Effect of Hall Currents on Transient Convective Heat and Mass Transfer Flow of a Viscous Fluid in a Vertical Wavy Channel with Tranveling Thermal Waves

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Abstract: ***In this paper, we investigate the effect of chemical reaction and radiation absorption on mixed convective heat and mass transfer flow of a viscous, electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat sources. The equations governing the flow, heat and mass transfer are solved by employing perturbation technique with aspect ratio δ as perturbation parameter. The velocity, temperature and concentration distributions are investigated for different values of G, M, m, k, Q1, N, α and $x+\gamma t$. The rate of heat and mass transfer are numerically evaluated for different variations of the governing parameters.***

Keywords: Effect of Hall currents, Heat and Mass transfer , Porous medium, Vertically wavy channel, Thermal waves, Chemical reactions, Temperature, Diffusion.

1. Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid.

We are particularly interested in cases in which diffusion and chemical reaction occur at roughly the same speed. When diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate; when diffusion is not much faster than reaction, the diffusion and kinetics interact to produce very different effects. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction. Due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight.

Muthucumaraswamy and Ganesan (32) studied effect of the chemical reaction and injection on flow characteristics in an instead upward motion of an unsteady upward motion of an isothermal plate. Deka et at. (13) studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant beat and mass transfer. Chamkha (5) studies the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The effect of foreign mass on the free-convection flow past a semi-infinite vertical plate were studied by Gebhart et al (19). Chamka (5) assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Raptis and Perdikis (41) studied the unsteady free convection flow of water near 4 C in the laminar boundary layer over a vertical moving porous plate.

In the theory of flow through porous medium, the role of momentum equations or force balance is occupied by the numerous experimental observations summerised mathematically as the Darcy's law. It is observed that the Darcy's law is applicable as long as the Reynolds number based on average grain (pore) diameter does not exceed a value between 1 and 10.But in general , the speed of specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generated in the fluid due to its viscous nature produces distortions in the velocity field. Also in the case of highly porous media such as fibre glass, pappus of dandilion etc., the viscous stress at the surface is able to penetrate into media and produce flow near the surface even in the absence of the pressure gradient. Thus Darcy's law which specifies a linear relationship between the specific discharge and hydraulic gradient is inadequate in describing high speed flows or flows near surfaces which may be either permeable or not. Hence consideration for non-Darcian description for the viscous flow through porous media is warranted. Saffman (45a) employing statistical method derived a general governing equations for the flow in a porous medium which takes into account the viscous stress. Later another modification has suggested by Brinkman (3)

$$o = -\nabla p - (\frac{\mu}{k})\bar{v} + \mu\nabla^2\bar{v}$$

in which $\mu\nabla^2\bar{v}$ is intended to account for the distortions of the velocity profiles near the boundary. The same equation was derived analytically by Tam (55) to describe the viscous flow at low Reynolds number pasr a swam of small particles. The generalization of the above study was presented by Yamamoto and Iwamura (61). The steady two-dimensional flow of viscous fluid through a porous medium bounded by porous surface subjected to a constant suction velocity by taking account of free convection currents (both velocity and temperature fields are constant along x-axis)was studied by Raptis et al (42). Combarnous and Bories (10), Chang (7, 8) and Combarnous (9) have recently proved extensive reviews of state of the art of free convection in fluid saturated porous medium.

There is an extensive literature on free convection in porous media, i.e., flows through a porous media under gravitational fields that are driven by gradients of fluid density caused by temperature gradient. Many studies, including most of the earlier work, have dealt with systems heated from below (20, 32, 37).Some attention has also been given to investigations of free convection in porous media introduced by a temperature gradient normal to the gravitational field. Raptis (43) has investigated unsteady free convective flow through a porous medium.

Convection fluid flows generated by traveling thermal waves have also received attention due to applications in physical problems. The linearised analysis of these flows has shown that a traveling thermal wave can generate a mean shear flow within a layer of fluid, and the induced mean flow is proportional to the square of the amplitude of the wave. From a physical point of view, the motion induced by traveling thermal waves is quite interesting as a purely fluid-dynamical problem and can be used as a possible explanation for the observed four-day retrograde zonal motion of the upper atmosphere of Venus. Also, the heat transfer results will have a definite bearing on the design of oil-or gas –fired boilers. Vajravelu and Debnath (57) have made an interesting and a detailed study of non-linear convection heat transfer and fluid flows, induced by traveling thermal waves. The traveling thermal wave problem was investigated both analytically and experimentally by Whitehead (59) by postulating series expansion in the square of the aspect ratio (assumed small) for both the temperature and flow fields. Whitehead (59) obtained an analytical solution for the mean flow produced by a moving source theoretical predictions regarding the ratio of the mean flow velocity to the source speed were found to be in good agreement with experimental observations in Mercury which therefore justified the validity of the asymptotic expansion a posterior

