Double Diffusive Convection on Peristaltic Transport of a Jeffrey Fluid

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Abstract: ***The present research paper concentrates on the peristaltic flow of a Jeffrey nanofluid with double diffusion under the assumption of long wavelength procedure. The importance of double diffusion is natural process and engineering applications. The influence of double diffusion is the heat and mass transfer occurs concurrently with the complicity of the fluid motion. The mathematical modeling of the present problem includes the nonlinear partial differential equations. Those equations are solved by using Homotopy Analysis Method. Solutions are described as velocity, temperature, solutal nanoparticle concentration and nanoparticle volume fraction profiles. Obtained results are displayed graphical form.***

Keywords: double diffusion,Nanofluids, Peristaltic Flow, Jeffrey fluid model, Channel

1. Introduction

Nowadays, Peristaltic flow is one of the most important pumping mechanisms. Peristaltic flow problems have been special attention because huge number of applications in numerous fields of chemical industries, biomedical, engineering, nuclear reactors and physiology. In particular peristaltic pumping mechanisms involved in many biological systems like urinary system, swallowing of food through the esophagus and movement of chime in the gastrointestinal tract. Peristalsis was first initiated by Latham [1] in 1966. Further, this mechanism has become an important concept of research to the above-mentioned applications. This work was further extended by Shapiro et al. [2] and Jaffrin et al. [3].

Homogeneous mixture of nanoparticles consist of a base fluids is called nanofluids. The nanoparticles employed in nanofluids are made of carbides, metals, oxides and carbon nanotubes. base fluids consist of oil, water and ethylene glycol. The nanofluids model was first initiated by Choi [4]. nanofluids can be enhanced by thermophysical properties compared to ordinary fluids. Nanofluids have so many applications in industries. Various types of nanoparticles such as oxide ceramics carbide ceramics metals and carbon nanotubes [5].Many researchers have studied both theoretical and practical aspects of the peristaltic flow of non-Newtonian fluids. Jeffrey fluid is a viscous fluid for which viscosity of fluid reduces with enhancing the shear stress. Hayat et al. [6] analyzed compressible Jeffrey fluid in a circular tube on peristaltic flow. Recently, some researchers [7-8] have worked on the area of peristaltic flow of Jeffrey fluid.

In recent times, the researcher has been focused on double-diffusive convection on peristalsis. The peristaltic flow of nanofluid diffusion investigated by Noreen et al. [9]. The heat transfer and also the mass transfer occurs concurrently with the complicity of the fluid motion is known as double diffusion. Double diffusion has important applications in solid-state physics, chemical engineering, geophysics, oceanography, astrophysics and biology [10] and also many engineering applications like natural gas storage tanks, solar ponds, metal solidification processes, and crystal manufacturing. The research on double diffusion was continued by Nield et al. [11] by considering the double diffusion-convection over a nanofluid. Kuznetsov [12] have also contributed a lot of work on double diffusion.

The problem considered in the current research work has potential biomedical, industrial and engineering applications. The investigation of double-diffusive convection on peristaltic flow is an innovative idea of research. In review the literature, we noticed that many researchers have worked on the peristaltic flow, separately. Especially, no investigation is carried out to demonstrate the Jeffrey fluid over a channel in the presence of nanofluids. Moreover, the concept of adding Double-diffusive problems has wide range of practical applications in oceanography, chemical engineering, geophysics, astrophysics and biology. Further, the nonlinear partial differential equations modeled in the current research work are overcomes by utilizing non-dimensional quantities. Those equations are solved by utilizing Homotopy Analysis Method [13].

2. Mathematical Analysis

Consider the peristaltic flow through a asymmetric channel.

$$Y' = h_1' = d_1 + a_1 \cos\left[\left(X' - ct'\right)\frac{2\pi}{\lambda}\right], \tag{1}$$

$$Y' = h_2' = -d_2 - b_1 \cos\left[\phi + \left(X' - ct'\right)\frac{2\pi}{\lambda}\right]. \tag{2}$$

Here λ is the wavelength, a_1 and b_1 are the amplitudes of the waves, ϕ is the phase difference with d_1, d_2 are satisfies the below condition.

