International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

An Inventory Model for Deteriorating Items with Two Parameter Weibull Demand under Inflation

R. Amutha

Assistant Professor of Mathematics, GOVT Arts and Science College, Arakkonam, Tamil Nadu, India

Abstract: ***In this paper we developed a general Inventory Model for perishable products by assuming two parameter Weibull demand and two parameters exponential distribution. We also considered the effect of inflation and the time value money. Shortages are allowed which are completely backlogged. Our aim is to obtain the economic order quantity for minimizing the average total cost per unit time. Numerical examples are also given to illustrate this Inventory model.***

Keywords: Inventory model, Deteriorating, Weibull Distribution, Inflation

1. Introduction

In recent decades, a huge attention has been paid in developing deteriorating inventory models by assuming various factors like demand, deterioration rate, allowing shortages or not, partial backlogging, trade credit, two - ware house system, inflation etc. Now- a -days the impact of inflation is considered widely. Inflation plays a vital role in developing inventory models when the value of money decreases with respect to time. Many inventory models have been developed under inflationary conditions.

Initially Buzcott (1975) incorporated the inflationary conditions and recorded the impact of cost variations. Sushil Kumar et al. (2016) developed a probabilistic inventory model for deterioration items with ramp type demand under inflation. Abolfazl Mirzazadeh (2010) evaluated an inflation - proportional demand rate inventory model. Furthur more both the internal and external inflation rate were considered as time dependent. S. Pradhan et al.(2016) expressed the impact of inflation with two parameter Weibull distribution rate for deterioration and power demand and also adopted a discount - cash - flow approach with trade credit. M.Valliathal et al(2013) determined the effects of inflation with Weibull deterioration rate and partial backlogging. Further the inventory model had been developed under two different types of backlogging rates. M. Palanivel et al (2017), considered a two-warehouse inventory model for a non-instantaneous deteriorating item with inflation and time value money over affixed planning horizon. Shital. S. Patel (2018) examined a different deterioration rate based inventory model with permissible delay in payments under inflation to obtain the maximum profit. Sanjey Kumar et al (2016) designed for stock dependent demand rate with permissible delay in payments under inflationary conditions. A new algorithm was developed to obtain the EOQ. Parvesh Kumar (2016) analyzed inventory models under the environment of inflation so far. Raman Patel et al (2014) investigated a two - ware house inventory model assuming time - varying holding cost under inflation and permissible delay in payments in which shortages are allowed and completely backlogged. Nita Shah et al (2010) worked a demand declining market under inflationary conditions when suppliers offer a permissible delay in payments for a large order that is greater than or equal to the pre - specified quantity. An integrating single supplier producer and single buyer to obtain the optimal number of deliveries and order lot-size, when the joint total cost of the supplier, the producer and the buyer is minimized was derived by N K. Gaur et al (2011). Sanjay Sharma (2011) scrutinized a two - ware house inventory, model with linear trend demand and inflation. Also time depending partial backlog rate consumed. S.R. Singh et al (2011) explained a two -ware house inventory model with partial backordering and time as well as stock dependent demand. The deterioration rate follows two parameter Weibull distribution. Manish Pal et al (2012) examined a dynamic inventory model where demand is price dependent and fixed permissible delay in payments under inflation. A power pattern demand with two-parameter Weibull distribution for deterioration and the influence of inflation inventory model was developed by Srichabdan Mishra et al (2012). Jayjayanti Roy (2014) studied an inflationary inventory model with stock - dependent demand shortages. PoojaD. Khatri et al (2017) described the inventory system for perishable products by considering two - parameter Weibull and Pareto - Type I distribution for deterioration rate along with power pattern demand under inflation.

In this paper, an inventory model has been formulated for perishable products where the demand rate follows a two parameter Weibull distribution under inflationary conditions. Shortages are allowed which are completely backlogged. The objective of the model is to minimize the total cost.

