A Model for Predicting Forest Growth and Harvesting Level

P. K. Nyarko[1], J. Acquah[2]

[1]University of Mines and Technology, P. O. Box 237, Tarkwa, Ghana
Corresponding Author Email: *pknyarko[at]umat.edu.gh*

[2]University of Mines and Technology, P. O. Box 237, Tarkwa, Ghana
Email: *jacquah[at]umat.edu.gh*

Abstract: ***The study of forest dynamics is concerned with the changes in forest structure and composition over time,and it is obvious that forests managed in a sustainable manner are able to produce both high quality wood products and other ecological goods and services which include water purification, wildlife habitat and carbon sequestration. Similarly, forest management practices that protect aquatic systems, minimise soil disturbance and erosion, and promote rapid forest regeneration are all components of sustainable forestry. These factors have therefore necessitated this study of modelling forest growth and its associated harvesting policy for determining future sequence in order not to compromise its benefits. A number of growth models such as The Malthusian Growth Model, The Logistic Growth Model, Verhulst Logistic Growth Scaled with a 'Delaying' Factor $\left[1+c\left(N/K\right)\right]^{-1}$, and Richard's Growth Equation were some of the growth models mentioned in this study of the dynamics of the forest stand.The two possible equilibrium values for the solution of the model are $P_e = 0$ and $P_e = 1000\left(1-5h\right)$. Whenever $h = r$, this will lead to extinction of the population and therefore as $h < 0.2 = r$, the population size will settle down to the stable equilibrium $P_e = 1-5h$. Hence for as long as $0 \le h < 0.2$ one can be confident that, the population won't die out. Again, $h = 0.1$ which is one-half of the growth rate r yields the maximum value of harvest, $h\left(1-5h\right)$ for any value of the harvesting rate h. The model is applicable to most forest reserves in Ghana for sustainable harvesting.***

Keywords: sustainable, Forest, Harvest, growth, ecological, carbon sequestration

1. Introduction

Forests are long-lived biological systems that are continuously changing. The study of forest dynamics is concerned with the changes in forest structure and composition over time, including its behaviour in response to anthropogenic and natural destructions. Tree growth and forest destructions are primary evidence of forest dynamics. They are determined by resources such as water, nutrients supply and environmental conditions which includes temperature, soil acidity, air pollution and human activities. Forest management decisions are made based on information about both current and future resource conditions. Inventories taken at one instant in time provide information on current wood volumes and related statistics. Growth and yield models describe forest dynamics (it's growth, mortality, reproduction and associated changes in the stand) over time. These models have been widely used in forest management because of their ability to update inventories, predict future yields and to explore management alternatives and silvicultural options, thus providing information for decision-making (Burkhart, 1990; Vanclay, 1994).

Human activities affect forest growth in many diverse ways. To begin with, they influence the composition, cover, age and density of the vegetation. The landscapes for these forests systems are altered by human activities. Thus, changing the kinds of stands present and their spatial arrangement, which influences the movement of wind, water, animals and soils. At the regional level, we introduce by-products into the air that may fertilize or kill forests. At the global scale, human consumption of fossil fuels has increased atmospheric carbon dioxide levels and changed the way that carbon is distributed in vegetation, soils and the atmosphere, with implications on global climate. The worldwide demand for forests products has stimulated not only the transfer of processed wood products from one country to another, but also the introduction of non-native tree species, along with associated pests, that threaten native forests and fauna. While the management of forested lands is becoming increasingly important, it is also becoming more contentious because less forested land is available for an increasing range of demands.

It is becoming increasingly clear that, forests that are managed in a sustainable manner are able to produce both high quality wood products and other ecological goods and services such as water purification, wildlife habitat and carbon sequestration. Forest management practices that protect aquatic systems, minimise soil disturbance and erosion, and promote rapid forest regeneration are all components of sustainable forestry. When forests are managed in a sustainable manner, the environmental values can be explained along the following lines of action:

1) The forests play an important role in our water cycle by pumping water from the soil back into the atmosphere through transpiration. This process also helps to cool the surrounding environment;
2) The forests stabilise the soil and reduce erosion and sedimentation into aquatic systems thereby help maintain water quality and

3) The forests remove airborne particles and ozone from our air and improve air quality.
4) In similar manner, forests have natural economic values that are often overlooked by society. When forests are degraded, there is a financial cost incurred by society to replace the lost ecological goods and services through the following:
5) Increased water treatment cost,
6) Increased illness and health care costs due to decreased water and air quality,
7) Decreased property value due to degraded aesthetic qualities, and also
8) Decreased revenues from tourism and other non-timber commercial activities associated with healthy ecosystems.