$$b_1^2 + 2a_1 b_1 \cos\phi + a_1^2 \le \left(d_2^2 + d_1^2\right). \tag{3}$$

The velocity V can be written in the form of vector components, where U' and V' are the velocity components

$$V = \left(U'\left(X', Y', t'\right), V'\left(X', Y', t'\right), 0\right). \tag{4}$$

Consider the Jeffrey fluid model is defined by

$$\tau = \frac{\mu}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right), \tag{5}$$

where

$$\dot{\gamma} = \nabla \vec{V} + \left(\nabla \vec{V}\right)^T. \tag{6}$$

The Jeffrey fluid stress tensor τ components are

$$\tau_{XX} = \frac{2\mu}{1+\lambda_1}\left[\lambda_2\left(V'\frac{\partial}{\partial Y'} + U'\frac{\partial}{\partial X'}\right)+1\right]\frac{\partial U'}{\partial X'}, \tag{7}$$

$$\tau_{XY} = \tau_{YX} = \frac{\mu}{1+\lambda_1}\left[\lambda_2\left(V'\frac{\partial}{\partial Y'} + U'\frac{\partial}{\partial X'}\right)+1\right]\left(\frac{\partial U'}{\partial Y'} + \frac{\partial V'}{\partial X'}\right), \tag{8}$$

$$\tau_{YY} = \frac{2\mu}{1+\lambda_1}\left[\lambda_2\left(V'\frac{\partial}{\partial Y'} + U'\frac{\partial}{\partial X'}\right)+1\right]\frac{\partial V'}{\partial X'}. \tag{9}$$

In the above equations, λ_1 is the ratio of relaxation to retardation time, $\dot{\gamma}$ is the vector quantity of the shear rate and dots denotes the differentiate with respect to time, λ_2 is the retardation time, μ is the dynamic viscosity.

The governing equations for Jeffrey nanofluid can be written as

$$\frac{\partial U'}{\partial X'} + \frac{\partial U'}{\partial Y'} = 0. \tag{10}$$

$$\rho_f\left(\frac{\partial U'}{\partial t'} + U'\frac{\partial U'}{\partial X'} + V'\frac{\partial U'}{\partial Y'}\right) = -\frac{\partial p'}{\partial X'} + \frac{\partial}{\partial Y'}\left(\tau_{YX}\right) + \frac{\partial}{\partial X'}\left(\tau_{XX}\right) - g\left(\rho_p - \rho_{f0}\right)\left(F' - F'_0\right) - \left(\rho_p - \rho_f\right) g\left(C' - C'_0\right) + \left(1-\phi_1\right)\rho_f g\beta\left(T' - T'_0\right), \tag{11}$$

$$\rho_f\left(\frac{\partial V'}{\partial t'} + U'\frac{\partial V'}{\partial X'} + V'\frac{\partial V'}{\partial Y'}\right) = -\frac{\partial p'}{\partial Y'} + \frac{\partial}{\partial Y'}\left(\tau_{YY}\right) + \frac{\partial}{\partial X'}\left(\tau_{XY}\right). \tag{12}$$

$$(\rho c)_f\left(\frac{\partial T'}{\partial t'} + U'\frac{\partial T'}{\partial X'} + V'\frac{\partial T'}{\partial Y'}\right) = k\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right) + D_{TC}\left(\frac{\partial^2 \varphi'}{\partial X'^2} + \frac{\partial^2 \varphi'}{\partial Y'^2}\right) (\rho c)_p D_B\left(\frac{\partial F'}{\partial X'}\frac{\partial T'}{\partial X'} + \frac{\partial F'}{\partial Y'}\frac{\partial T'}{\partial Y'}\right) + (\rho c)_p \frac{D_T}{T_m}\left[\left(\frac{\partial T'}{\partial X'}\right)^2 + \left(\frac{\partial T'}{\partial Y'}\right)^2\right], \tag{13}$$

$$\frac{\partial \varphi'}{\partial t'} + U'\frac{\partial \varphi'}{\partial X'} + V'\frac{\partial \varphi'}{\partial Y'} = D_S\left(\frac{\partial^2 \varphi'}{\partial X'^2} + \frac{\partial^2 \varphi'}{\partial Y'^2}\right) + D_{CT}\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right). \tag{14}$$