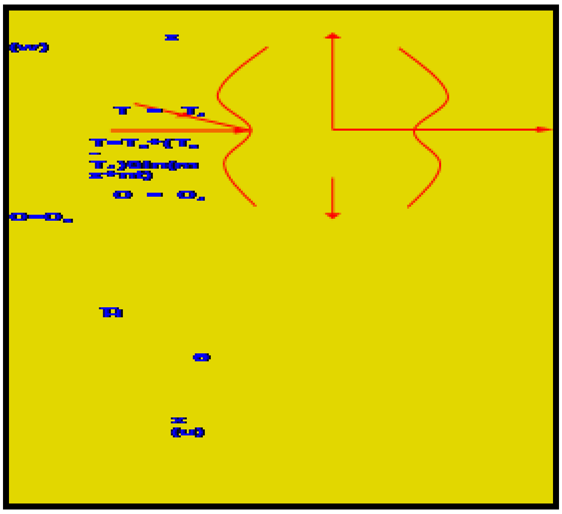
Heat generation in a porous media due to the presence of temperature dependent heat sources has number of applications related to the development of energy resources. It is also important in engineering processes pertaining to flows in which a fluid supports an exothermic chemical or nuclear reaction. Proposal of disposing the radioactive waste material b burying in the ground or in deep ocean sediment is another problem where heat generation in porous medium occurs, Foroboschi and Federico [18] have assumed volumetric heat generation of the type

$$\theta = \theta_o (T - T_0) \quad \text{for} \quad T \geq T_0$$
$$= 0 \quad \text{for} \quad T < T_0$$

David Moleam [14] has studied the effect of temperature dependent heat source $\theta = 1/ a + bT$ such as occurring in the electrical heating on the steady state transfer within a porous medium. Chandrasekahr [6], Palm [38] reviewed the extensive work and mentioned about several authors who have contributed to the force convection with heat generating source. Mixed convection flows have been studied extensively for various enclosure shapes and thermal boundary conditions. Due to the super position of the buoyancy effects on the main flow there is a secondary flow in the form of a vortex recirculation pattern.

In all these investigations, the effects of Hall currents are not considered. However, in a partially ionized gas, there occurs a Hall current [4] when the strength of the impressed magnetic field is very strong. These Hall effects play a significant role in determining the flow features. Sato [48], Yamanishi [60], Sherman and Sutton [51] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. These effects in the unsteady cases were discussed by Pop [39]. Debnath [15, 16] has studied the effects of Hall currents on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow is investigated. Alam *et. al.*, [1] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects in to account Krishan *et. al.*, [27, 28] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao *et. al.*, [53] have analyzed Hall effects on unsteady Hydromagnetic flow. Siva Prasad *et. al.*, [44) have studied Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel. Recently Seth *et. al.*, [50] have investigated the effects of Hall currents on heat transfer in a rotating MHD channel flow in arbitrary conducting walls. Sarkar *et. al.*, [46] have analyzed the effects of mass transfer and rotation and flow past a porous plate in a porous medium with variable suction in slip flow region.

In this chapter, we investigate the effect of chemical reaction and radiation absorption on mixed convective heat and mass transfer flow of a viscous , electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat sources. The equations governing the flow , heat and mass transfer are solved by employing perturbation technique with aspect ratio δ as perturbation parameter. The velocity , temperature and concentration distributions are investigated for different values of G, M, m, k, Q1, N, α and x +γt. The rate of heat and mass transfer are numerically evaluated for different variations of the governing parameters.



Configuration of the Problem

2. Formulation and Solution of the Problem

We consider the unsteady flow of an incompressible, viscous , electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity Ho lying in the plane (x-z).The magnetic field is inclined at an angle α_1 to the axial direction and hence its components are $(0, H_0 Sin(\alpha_1), H_0 Cos(\alpha_1))$.In view of the traveling thermal wave imposed on the wall $x = +Lf(mz)$ the velocity field has components (u, 0, w)The magnetic field in the presence of fluid flow induces the current ($(J_x, 0, J_z)$ We choose a rectangular cartesian co-ordinate system O (x, y, z) with z-axis in the vertical direction and the walls at $x = \pm Lf(mz)$.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} x \bar{H} = \sigma(\bar{E} + \mu_e \bar{q} x \bar{H}) \qquad (2.1)$$

where $\bar{q}$ is the velocity vector. $\bar{H}$ is the magnetic field intensity vector. $\bar{E}$ is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and μ_e is the magnetic permeability. Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field E=0, equation (2.1) reduces

$$j_x - m H_0 J_z Sin(\alpha_1) = -\sigma \mu_e H_0 w Sin(\alpha_1) \qquad (2.2)$$

$$J_z + m H_0 J_x Sin(\alpha_1) = \sigma \mu_e H_0 u\, Sin(\alpha_1) \qquad (2.3)$$

where m= $\omega_e \tau_e$ is the Hall parameter.