2. Assumptions and Notations

The following are the assumptions and notations used in this inventory model.

- Single inventory system is followed.
- Lead time is zero.
- Shortages are allowed and are completely backlogged.
- Replenishment rate is infinite.
- Time horizon is finite.
- The time value of money and inflation are considered.
- T is the duration of a cycle.
- Q is the initial inventory level.
- The demand rate of any time is $\alpha\beta t^{\beta-1}$ two parameter Weibull distribution, where 0<0 $< \alpha \leq 1$, $\beta > 0$ are called scale and shape parameter respectively.

Licensed Under Creative Commons Attribution CC BY

- $\theta(t) = \frac{1}{\theta}$ is the distribution rate which follows the two parameter exponential distribution $f(t;\mu,\theta) = \frac{1}{\theta} e^{\frac{-(t-\mu)}{\theta}}$, $t \geq \mu$ and $t \geq 0$, where μ is the location parameter, and $\theta > 0$ is scale parameter.
- A is set up cost per order.
- C_1 is deterioration cost per unit time.
- C_2 is shortages cost per unit time.
- I(t) is Inventory level at time t.
- T_1 is the time at which the inventory level reaches zero.
- K(T) is the total cost per unit time.
- R = r-i is the net discount rate and rate of inflation is constant
- i is the inflation rate per unit time
- r is the discount rate representing the time value of money.
- $T_1^*, K(T_1^*)$ and Q^* are the optimal values of length, total average inventory cost and order quantity.

3. Mathematical Model

The cycle starts with the initial inventory level Q. Due to demand or deterioration the inventory level becomes zero when t = T_1. The differential equation governing the instantaneous state (0, T) are given by

$$\frac{dI(t)}{dt} + \frac{1}{\theta} I(t) = -\alpha\beta t^{\beta-1} \quad , \quad 0 \leq t \leq T_1 \quad \text{---------(1)}$$

$$\frac{dI(t)}{dt} = \alpha\beta t^{\beta-1} \quad , \quad T_1 \leq t \leq T \quad \text{---------(2)}$$

With boundary conditions $I(T_1) = 0$ and $I(0) = Q$

The solution of equations (1) and (2)with boundary conditions $I(T_1) = 0$ and $I(0) = Q$ is

$$I(t)=\alpha\beta\left(\frac{T_1-t^{\beta}}{\beta}+\frac{T^{\beta+1}{}_1-t^{\beta+1}}{\theta(\beta+1)}+\frac{T_1^{\beta+2}-t^{\beta+2}}{2\theta^2(\beta+2)}\right) - \frac{\alpha\beta}{\theta}\left(\frac{tT_1^{\beta}-t^{\beta+1}}{\beta}+\frac{tT_1^{\beta+1}-t^{\beta+2}}{\theta(\beta+1)}+\frac{tT_1^{\beta+2}-t^{\beta+3}}{2\theta^2(\beta+2)}\right)+$$
$$\frac{\alpha\beta}{2\theta^2}\left(\frac{t^2T_1^{\beta}-t^{\beta+2}}{\beta}+\frac{t^2T_1^{\beta+1}-t^{\beta+3}}{\theta(\beta+1)}+\frac{t^2T_1^{\beta+2}-t^{\beta+4}}{2\theta^2(\beta+2)}\right) \quad , \quad 0 \leq t \leq T_1 \text{--------------(3)}$$

$$I(t)=\alpha(T_1^{\beta}-t^{\beta}) \quad , \quad T_1 \leq t \leq T \text{---------------(4)}$$

The initial order quantity at t = 0 is

$$Q=\alpha\beta\left(\frac{T_1^{\beta}}{\beta}+\frac{T_1^{\beta+1}}{\theta(\beta+1)}+\frac{T_1^{\beta+2}}{2\theta^2(\beta+2)}\right) \text{-----------------(5)}$$