$$\frac{\partial F'}{\partial t'} + U'\frac{\partial F'}{\partial X'} + V'\frac{\partial F'}{\partial Y'} = D_B\left(\frac{\partial^2 F'}{\partial X'^2} + \frac{\partial^2 F'}{\partial Y'^2}\right) + \frac{D_T}{T_m}\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right). \tag{15}$$

where D_{CT} and D_{TC} are the Soret diffusivity and Dufour diffusivity, g is the acceleration due to gravity, $(\rho c)_p$ and $(\rho c)_f$ are the effective heat capacity of the nanoparticle material and heat capacity of the fluid, k is the thermal conductivity of the fluid and F' is nanoparticle volume fraction. Further ρ_f is the effective density, D_S is the solutal diffusivity

Corresponding Boundary conditions are

$$\left.\begin{aligned} &\psi' = \frac{q}{2}, u' = \frac{\partial \psi'}{\partial \eta'} = -c, T' = T'_0, \varphi' = \varphi'_0, F' = F'_0 \quad \text{at} \quad Y' = h'_1 = d_1 + a_1\cos\left((X' - ct')\frac{2\pi}{\lambda}\right), \\ &\psi' = -\frac{q}{2}, u' = \frac{\partial \psi'}{\partial \eta'} = -c, T' = T'_1, \varphi' = \varphi'_1, F' = F'_1 \quad \text{at} \quad Y' = h'_2 = -d_2 - b_1\cos\left(\phi + (X' - ct')\frac{2\pi}{\lambda}\right). \end{aligned}\right\} \tag{16}$$

Relationship between the wave frame and laboratory frame are introduced through

$u' = U' - c, \qquad x' = X' - ct', \qquad y' = Y', \qquad v' = V'.$

The following dimensionless physical parameters and variables

$$\left.\begin{aligned}
&X=\frac{X'}{\lambda},\psi=\frac{\psi'}{ca},\ Y=\frac{Y'}{d_1},\ x=\frac{x'}{\lambda},\ y=\frac{y'}{d_1},\ t=\frac{ct'}{\lambda},\ \ \delta=\frac{d_1}{\lambda},u=\frac{u'}{c},\alpha=\frac{k}{(\rho c)_f},\ \theta=\frac{T'-T'_0}{T'_1-T'_0},d=\frac{d_1}{d_2},\\
&\varphi=\frac{\varphi'-\varphi'_0}{\varphi'_1-\varphi'_0}\ \ \gamma=\frac{F'-F'_0}{F'_1-F'_0},h_1=\frac{h'_1}{a_1},v=\frac{v'}{c},\ h_2=\frac{h'_2}{a_2},\ N_{CT}=\frac{D_{CT}\left(T'_1-T'_0\right)}{D_S},a=\frac{a_1}{d_1},b=\frac{b_1}{d_1},\ \mathrm{Re}=\frac{\rho_f c d_1}{\mu},\\
&F^*=\frac{q}{ca},N_{TC}=\frac{(\rho c)_f D_{TC}\left(\varphi'_1-\varphi'_0\right)}{k^*\left(T'_1-T'_0\right)},Gr_T=\frac{\left(1-\phi_1\right)\rho_f g\beta b^2\left(T'_1-T'_0\right)}{c\mu},\ Gr_C=\frac{\left(\rho_p-\rho_f\right)\left(\varphi'_1-\varphi'_0\right)}{\left(1-\phi_1\right)\beta\left(T'_1-T'_0\right)\rho_f},\\
&Nb=\frac{(\rho c)_p D_B\left(\varphi'_1-\varphi'_0\right)}{(\rho c)_f\nu},\ Nt=\frac{(\rho c)_p D_T\left(T'_1-T'_0\right)}{(\rho c)_f T_m\nu},Gr_F=\frac{\left(\rho_p-\rho_{f0}\right)gb^2\left(F'_1-F'_0\right)}{c\mu},\Pr=\frac{\mu}{\alpha},
\end{aligned}\right\} \quad (17)$$

where Gr_F is the nanoparticle Grashof number, Re is Reynolds number, Nt is thermophoresis parameter, N_{CT} is the Soret parameter, Gr_C is the solutalGrashof number, Pr is Prandtl number, Gr_T is the thermal Grashof number, Nb is Brownian motion parameter, N_{TC} is Dufour parameter.