On solving equations (2.2)& (2.3) we obtain

$$j_x = \frac{\sigma \mu_e H_0 Sin(\alpha_1)}{1 + m^2 H_0^2 Sin^2(\alpha_1)} (m H_0 Sin(\alpha_1) - w) \qquad (2.4)$$

$$j_z = \frac{\sigma \mu_e H_0 Sin(\alpha_1)}{1 + m^2 H_0^2 Sin^2(\alpha_1)} (u + m H_0 w\, Sin(\alpha_1)) \qquad (2.5)$$

where u, w are the velocity components along x and z directions respectively,

The Momentum equations are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \mu_e(-H_0 J_z Sin(\alpha_1)) \qquad (2.6)$$

$$\frac{\partial w}{\partial t} u \frac{\partial W}{\partial x} + w \frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) + \mu_e(H_0 J_x Sin(\alpha_1)) \qquad (2.7)$$

Substituting J_x and J_z from equations (2.4)& (2.5)in equations (2.6)& (2.7) we obtain

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + $$

$$- \frac{\sigma \mu_e H_0^2 Sin^2(\alpha_1)}{1 + m^2 H_0^2 Sin^2(\alpha_1)} (u + m H_0 w Sin(\alpha_1)) - \rho g \qquad (2.8)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial W}{\partial x} + w \frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right)$$

$$- \frac{\sigma \mu_e H_0^2 Sin^2(\alpha_1)}{1 + m^2 H_0^2 Sin^2(\alpha_1)} (w - m H_0 u Sin(\alpha_1)) \qquad (2.9)$$

The energy equation is

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z}\right) = k_f \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q + Q_1'(C - C_0) \qquad (2.10)$$

The diffusion equation is

$$(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+w\frac{\partial C}{\partial z}=D_{1f}(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2})-k_1(C-C_0) \qquad (2.11)$$

The equation of state is

$$\rho-\rho_0=-\beta(T-T_o)-\beta^{\bullet}(C-C_0)) \qquad (2.12)$$

Where T, C are the temperature and concentration in the fluid. k_f is the thermal conductivity, Cp is the specific heat constant pressure, k is the permeability of the porous medium, β is the coefficient of thermal expansion, $\beta^{\bullet}$ is the volumetric coefficient of expansion with mass fraction coefficiemnt, $D_1$is the molecular diffusivity, Q is the strength of the heat source, k1 is the chemical reaction coefficient, Q_1' is the radiation absorption coefficient.,

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q=\frac{1}{L}\int_{-Lf}^{Lf} w dz \qquad (2.13)$$

The boundary conditions are

u= 0 , w=0 T=T_1 , C=C_1 on $x=-Lf(mz)$ (2.14)

w=0, w=0, T=T_2 + ((T_1-T_2)Sin (mz+nt), C=C_2 on $x=-Lf(mz)$ (2.15)

Eliminating the pressure from equations (2.8)& (2.9) and introducing the Stokes Stream function ψ as

$$u=-\frac{\partial\psi}{\partial z}\quad , w=\frac{\partial\psi}{\partial x} \qquad (2.16)$$

the equations (2.8), (2.9) & (2.10) in terms of ψ are

$$\frac{\partial(\nabla^2\psi)}{\partial t}-\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}=\mu\nabla^4\psi+\beta g\frac{\partial(T-T_e)}{\partial x} + \beta^{\bullet} g\frac{\partial(C-C_e)}{\partial x}-(\frac{\sigma\mu_e^2 H_0^2 Sin^2(\alpha_1)}{1+m^2H_0^2\, Sin^2(\alpha_1)})\nabla^2\psi \qquad (2.17)$$

$$\rho C_p(\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x}=k_f(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2})+Q+Q_1'(C-C_o) \qquad (2.18)$$

$$(\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial c}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial C}{\partial x}=D_1(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2})-k_1(C-C_0) \qquad (2.19)$$

On introducing the following non-dimensional variables

$$(x',z')=(xz,z/L),\ \psi'=\frac{\psi}{qL},\ \theta=\frac{T-T_2}{T_1-T_2},C'=\frac{C-C_2}{C_1-C_2}$$

the equation of momentum and energy in the non-dimensional form are

$$\nabla^4\psi-M_1^2\nabla^2\psi+\frac{G}{R}(\frac{\partial\theta}{\partial z}+N\frac{\partial C}{\partial z})=\delta R(\delta\frac{\partial}{\partial t}(\nabla^2\psi)+ (\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}) \qquad (2.20)$$

$$\delta P(\delta\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x})=(\frac{\partial^2\theta}{\partial x^2}+\delta^2\frac{\partial^2\theta}{\partial z^2})+\alpha+Q_1C \qquad (2.21)$$

$$\delta Sc(\delta\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial C}{\partial x})=(\frac{\partial^2 C}{\partial x^2}+\delta^2\frac{\partial^2 C}{\partial z^2})-KC \qquad (2.22)$$

$$\nabla^2 = \frac{\partial}{\partial x^2} + \delta^2 \frac{\partial}{\partial z^2}$$

The corresponding boundary conditions are

$$\psi(1) - \psi(-1) = 1$$

$$\frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = 1, C = 1 \quad at \;\; x = -f(z)$$

$$\frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = Sin(z + \gamma t), \;\; C = 0 \;\; at \;\; x = +f(z) \tag{2.23}$$