The total cost consists of the following components SC, DC, SHC and HC

Setup cost per unit time

$$SC = \frac{A}{T} \text{---------------(6)}$$

Deterioration Cost per unit time

$$DC=\frac{C_1}{T}\left[Q-\int_0^{T_1} D(t)\, e^{-Rt}\, dt\right] \quad DC=\frac{C_1}{T}\left[\frac{\alpha\beta(Rt+1)T_1^{\beta+1}}{\theta(\beta+1)}+\frac{\alpha\beta T_1^{\beta+2}}{2\theta^2(\beta+2)}\right] \text{---------------(7)}$$

Shortages cost per unit time

$$SHC=\frac{-C_2}{T}\left[\int_{T_1}^{T}\alpha(T_1^{\beta}-t^{\beta})e^{-Rt}\, dt\right]$$

$$SHC=\frac{-C_2}{T}\left[\alpha T_1(T-T_1)+\frac{\alpha(T^{\beta+1}{}_1-T_1^{\beta+1})}{\beta+1}+\frac{\alpha RT_1^{\beta}(T_1^2-T)}{2}+\frac{\alpha R(T^{\beta+2}-T_1^{\beta+2})}{\beta+2}\right] \text{-------(8)}$$

Inventory Holding Cost per unit time

$$HC=\frac{h}{T}\int_0^{T_1} I(t)e^{-Rt}\, dt$$

$$HC=\frac{h}{T}\left[\alpha\beta\left(\frac{T_1^{\beta+1}}{(\beta+1}+\frac{T_1^{\beta+2}}{\theta(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta^2(\beta+3)}\right)-\frac{\alpha\beta}{\theta}\left(\frac{T_1^{\beta+2}}{2(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta(\beta+3)}+\frac{T_1^{\beta+4}}{4\theta^2(\beta+4)}\right)\right.$$

Licensed Under Creative Commons Attribution CC BY

$$+\frac{\beta\alpha}{2\theta^2}\left(\frac{T_1^{\beta+3}}{3(\beta+3)}+\frac{T_1^{\beta+4}}{3\theta(\beta+4)}+\frac{T_1^{\beta+5}}{6\theta^2(\beta+5)}\right)-\left(\frac{T_1^{\beta+2}}{2(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta(\beta+3)}+\frac{T_1^{\beta+4}}{4\theta^2(\beta+4)}\right)$$

$$+\frac{R\alpha\beta}{\theta}\left(\frac{T_1^{\beta+3}}{3(\beta+3)}+\frac{T_1^{\beta+4}}{3\theta(\beta+4)}+\frac{T_1^{\beta+5}}{6\theta^2(\beta+5)}\right)-\frac{R\alpha\beta}{2\theta^2}\left(\frac{T_1^{\beta+4}}{4(\beta+4)}+\frac{T_1^{\beta+5}}{4\theta(\beta+5)}+\frac{T_1^{\beta+6}}{8\theta^2(\beta+6)}\right)]\text{----(9)}$$

Using equations(6), (7) , (8) and (9) we obtain the average total cost per unit time $K(T_1)$ as $K(T_1)$ = {Setup Cost + Deterioration Cost + Holding Cost + Shortages Cost}

$$K(T_1) = \frac{A}{T}+\frac{C_1}{T}\left[\frac{\alpha\beta(Rt+1)T_1^{\beta+1}}{\theta(\beta+1)}+\frac{\alpha\beta T_1^{\beta+2}}{2\theta^2(\beta+2)}\right]$$

$$-\frac{C_2}{T}\left[\alpha T_1(T-T_1)+\frac{\alpha(T^{\beta+1}{}_1-T_1^{\beta+1})}{\beta+1}+\frac{\alpha R T_1^{\beta}(T_1^2-T)}{2}+\frac{\alpha R(T^{\beta+2}-T_1^{\beta+2})}{\beta+2}\right]$$