Velocity can be written in the form of stream function $\left(v=-\delta\frac{\partial\psi}{\partial x}\text{ and } u=\frac{\partial\psi}{\partial y}\right)$ manipulate the equations (11)-(12) and also applying long wavelength procedure, equations (10)-(17) can be written as

$$\frac{\partial p}{\partial x}=\frac{1}{1+\lambda_1}\frac{\partial^3\psi}{\partial y^3}+Gr_T\theta+Gr_C\varphi-Gr_F\gamma, \quad (18)$$

$$\frac{\partial p}{\partial y}=0, \quad (19)$$

$$\frac{1}{1+\lambda_1}\frac{\partial^4\psi}{\partial y^4}+Gr_T\frac{\partial\theta}{\partial y}+Gr_C\frac{\partial\varphi}{\partial y}-Gr_F\frac{\partial\gamma}{\partial y}=0, \quad (20)$$

$$\frac{\partial^2\theta}{\partial y^2}+Nt\Pr\left(\frac{\partial\theta}{\partial y}\right)^2+N_{TC}\Pr\frac{\partial^2\varphi}{\partial y^2}+Nb\Pr\frac{\partial\theta}{\partial y}\frac{\partial\gamma}{\partial y}=0, \quad (21)$$

$$\frac{\partial^2\varphi}{\partial y^2}+N_{CT}\frac{\partial^2\theta}{\partial y^2}=0, \quad (22)$$

$$\frac{\partial^2\gamma}{\partial y^2}+\frac{Nt}{Nb}\frac{\partial^2\theta}{\partial y^2}=0. \quad (23)$$

The corresponding non dimensional boundary conditions are

$$\left.\begin{aligned}
&\psi=\frac{F^*}{2},\quad \frac{\partial\psi}{\partial y}=-1,\quad \theta=0,\quad \varphi=0,\quad \gamma=0\quad \text{at}\quad y=h_1=1+a\cos x,\\
&\psi=-\frac{F^*}{2},\quad \frac{\partial\psi}{\partial y}=-1,\quad \theta=1,\quad \varphi=1\quad \gamma=0\quad \text{at}\quad y=h_2=-d-b\cos(x+\phi).
\end{aligned}\right\} \quad (24)$$

3. Results and Discussion

From the above mentioned description of the considered problem the system of non linear coupled partial differential equations is obtained. Such system is difficult to solve explicitly to get exact solutions. However the advancement of techniques during last few decades has provided more efficient ways to solve the complex non-linear models. Thus the problem in hand is approximated semi-analytical method (Homotopy Analysis Method) in Mathematica software and graphical results are drawn in Origin. Therefore this section comprises the development of velocity, temperature, solutal (species) concentration and nanoparticle volume fraction corresponding to variation of Gr_c is the solutal Grashof number, Gr_T is the thermal Grashof number, Gr_F is the nanoparticle Grashof number, Nb is Brownian motion parameter, Nt is thermophoresis parameter, N_{TC} is Dufour parameter and N_{CT} is the Soret parameter.

The effects of Gr_C, Gr_F, Gr_T and λ_1 on the velocity profile $u(y)$ are representing through the figures 1 to 4. Figure 1 investigates the effect of Solutal Grshof number on the velocity profile. The graphical result indicates that increases in the values of Gr_C on velocity Increases. However the effect Gr_C more pronounced. It is apparent that increases in Gr_C reduce the velocity of the wall which causes an enhanced the thermal boundary layer. Figure 2 is developed to examine the nanoparticle Grashof number effect on velocity profile. The displayed results portray the enhancing Gr_F on velocity profile. Since, an increase in Gr_F causes reduction in viscosity. Figure 3 demonstrates the effect of Gr_T on velocity profile. The displayed results is increase in Gr_T tends to reduce velocity. Since, Gr_T with its increasing values of enhances the fluid concentration. Figure 4 includes the effect of Jeffrey fluid parameter λ_1. Velocity profile appears to increases in all the region of the peristaltic pumping. When λ_1 is enhanced.

