk	50	100	45 80	120

Objective Value = 26725 (minimization)

4.1The optimal solution

From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
Jeddah	Deba	50	140	7000	0
Jeddah	Mecca	150	50	7500	0
Damam	Qaryat	45	100	4500	0
Damam	Khobar	65	45	2925	0
Damam	Deba	10	120	1200	0
Tabuk	Deba	80	45	3600	0
Total Objective function value = 26725					

4.3 Discussion and conclusion

It is clear that from iteration one, To Attain the objective value which is 29975 The following shipments should be achieved:

- Shipping 50 units from Jeddah to Khobar with cost of 130 SR and total cost of 6500 SR.
- Shipping 150 units from Jeddah to Mecca with cost of 50SR per unit with total cost of 7500 SR.
- Shipping 45 units from Dammam to Qaryat with cost of 100 SR per unit and total cost of 4500 SR.
- Shipping 15 units from Dammam to Khobar with cost of 45 SR per unit and total cost of 675 SR.
- Shipping 60 units from Dammam to Deba with cost of 120 per unit and total cost of 7200 SR
- Shipping 80 units from Tabuk to Deba with cost of 45 SR per unit and total cost of 3600 SR.
- By checking optimality, we realize that there is a possibility of enhancing the solution by entering a shipment from Jeddah to Deba and leaving a shipment from Jeddah to Khobar.
- The optimal solution attained in Table (4.1) with objective value of 26725 SR

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