3. Analysis of the Flow

On introducing the transformation

$$\eta = \frac{x}{f(z)} \tag{3.1}$$

the equations (2.20)- (2.22) reduce to

$$F^4\psi - (M_1^2 f^2)F^2\psi + \left(\frac{Gf^3}{R}\right)\left(\frac{\partial \theta}{\partial z} + N\frac{\partial C}{\partial z}\right) = 9\delta Rf)\left(\delta\frac{\partial}{\partial t}(F^2\psi) + \left(\frac{\partial \psi}{\partial z}\frac{\partial (F^2\psi)}{\partial \eta} - \frac{\partial \psi}{\partial \eta}\frac{\partial (F^2\psi)}{\partial z}\right)\right. \tag{3.2}$$

$$(\delta Pf)\left(\delta\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial \eta}\frac{\partial \theta}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial \theta}{\partial \eta}\right) = \left(\frac{\partial^2 \theta}{\partial \eta^2} + \delta^2 f^2\frac{\partial^2 \theta}{\partial z^2}\right) + (\alpha f^2) + (Q_1 f^2)C \tag{3.3}$$

$$(\delta Scf)\left(\delta f^2\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial \eta}\frac{\partial C}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial C}{\partial \eta}\right) = \left(\frac{\partial^2 C}{\partial \eta^2} + \delta^2 f^2\frac{\partial^2 C}{\partial z^2}\right) - (Kf^2)C \tag{3.4}$$

Assuming the aspect ratio δ to be small we take the asymptotic solutions as

$$\psi(x,z,t) = \psi_0(x,z,t) + \delta\psi_1(x,z.,t) + \delta^2\psi_2(x,z,t) + ...$$

$$\theta(x,z,t) = \theta_o(x,z,t) + \delta\theta_1(x,z,t) + \delta^2\theta_2(x,z,t) + ... \tag{3.5}$$

$$C(x,z,t) = C_o(x,z,t) + \delta C_1(x,z,t) + \delta^2 C_2(x,z,t) + ...$$

Substituting (3.5) in equations (3.2)- (3.4) and equating the like powers of δ the equations and the respective boundary conditions to the zeroth order are

$$\frac{\partial^2 \theta_0}{\partial \eta^2} = -(\alpha f^2) = -(Q_1 f^2)C_o \tag{3.6}$$

$$\frac{\partial^2 C_0}{\partial \eta^2} - (kf^2)C_0 = 0 \tag{3.7}$$

$$\frac{\partial^4 \psi_0}{\partial \eta^4} - (M_1^2 f^2)\frac{\partial^2 \psi_0}{\partial \eta^2} = -\left(\frac{Gf^3}{R}\right)\left(\frac{\partial \theta_0}{\partial z} + N\frac{\partial C_0}{\partial z}\right) \tag{3.8}$$

with

$$\psi_0(+1)-\psi_0(-1)=1$$

$$\frac{\partial\psi_0}{\partial\eta}=0,\ \frac{\partial\psi_0}{\partial\bar{z}}=0,\quad \theta_0=1,\ C_0=1\ \ at\ \ \eta=-1 \tag{3.9}$$

$$\frac{\partial\psi_0}{\partial\eta}=0,\ \frac{\partial\psi_0}{\partial\bar{z}}=0,\quad \theta_0=Sin(z+\gamma t),\ C_0=0\ \ at\ \ \eta=+1$$

and to the first order are

$$\frac{\partial^2\theta_1}{\partial\eta^2}=(PRf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial\theta_0}{\partial\bar{z}}-\frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial\theta_0}{\partial\eta}\right) \tag{3.10}$$

$$\frac{\partial^2 C_1}{\partial\eta^2}-(kf^2)C_1=(ScRf)\left(\frac{\partial\psi_0}{\partial\mu}\frac{\partial C_0}{\partial\bar{z}}-\frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial C_0}{\partial\eta}\right) \tag{3.11}$$

$$\left.\begin{aligned}&\frac{\partial^4\psi_1}{\partial\eta^4}-(M_1^2f^2)\frac{\partial^2\psi_1}{\partial\eta^2}=-\left(\frac{Gf^3}{R}\right)\left(\frac{\partial\theta_1}{\partial z}+N\frac{\partial C_1}{\partial z}\right)+\\&\qquad+(Rf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial z^3}-\frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial^3\psi_0}{\partial\eta\partial z^2}\right)\end{aligned}\right| \tag{3.12}$$

With

$$\left.\begin{aligned}&\psi_1(+1)-\psi_1(-1)=0\\&\frac{\partial\psi_1}{\partial\eta}=0,\ \frac{\partial\psi_1}{\partial\bar{z}}=0,\quad \theta_1=0,\ C_1=0\ \ at\ \ \eta=-1\\&\frac{\partial\psi_1}{\partial\eta}=0,\ \frac{\partial\psi_1}{\partial\bar{z}}=0,\quad \theta_1=0,\ C_1=0\ \ \ at\ \ \eta=+1\end{aligned}\right| \tag{3.13}$$