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The effect of temperature profile has been shown through figures 5 to 8. The variation of temperature profile against the Brownian motion parameter Nb is displayed in figure 5. Figure 5 concludes that temperature profile rising with the increase in the Brownian motion parameter. Figure 6 discloses that the temperature profile is increasing with the thermophoresis parameter Nt. Figure 7 represents the impact of Soret parameter N_{CT} on the temperature profile. Figure 7 depicts that increasing the Soret parameter increases the temperature. Figure 8 observes the effect of the Dufour effect N_{TC} on the temperature profile. Here temperature profile increases with an increasing the Dufour effect.

Figures 9 to 10 are shown to examine the solutal (species) concentration profile via N_{CT} and N_{TC}. effects N_{CT} and N_{TC}. are studied via figures 9 to 10. From these two figures, one can see that the solutal (species) concentration profile have the similar behavior on both N_{CT} and N_{TC}.

Figures 11 to 12 depict the nanoparticle concentration visualizations for the influence of Nb and Nt of peristaltic flow in the presence of nanofluid. Fig. 11 shows that nanoparticle volume fraction of fluid enhanced with enhancing the of Brownian motion parameter. It is due to the fact the Nb forces. The particles in the opposite results and makes the nanoparticles are more homogeneous. However, the opposite results are seen in the case of the thermophoresis parameter (see figure 12).

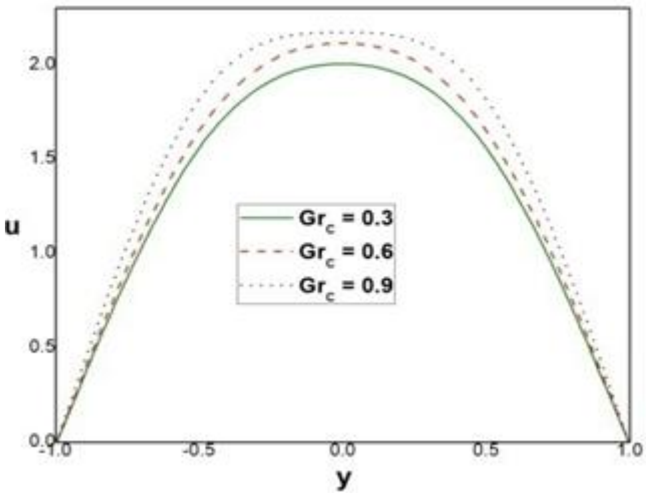

Figure 1: Impacts of embedded parameters on velocity profile various values $Gr_F = 1.2, Gr_T = 1.2$ and $\lambda_1 = 0.5$;

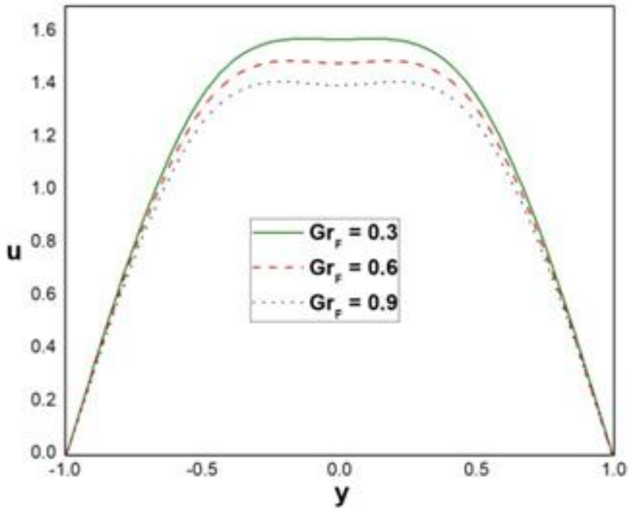

Figure 2: Impacts of embedded parameters on velocity profile various values $Gr_T = 1.2$, $Gr_C = 1.2$ and $\lambda_1 = 0.5$;