$$+\frac{h}{T}\left[\alpha\beta\left(\frac{T_1^{\beta+1}}{(\beta+1}+\frac{T_1^{\beta+2}}{\theta(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta^2(\beta+3)}\right)-\frac{\alpha\beta}{\theta}\left(\frac{T_1^{\beta+2}}{2(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta(\beta+3)}+\frac{T_1^{\beta+4}}{4\theta^2(\beta+4)}\right)\right.$$

$$+\frac{\beta\alpha}{2\theta^2}\left(\frac{T_1^{\beta+3}}{3(\beta+3)}+\frac{T_1^{\beta+4}}{3\theta(\beta+4)}+\frac{T_1^{\beta+5}}{6\theta^2(\beta+5)}\right)-\left(\frac{T_1^{\beta+2}}{2(\beta+2)}+\frac{T_1^{\beta+3}}{2\theta(\beta+3)}+\frac{T_1^{\beta+4}}{4\theta^2(\beta+4)}\right)$$

$$+\frac{R\alpha\beta}{\theta}\left(\frac{T_1^{\beta+3}}{3(\beta+3)}+\frac{T_1^{\beta+4}}{3\theta(\beta+4)}+\frac{T_1^{\beta+5}}{6\theta^2(\beta+5)}\right)-\frac{R\alpha\beta}{2\theta^2}\left(\frac{T_1^{\beta+4}}{4(\beta+4)}+\frac{T_1^{\beta+5}}{4\theta(\beta+5)}+\frac{T_1^{\beta+6}}{8\theta^2(\beta+6)}\right)]$$

----------------(10)

Now our objective is to minimize the total average cost per unit time $K(T_1)$. The necessary and sufficient conditions for total cost $K(T_1)$ to be minimum are $\frac{\partial K(T_1)}{\partial T_1}=0 \; and \; \frac{\partial^2 K(T_1)}{\partial T_1^2}\geq 0$

Hence,

$$\frac{\partial K(T_1)}{\partial T_1}=\frac{C_1}{T}\left(\frac{\alpha\beta(R\theta+1)T_1^{\beta}}{\theta}+\frac{\alpha\beta T_1^{\beta+1}}{2\theta^2}\right)-\frac{C_2}{T}\left(\alpha\beta T T_1^{\beta-1}-\alpha(\beta+1)T_1^{\beta}+\right.$$

$$\alpha T_1^{\beta}+\frac{\alpha R(\beta+2)T_1^{\beta+1}-\alpha\beta R T^2 T_1^{\beta-1}}{2}-\alpha R T_1^{\beta+1}\Big)+\frac{h}{T}\left[\alpha\beta\left(T_1^{\beta}+\frac{T_1^{\beta+1}}{\theta}+\frac{T_1^{\beta+2}}{2\theta^2}\right)-\right.$$

$$\frac{\alpha\beta}{\theta}\left(\frac{T_1^{\beta+1}}{2}+\frac{T_1^{\beta+2}}{2\theta}+\frac{T_1^{\beta+3}}{4\theta^2}\right)+\frac{\alpha\beta}{2\theta^2}\left(\frac{T_1^{\beta+2}}{3}+\frac{T_1^{\beta+3}}{3\theta}+\frac{T_1^{\beta+4}}{6\theta^2}\right)-R\alpha\beta\left(\frac{T_1^{\beta+1}}{2}+\frac{T_1^{\beta+2}}{2\theta}+\frac{T_1^{\beta+3}}{4\theta^2}\right)+$$

$$\frac{R\alpha\beta}{\theta}\left(\frac{T_1^{\beta+2}}{3}+\frac{T_1^{\beta+3}}{3\theta}+\frac{T_1^{\beta+4}}{6\theta^2}\right)-\frac{R\alpha\beta}{2\theta^2}\left(\frac{T_1^{\beta+3}}{4}+\frac{T_1^{\beta+4}}{4\theta}+\frac{T_1^{\beta+5}}{8\theta^2}\right)\Big]=0 \text{ --------------------------(11)}$$