4. SOLUTIONS OF THE PROBLEM

Solving the equations (3.6)- (3.8) and (3.10) – (3.12) subject to the boundary conditions (3.9) & (3.13) we obtain

$$\theta_{0_0}=0.5\alpha(x^2-1)+0.5Sin(z+\gamma t)(1+x)+0.5(1-x)$$

$$C_0=0.5\left(\frac{Ch(\beta_1 x)}{Ch(\beta_1)}-\frac{Sh(\beta_1 x)}{Sh(\beta_1)}\right)+a_3\left(\frac{Ch(\beta_1 x)}{Ch(\beta_1)}-1\right)$$

$$\psi_0=a_9Cosh(M_1x)+a_{10}Sinh(M_1x)+a_{11}x+a_{12}+\phi_1(x)$$

$$\phi_1(x)=-a_6x+a_7x^2-a_8x^3$$

Similarly the solutions to the first order are

$$\begin{aligned}\theta_1&=a_{36}(x^2-1)+a_{37}(x^3-x)+a_{38}(x^4-1)+a_{39}(x^5-x)+a_{40}(x^6-1)+\\&+(a_{41}+xa_{43})(Ch(M\ \ x)-Ch(M_1)+a_{42}(Sh(M_1x)-xSh(M_1))+\\&+a_{44}(xSh(M_1x)-Sh(M_1))\end{aligned}$$

$$C_1 = a_{47}(1-\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) + a_{48}(x-\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) + a_{49}(x^2-\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) +$$

$$+a_{50}(x^3-\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) + a_{51}(x^4-\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) + a_{52}(Ch(M_1 x) - Ch(M_1)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) +$$

$$+a_{53}(Sh(M_1 x) - Sh(M_1)\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) + a_{54}(xCh(M_1 x) - Ch(M_1)\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) +$$

$$+a_{55}(xSh(M_1 x) - Sh(M_1)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) + b_3(Sh(\beta_2 x) - Sh(\beta_2)\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) +$$

$$+b_4(Sh(\beta_3 x) - Sh(\beta_3)\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) + b_5(Ch(\beta_2 x) - Ch(\beta_2)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) +$$

$$+b_6(Ch(\beta_3 x) - Ch(\beta_2)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) + b_7(xSh(\beta_1 x) - Sh(\beta_1)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) +$$

$$+b_8(x^2 Sh(\beta_1 x) - Sh(\beta_1)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) + b_9(x^3 Sh(\beta_1 x) - Sh(\beta_1)\frac{Ch(\beta_1 x)}{Ch(\beta_1)}) +$$

$$+b_{11}(xCh(\beta_1 x) - Ch(\beta_1)\frac{Sh(\beta_1 x)}{Sh(\beta_1)}) + b_{12}(x^2 Ch(\beta_1 x) - Ch(\beta_1)) +$$

$$+b_{13}(x^3 Ch(\beta_1 x) - Ch(\beta_1)\frac{Sh(\beta_1 x)}{Sh(\beta_1)})$$

$$\psi_1 = d_2 Cosh(M_1 x) + d_3 Sinh(M_1 x) + d_4 x + d_5 + \phi_4(x)$$

$$\phi_4(x) = b_{65}x + b_{66}x^2 + b_{67}x^3 + b_{68}x^4 + b_{69}x^5 + b_{70}x^6 + b_{71}x^7 + (b_{72}x +$$

$$+b_{74}x^2 + b_{77}x^3)Cosh(M_1 x) + (b_{73}x + b_{75}x^2 + b_{76}x^3)Sinh(M_1 x) +$$

$$+b_{78}Cosh(\beta_1 x) + b_{79}Sinh(\beta_{1x} x)$$

Nusselt Number and Sherwood Number

The rate of heat transfer (Nusselt Number) on the walls has been calculated using the formula

$$Nu = \frac{1}{(\theta_m - \theta_w)}(\frac{\partial \theta}{\partial x})_{x=\pm 1}$$

where

$$\theta_m = 0.5\int_{-1}^{1} \theta\, dx$$

and the corresponding expressions are

$$(Nu)_{x=+1} = \frac{1}{\theta_m - Sin(z+\gamma t)}(b_{24} + \delta b_{22})$$

$$(Nu)_{x=-1} = \frac{1}{(\theta_m - 1)}(b_{25} + \delta b_{23})$$

$$\theta_m = b_{26} + \delta\, b_{27}$$

The rate of mass transfer (Sherwood Number) on the walls has been calculated using the formula

$$Sh = \frac{1}{(C_m - C_w)}(\frac{\partial C}{\partial x})_{x=\pm 1}$$

where

$$C_m = 0.5\int_{-1}^{1} C\, dx$$

and the corresponding expressions are

$$(Sh)_{x=+1} = \frac{1}{C_m}(b_{18} + \delta\, b_{16})$$

$$(Sh)_{x=-1} = \frac{1}{(C_m - 1)}(b_{19} + \delta b_{17})$$

$$C_m = b_{20} + \delta b_{21}$$

where a_1, a_2, …………, a_{90}, b_1, b_2, …………, b_{79} are constants given in the appendix

4. Results and Discussion of the Numerical Results

In this analysis we investigate the effect of Hall Current on convective heat and mass transfer flow in a vertical wavy channel with walls taken at $\eta = 1 + \beta \exp(-x^2)$. $\beta>0$ corresponds to dilation of channel walls and $\beta < 0$ represents a constricted channel. In this case we consider the case $\beta > 0$. We apply a magnetic filed of strength H_o, making an angle α to the horizontal .The equations governing the flow, heat and mass transfer which are non-linear and coupled are solved by employing a regular perturbation technique with aspect ratio δ of traveling thermal wave as a perturbation parameter.