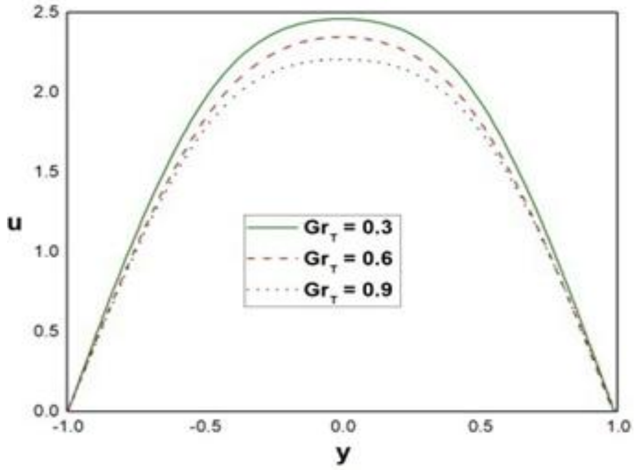

Figure 3: Impacts of embedded parameters on velocity profile various values $Gr_F = 1.2$, $Gr_C = 1.2$ and $\lambda_1 = 0.5$;

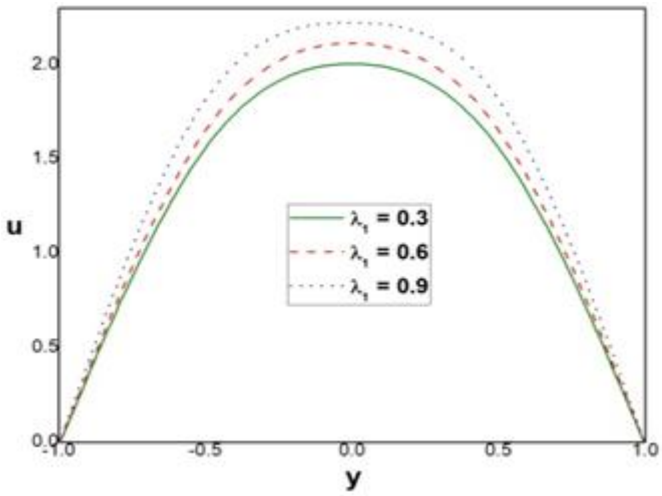

Figure 4: Impacts of embedded parameters on velocity profile various values $Gr_T = 1.2$, $Gr_C = 1.2$ and $Gr_F = 1.2$.

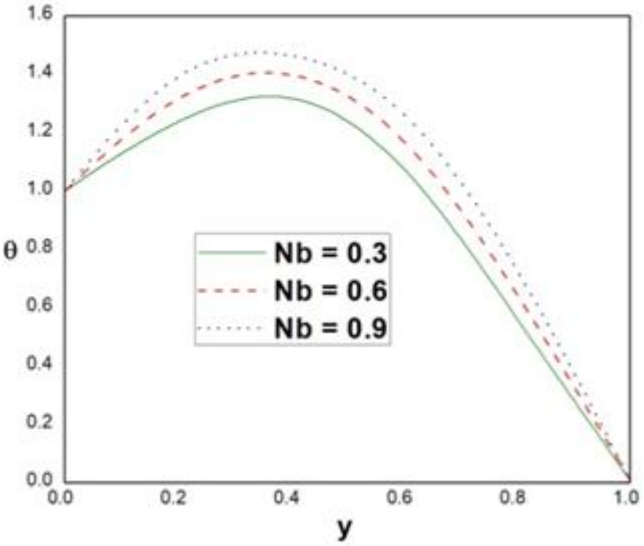

Figure 5: Impacts of physical parameters on temperature profile for various values $Nt = 1.2, N_{TC} = 1.2$ and $N_{CT} = 1.2$;

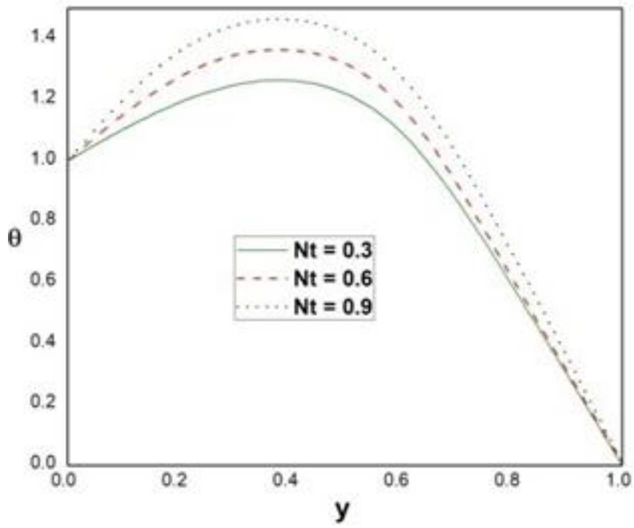

Figure 6: Impacts of physical parameters on temperature profile for various values $Nb = 1.2, N_{TC} = 1.2$ and $N_{CT} = 1.2$;