And

$$\frac{\partial^2 K(T_1)}{\partial T_1^2}=\frac{C_1}{T}\left(\frac{\alpha\beta^2(R\theta+1)T_1^{\beta-1}}{\theta}+\frac{\alpha\beta(\beta+1)T_1^{\beta}}{2\theta^2}\right)-\frac{C_2}{T}\left(\alpha\beta(\beta-1)T T_1^{\beta-2}-\alpha(\beta+1)\beta T_1^{\beta-1}+\right.$$

$$\alpha\beta T_1^{\beta-1}+\frac{\alpha R(\beta+2)(\beta+1)T_1^{\beta}-\alpha\beta(\beta-1)R T^2 T_1^{\beta-2}}{2}-\alpha R(\beta+1)T_1^{\beta}\Big)+$$

$$\frac{h}{T}\left[\alpha\beta\left(\beta T_1^{\beta-1}+\frac{(\beta+1)T_1^{\beta}}{\theta}+\frac{(\beta+2)T_1^{\beta+1}}{2\theta^2}\right)-\right.$$

Licensed Under Creative Commons Attribution CC BY

$$\frac{\alpha\beta}{\theta}\left(\frac{(\beta+1)T_1^{\beta}}{2}+\frac{(\beta+2)T_1^{\beta+1}}{2\theta}+\frac{(\beta+3)T_1^{\beta+2}}{4\theta^2}\right)+\frac{\alpha\beta}{2\theta^2}\left(\frac{(\beta+2)T_1^{\beta 12}}{3}+\frac{(\beta+3)T_1^{\beta+2}}{3\theta}+\frac{(\beta+4)T_1^{\beta+3}}{6\theta^2}\right)-$$

$$R\alpha\beta\left(\frac{(\beta+1)T_1^{\beta}}{2}+\frac{(\beta+2)T_1^{\beta+1}}{2\theta}+\frac{(\beta+3)T_1^{\beta+2}}{4\theta^2}\right)+$$

$$\frac{R\alpha\beta}{\theta}\left(\frac{(\beta+2)T_1^{\beta+1}}{3}+\frac{(\beta+3)T_1^{\beta+2}}{3\theta}+\frac{(\beta+4)T_1^{\beta+3}}{6\theta^2}\right)-\frac{R\alpha\beta}{2\theta^2}\left(\frac{(\beta+3)T_1^{\beta+2}}{4}+\frac{(\beta+4)T_1^{\beta+3}}{4\theta}+\frac{(\beta+5)T_1^{\beta+4}}{8\theta^2}\right)]\succ 0$$

------------------------(12)

Numerical Example

To Illustrate this Inventory model the following parameter values have been taken as $\alpha = 0.2$, $\beta = 1.2$, A= 50 , T= 0.2 , R = 0.4 , $\theta = 5$, h= 7 , C_1 = 20 , C_2 = 15 .The optimal values are

$T_1^* = 0.1935$, $K(T_1^*) = 250.2803$ and $Q^* = 0.0285$.

4. Conclusions

In this study, an inventory model for deteriorating items with shortages which are completely backlogged under inflationary conditions is presented and give analytical solution of the model that minimize the total cost. A Numerical example has been given to illustrate this proposed model. Though shortages are completely backlogged, this model can be extended by allowing partial backlogging in future.