The axial velocity 'w' is shown in fig (1) – (8) for different values of G , R M, N, α, Q_1, k, Sc, λ, β and x+γt. The actual axial flow is in the vertically downward direction and hence w>0 represents the reversal flow. Fig (1) represents w with G and R. It is found that 'w' exhibits reversal flow for G < 0 and the region of reversal flow enlarges with increase in G < 0. The maximum |w| enhances with increase in |G| with maximum attained at η = 0. An increase in the Reynolds Number 'R' reduces |w| in entire flow region. The variation of 'w' with 'Hartman Number M' shows that for higher the Lorentz force, w exhibits a reversal flow in the left half and the region of reversal flow enhances with increase M. |w| depreciates with M ≤ 5 and enhances with higher M ≥ 7. The variation of w with Hall parameter m shows that, w exhibits a reversal flow with increase in m ≥ 2.5 and the region of a reversal flow shrinks with increase in m. |w| depreciates with m ≤ 1.5, enhances at m = 2.5 and again depreciates at higher value of M = 3.5 (fig (2).The variation of 'w' with buoyancy ratio 'N' shows that when the molecular buoyancy force dominates over the thermal buoyancy force |w| enhances in the flow region when the buoyancy forces act in the same direction and for the forces acting in opposite directions |w| depreciates in the region. With respect to heat source parameter (α), we find that 'w' exhibits a reversal flow in the right half of the channel with $\alpha \geq 4$. |w| depreciates with increase in $\alpha \leq 4$ and enhances with increases with $\alpha \geq 6$ (fig (3)). Fig (4) represents the variation of w with Chemical reaction parameter 'k', and radiation absorption parameter Q_1. It is found that |w| depreciates with increase in k ≤ 1.5 and enhances with higher k ≥ 2.5. Also |w| depreciates with increase in Q_1 ≤ 1.5 and enhances wit higher Q_1 ≥ 2.5 everywhere in the flow region. The variation of 'w' with Schmidt number 'Sc' shows that lesser the molecular diffusivity larger |w| in the flow region (fig (5)). Fig (6) represents the variation of 'w' with inclination λ of the magnetic field. It is found that 'w' exhibits a reversal flow in the entire flow region with increase in $\lambda \leq \pi/2$. The region of reversal flow enhances with $\lambda \leq \pi$ and reduces with higher $\lambda = 2\pi$. Also |w| depreciates $\lambda \leq \pi/2$, enhances at $\lambda = \pi$ and again depreciates at $\lambda = 2\pi$. The variation w with β is shown in fig (7). It is found that higher the dilation of the channel walls smaller |w| in the flow region. Fig.8 represents 'w' with the phase 'x+γt' of the boundary temperature. It is found that |w| enhances with x+γt ≤ π/2, depreciates at x+γt = π and again enhances at higher x+γt = 2π.

The secondary velocity 'u' is shown in fig (9)- (16) for different parametric values. Fig .9 represents 'w' with G and R. It is found that for G > 0, w is towards the midregion in the left half and is towards the boundary in right half, while a reversed effect is observed for G <0. |w| enhances with increase in |G|. An increasing R depreciates |u| in entire flow region. The variation of u with M shows that for M = 2, u is towards the midregion in the left half and is towards the boundary in the right half. For higher value of M > 5, this behaviour gets reversed. Also |u| depreciates with increase in M. With respect to Hall parameter m ≤ 1.5, u is towards the midregion in the left half and is towards the boundary in the right half, while for m ≤ 2.5, u is towards the boundary in left half and is towards the midregion in the right half . |u| depreciates with m ≤ 1.5 and for |m| =2.5, |u| enhances in left half and reduces in right half and for higher m = 3.5, we notice a depreciation in |u| in entire flow region (fig (10)). Fig.11 represents the variation of u with buoyancy ratio N and Heat source parameter α. It is found that |u| enhances with increase N > 0 and depreciate with |N| in the entire flow region. An increase in the heat source parameter 'α' results in a depreciation in |u|. Fig.12 represents 'u' with k and Q_1. It is found that |u| depreciates with increase in k ≤ 1.5, and enhances with higher k ≥ 2.5. |u| enhances with increase Q_1 ≤ 1.5 and depreciate wit higher Q_1 ≥ 2.5. The variation of 'u' with 'Sc' shows that lesser the molecular diffusivity, larger |u| in the flow region (fig.3). Fig .14 represents u with inclination λ of the magnetic field. It is found that for $\lambda = \pi/4$, u is towards the mid region in the left half and is towards the boundary in the right half. And at $\lambda = \pi/2$, 'u' is towards the midregion except in a narrow region adjacent to η = 1. For still higher $\lambda \geq \pi$, u is towards the boundary in the entire region. |u| enhances in the left half and depreciates in the right half of the channel and for still higher $\lambda = 2\pi$, |u| enhances in the flow region except in the central region 0.2 ≤ η ≤ 0.4. Where it depreciates. The variation of u with β shows that higher the dilation of the channel walls larger |u| in the flow region (fig (15)). The variation of 'u' with phase x+γt shows that |u| depreciates in left half and enhances in the right half with increase in the phase x+γt of the boundary temperature (fig (16)).