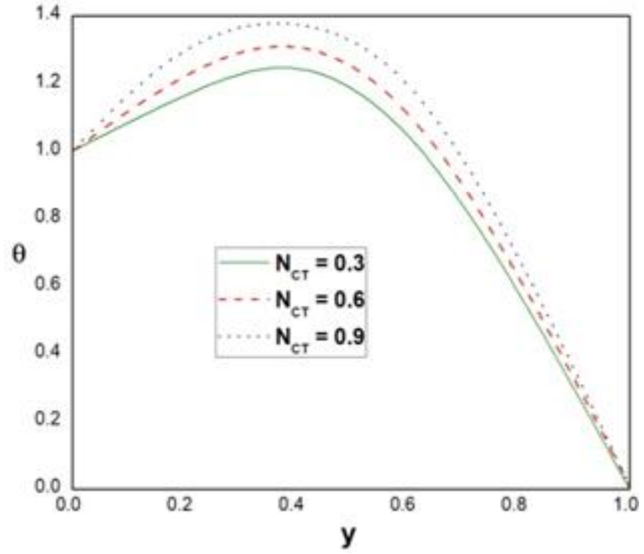

Figure 7: Impacts of physical parameters on temperature profile for various values $Nt = 1.2, Nb = 1.2$ and $N_{CT} = 1.2$;

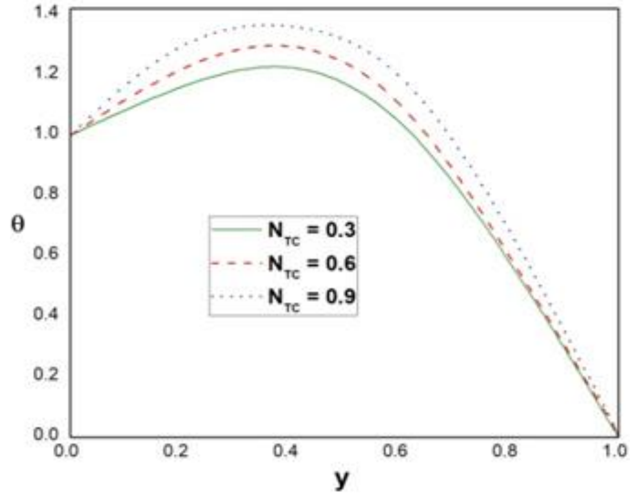

Figure 8: Impacts of physical parameters on temperature profile for various values $Nt = 1.2, Nb = 1.2$ and $N_{CT} = 1.2$.

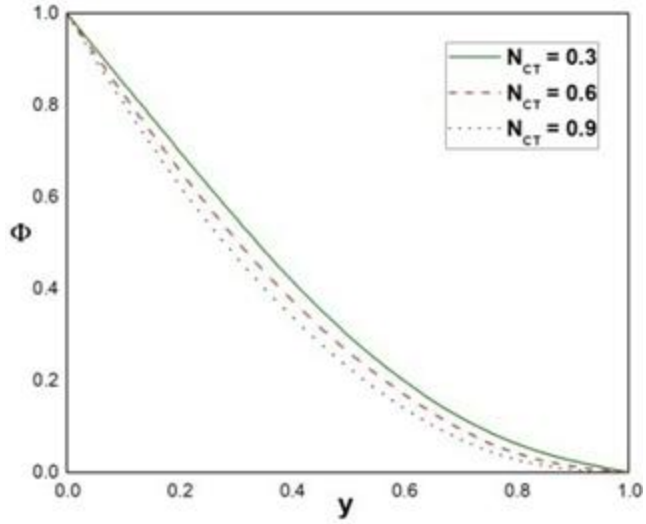

Figure 9: Impacts of physical parameters on solutal (species) concentration profile with $N_{TC} = 1.2$;