References

[1] Abolfazl Mirzazadeh, "Effects of variable Inflationary conditions on an Inventory model with inflation - proportional demand rate", Journal of Applied Sciences 10(7) ;551-557, 2010

[2] Amutha. R and Chandrasekaran. E, "Deteriorating Inventory Model for Two Parameter Weibull demand and shortages", Mathematical Theory and Modeling, Vol3, No.4(20130

[3] Buzacott JA, "Economic order quantity with Inflation"Operational Research Quarterly, 26 , 553-558

[4] Dhir Singh and R.Singh, "Review of Literature Surveys for the Developed Inventory Models", Int.J.Math. and Appl., 6(1-D) , 673-685 (2018)

[5] Gaur N.K, Sathish Kumar and Singh S.R., "An integrated Production - Inventory model for deteriorating items under the effect of inflation ", International Transactions in Applied Sciences, Vol.3 , No.3, pp 543-562 (2011)

[6] Jayjayanti Roy, "A study of an Inflationary Inventory model with stock - dependent demand shortages "International Journal of Mathematics trends and Technology, voi.10, (2014)

[7] Manish Pal, Hareand Krishna Maity, "An Inventory Model for deteriorating Item with permissible delay in payments and Inflation under price dependent demand ", Statistics in the twenty -First century, Special Volume, Vol 8, No.3, pg 583-592 (2012)

[8] Nita H. Shah and Kunal T. Shukla, "Deteriorating Inventory model in demand declining market under Inflation when supplier credits linked to order quantity", Revista Investigacion Operacional, Vol31, N0.2, 95-108 (2010)

[9] Palinivel. M, Priyan.S and Mala. P, "Two - Ware house system for non- instantaneous deterioration products with promotional effort and inflation over a finite time horizon", Journal of Industrial Engineering International , 14;603-612 (2018)

[10] Pooja D. Khatri and Gothi. U.B, "An Inventory model for deteriorating items under Inflation and discount rate with power demand time depending holding cost", International Journal of Advanced Research in Science, Engineering and Technology, Vol 4, Issue 3, March (2017)

[11] Pradhan.S, Shital. R and Tripathy. P.K, "An Inventory model with power demand pattern under Inflation", Revista Investigation Operacional - Vol 37, No: 3, 281-291, (2016)

[12] Praveen Kumar Sharma, "Survey of Inventory models with Inflation", Indian Journal of Research, Paripex, ISSN - 2250-1991(2016)

[13] Raman Patel and Reena U. Parekh, "Inventory model for variable deteriorating items with two - ware house under storages, time varying holding cost, Inflation and Permissible Delay in Payments ", International Referred Journal of Engineering and Science, Vol3, Issue8, pp 06-17

[14] Ruxian Li, Hongjie Lan and John R.Mawwhinney "A Review on deteriorating Inventory Study, Service Sciences & Management, pp117-129

[15] Sanjay Sharma, "An Inventory model for deteriorating products with inflation, lost sales and stock and time - dependent demand", International Transaction in Applied Sciences, vol2, No.1, pp371-384 (2011)

[16] Sanjey Kumar and Neeraj Kumar, "An Inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm", Cogent Business and Management , 3:1 , 1239605(2016)

[17] Shital S.Patel , " Different deterioration rates Two - Ware house Inventory model with time and price dependent demand under Inflation and permissible delay in payments and shortages" , International Journal of Engineering Research and Development , Vol 14, Issue 3, (2018)

[18] Singh S.R, Malik A.K and Sathish Kumar, "Two - Ware house Inventory model with partial backordering and multi - variable demand under inflation", International Journal of Operations Research and Optimization, Vol2, No.2, pp 371-34

[19] Srichandan Mishra, RajuL.K, Misra U.K and Misra G "Astudy of EOQ model with power demand of

Licensed Under Creative Commons Attribution CC BY

deteriorating items under the influence of Inflation", Gen Math Notes, Vol.10, No.1 pp41-50 (2012)

[20] Sushil Kumar and Rajput U.S "A probabilistic Inventory model for deteriorating items with ramp type demand rate under inflation", American Journal of Operations Research , 6(1), pp:16-31 (2016)

[21] Valliathal M and UthayakumarR, "A study of Inflation effects in an EOQ model for Weibull deteriorating, ameliorating items with ramp type of demand and shortages", Yugoslav Journal of Operations Research 23, No.3, 441-455

Licensed Under Creative Commons Attribution CC BY