The non-dimensional temperature 'θ' is shown in figs (17)- (25) for different parametric values. Fig (17) represents 'θ' with G and R. It is found that the actual temperature enhances in the left half and reduces in the right half with increase in |G|. An increase in R reduces the actual temperature everywhere in fluid region. Higher the Lorentz force, larger the actual temperature in fluid region. An increase in Hall parameter m≤1.5, reduces the temperature in the flow region except in a narrow region adjacent to η =+1 and for higher m = 2.5 we notice an enhancement in θ and for still higher m = 3.5 we notice a depreciation in θ (fig (18)). The variation of 'θ' with radiation parameter 'N1 shows that, the actual temperature reduces in the left half and enhance in the right half with increase in N>0 with buoyancy forces acting in the same direction. and for the forces acting in opposite directions we notice a depreciation in the temperature. An increase in the heat source parameter $\alpha \le 4$ depreciates the actual temperature and for higher $\alpha \ge 6$ the actual temperature enhances in the vicinity of $\eta = \pm 1$ and reduces in the central region (fig (19)). Fig (20) represents 'θ' with increase in the chemical reaction parameter k, and radiation absorption parameter Q_1. It is found that the actual temperature depreciates in the left half and enhances in the right half with increase in $k \le 1.5$ and for higher $k \ge 2.5$, we notice an enhancement in the left half and depreciation in the right half and for still higher k = 3.5 the temperature enhances in the entire flow region. An increase in the radiation absorption parameter Q_1, results in a depreciation in the actual temperature.Fig.21 represents θ with Sc. It is found that lesser the molecular diffusivity smaller the actual temperature in the flow region. The variation of 'θ' with inclination λ of magnetic field is shown in fig (22). It is observed that an increase in $\lambda \le \pi$ results in an enhancement in the flow region and for still higher $\lambda = 2\pi$, the actual temperature depreciates except in a narrow region adjacent to $\eta = 1$ where it enhances. From fig (23) we find that higher the dilation of the channel walls smaller the actual temperature in the fluid region. The variation of 'θ' with phase $x+\gamma t$ of the boundary temperature shows that the actual temperature enhances with increase in $x+\gamma t \le \pi/2$, and for higher $x+\gamma t = \pi$ it depreciates and for still higher $x+\gamma t = 2\pi$, the actual temperature enhances in the flow region (fig (24)). The variation of 'θ' with Prandtl number (P) is shown in fig (25).It shows that lesser the thermal conductivity larger the actual temperature in the region $-0.8 \le \eta \le 0.4$ and we notice a depreciation in the remaining flow region.

The non-dimensional concentration distribution 'C' is shown in fig (26)- (33) for different parametric values. We follow the convention that the non- dimensional concentration is positive or negative according as the actual concentration is greater/lesser than the equilibrium concentration. Fig (26) represents the concentration C with G and R. It is found that the actual concentration enhances with G>0 and depreciates with G<0. An increase in R enhances the actual concentration in the flow region. The variation of 'C' with M and m shows that the actual concentration depreciates in the entire flow region with $M \le 5$ and for higher $M \ge 7$ we notice an enhancement in left half and depreciation in the right half of the channel. An increase in the Hall parameter $m \le 1.5$ results in a depreciation in the actual concentration and for higher m = 2.5, we notice a depreciation in C in the vicinity of $\eta = -1$ and enhances in the right half and for still higher m = 3.5, we find an enhancement in 'θ' in the flow region (fig .27). The variation of C with buoyancy ratio 'N' and Heat source parameter 'α' shows that when the molecular buoyancy force dominates over the thermal buoyancy force, the actual concentration enhances when the buoyancy forces act in the same direction. and for the forces acting in opposite directions the actual concentration depreciates in the flow region. Also an increase in 'α' leads to a depreciation in the actual concentration (fig .28). Fig.29 represents 'C' with chemical reaction parameter k and radiation absorption parameter Q_1. It is found that the actual concentration depreciates with increase $k \le 2.5$ and enhances with higher k = 3.5. An increase in $Q_1 < 1.5$ reduces the actual concentration , enhances with $Q_1 = 2.5$ and again depreciates with higher $Q_1 = 3.5$ (fig.30), We find that lesser the molecular diffusivity larger the actual concentration and for further lowering of the diffusivity we notice a depreciation in the actual concentration (fig .30). The effect of inclination of the magnetic field on C is shown in fig.31. It is found that an increase in $\lambda \le \pi/2$ reduces the actual concentration and for higher $\lambda = \pi$, the actual concentration enhances and for still higher $\lambda = 2\pi$, we notice a depreciation in the actual concentration. The variation of 'C' with 'β' is shown in fig.32. It is found that the dilation of the channel walls larger the concentration and for higher dilation. we notice a depreciation in the concentration everywhere in the fluid region. Form fig.33 we find that an increase in the phase $x+\gamma t$ of the boundary temperature results in an enhancement in the actual concentration everywhere in the flow region.