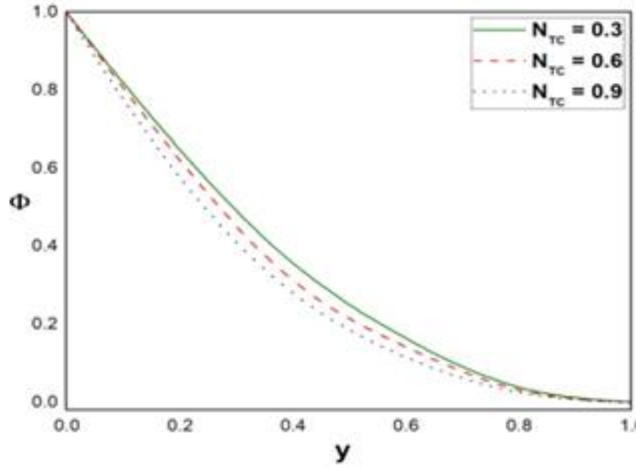

Figure 10: Impacts of physical parameters on solutal(species) concentration profile with $N_{CT} = 1.2$.

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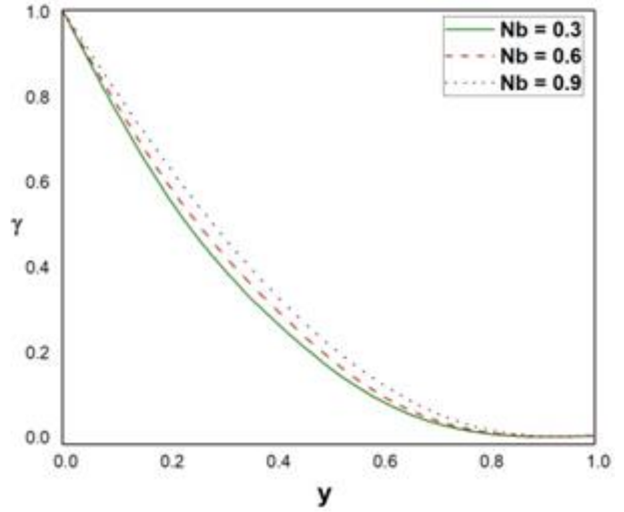

Figure 11: Impacts of physical parameters on nanoparticle volume fraction profile with $Nt = 1.2$;

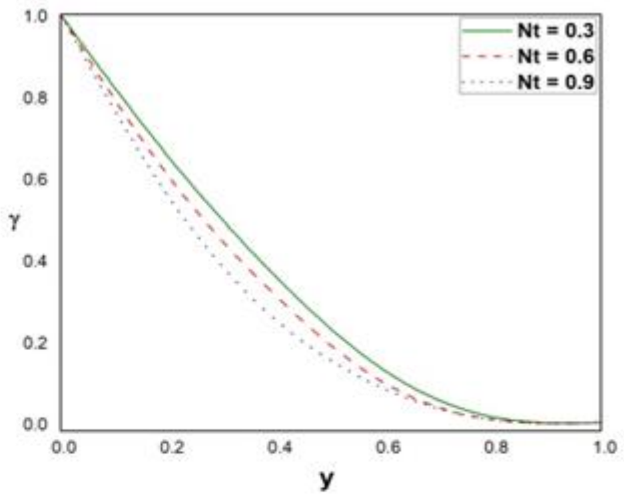

Figure 12: Impacts of physical parameters on nanoparticle volume fraction profile with $Nb = 1.2$.

4. Conclusion

This paper analyzes the double diffusion on peristaltic flow of Jeffrey fluid model. The important points are listed below

- The presence of temperature and one concentration diffusion together to create buoyancy is known as double diffusion convection.
- Velocity increases for larger heat transfer solutal Grashof number however reverse the effect is noted for increasing mass transfer thermal Grashof number and velocity is decreased by nanoparticle Grashof number.
- Velocity profile is increasing function of relaxation to retardation time λ_1. Behavior of Gr_T and Gr_C on temperatureare too similar and increase in N_{TC} and N_{CT} slightly enhanced the temperature of the wall surface.
- The similar behavior of N_{TC} and N_{TC} on solutal (species) concentration profile. Here solutal concentration profile reduced with higher values of N_{TC} and N_{CT}. Opposite behavior of Nb and Nt on nanoparticle volume fraction profile.

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