The rate of heat transfer at $\eta = \pm 1$, is shown in tables (1) – (6) for different values of G, M, m, N α, k, Q_1 and $x+\gamma t$. It is found that the rate of heat transfer enhances at $\eta = +1$ and depreciates at $\eta = -1$ with increase in G>0 while a reversed effect is observed in the behaviour of |Nu| with increase in G<0. The variation of Nu with M shows that at $\eta = +1$, the Nusselt Number reduces with $M \le 4$ and enhances with $M \ge 6$, while at $\eta = -1$ it enhances with M.An increase in the Hall parameter m<2.5 enhances |Nu| at $\eta=+1$ and reduces at $\eta+-1$ and for higher m>3.5 the rate of heat transfer reduces at $\eta = +1$ for all G and at $\eta = -1$ it enhances in the heating case and reduces in the cooling case. Lesser the molecular diffusivity larger |Nu| at $\eta = \pm 1$. The variation of 'Nu' with β shows that higher the dilation of the channel walls smaller |Nu| at both the walls and for further higher dilation larger |Nu| (tables (1) & (4)). The variation of Nu with 'α' shows that the rate of heat transfer at $\eta = +1$ reduces with $\alpha \le 4$ and enhances with higher $\alpha \ge 6$ .At $\eta = +1$, |Nu| reduces with 'α' for all G. The variation with buoyancy ratio N shows with when molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer depreciates at $\eta = -1$ and $\eta = +1$ it enhances for G>0 and depreciates for G<0, when the buoyancy forces act in the same direction and for the forces acting in opposite directions, |Nu| enhances at both the walls. WIth respect to chemical reaction parameter k, we find that the rate of heat transfer at $\eta = +1$, enhance with increase in $k \le 2.5$ and depreciates at higher $k \ge 3.5$, while at $\eta = -1$ it depreciates with k for all G. An increase in the radiation absorption parameter Q_1 enhances |Nu| at both the walls (tables (2) & (5)). An increase in smaller and higher values of $x+\gamma t$ results in a depreciation at $\eta = -1$ and enhances at $\eta = +1$ and for intermediate value of $x+\gamma t = \pi$, we notices an enhancement at $\eta = +1$ and depreciation at $\eta = -1$ (tables (3) & (6)).

The rate of mass transfer (Sh) at $\eta = \pm 1$ is shown in tables (7)- (12) for different parametric values. It is found that the rate of mass transfer depreciates at $\eta = -1$ and enhances at η

= +1 with increase in G>0 while for G<0 we notice an enhancement in |Sh| at both the walls. The variation of 'Sh' with magnetic parameter 'M' shows that the rate of mass transfer at η = +1 depreciates in heating case and enhances in the cooling case with M≤4 and for higher M≥10 we notice an enhancement in |Sh| .At η = -1 it enhance with M for all G. With respect to Hall parameter m, we find that for smaller and higher values of 'm', |Sh| depreciates and enhances with higher M = 3.5 at η = +1 while at η = -1 it enhances for G>0 and depreciates for G<0 with increase in M, Lesser the permeability of the of porous media larger |Sh| at η = +1 and for further lowering of the diffusivity smaller |Sh| at η = -1, |Sh| depreciates in the heating case and enhance in the cooling case with increase in Sc. Channel walls, smaller |Sh| for G>0 and larger |Sh| for G<0 and for still higher β = 0.7 we notice a reversal effect in the behaviour |Sh|. At η = -1 higher the dilation of the walls, smaller |Sh| for all G (tables. (7) & (10)).The variation of Sh with heat source parameter 'α' shows that the rate of mass transfer at η = +1 depreciates with α≤4 and higher M≥6, we notice a depreciation in |Sh|. The rate of mass transfer enhances at η = +1 and depreciates at η = -1 in the heating case and in the cooling case it depreciates at η = +1 and enhances at η = -1. When the buoyancy forces act in the same direction and for the force acting in opposite directions the rate of mass transfer experiences a depreciation at η = +1 and at η = -1 it enhances in the heating case and depreciates in the cooling case. An increase in k≤1.5 reduces |Sh| at η =±1 and enhances it for higher k = 2.5. The variation of Sh with 'Q_1' shows that the rate of mass transfer at η = +1 enhances with increase Q_1 while at η = -1, it depreciates with Q_1≤1.5 and enhances with high Q_1≥2.5. The variation of Sh with x+γt shows that the rate mass transfer enhances for smaller and higher values of x+γt and depreciates at intermediate value of x+γt = π. At η = -1 it depreciates with x+γt ≤ π/2 and enhances with high x+γt ≥ π (tab (9) & (12)).

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