It is therefore worth noting that "the contribution of forests to a country's economy, environment and social well-being is significant. Our forests therefore form an important part of the roots as a nation and a big part of our future. Taking care of them and ensuring their ongoing health, is a key priority" (Natural Resources Canada, 2006).

Despite these numerous benefits of our forest to the country's economy, environment and social well-being, the level of degradation of the forest reserves is high. Studies on the forests therefore cannot be over emphasized due to its economic, environmental and health importance to the society.

The early studies on forests growth were basically on continuous population dynamics and the original research on growth models was attributed to Thomas Malthus (1798). He was therefore considered as the originator of growth models.

Nye and Spiers (1964) developed the partial differential equations used to describe simultaneous mass flow and diffusion for nutrient uptake by a unit length of root. Nye and Marriot (1969) defined boundary conditions for the equations and solved them numerically, while Baldwin *et al.* (1973), on the other hand, solved the equations analytically with steady state approximations. Their work became the foundation for mechanistic nutrient uptake models. Building on this, Claassen and Barber (1976), Nye and Tinker (1977), Barber and Cushman (1981), Claassen *et al.* (1986), Smethurst and Comerford (1993b), Yanai (1994), Smethurst *et al.* (2004), and Comerford *et al.* (2006) proposed model revisions to cover the major sub-processes of nutrient uptake and to accommodate a variety of additional conditions. Other researchers such as Wu *et al.* (1985), Wu *et al.* (1994) and Sharpe *et al.* (1985) modelled the physical growth of the forest by considering the influence of stem, crown and roots. Others just considered the effect of either one of the following: availability of light, surface water or nutrients to the growth of the tree and subsequently to the growth of the forest.All in the quest to delve into the growth of the forest and how best to manage it.The timber in our forest reserves must be utilised despite its environment, economic and social benefitssince our very existence demands that we use the products of timber. Lumber and other forests products are therefore needed in our daily lives. This indicates that, the forest must be exploited. It is therefore necessary to formulate and institute appropriate management policies to check over exploitation of the timber in the forests reserves whose total depletion will have an adverse effect on the very existence of man (Nyarko, *et al*., 2010). These and many other factors have therefore necessitated the study into the modelling using non -linear models for determining future sequence of harvests from the forest. It has been an inherent part of forest management planning and decision-making to determine the optimal harvesting schedules for the forestry industry.

2. Materials and Methods

Many growth models have been applied to a number of growth data to determine the dynamics of the strand. A number of candidate models of continuous population dynamics have been outlined in this study. The essence is to determine a feasible growth model which can incorporate harvesting parameter (h) for predicting harvesting levels.

2.1 The Malthusian Growth Model

Thomas Malthus proposed an exponential growth model and assumed that, if $N(t)$ is the number of individuals in a population at time t, and let b and d be the average per capita birth and death rates, respectively, then in a short time Δt, the number of births in the population is $b\Delta tN(t)$, and the number of deaths is $d\Delta tN(t)$. Thus, the change in population between the times t and $(t+\Delta t)$ is determined by the relation $N(t+\Delta t)-N(t)=\Delta t(b-d)N(t)$ which can be rearranged as $(N(t+\Delta t)-N(t))/\Delta t=(b-d)N(t)$. Now, the limit as $\Delta t\to 0$, this expression was obtained as Equation (1)

$$\frac{dN(t)}{dt}=rN(t) \tag{1}$$

with the integral form which proposes an exponentially growth as Equation (2)

$$N(t)=N_0e^{rt} \tag{2}$$

where, N_0 is the initial population, $N(t)$ is the population after some time t and $r=b-d$ being the intrinsic growth rate.

2.2 The Logistic Growth Model

One of the two regulation models to the Malthus exponential growth model is the logistic growth model by Verhulst. Verhulst's findings in 1838 revealed that, Malthus exponential growth for population size is unrealistic over a long period since growth will eventually be checked by over-consumption of resources. He therefore proposed a model called the Logistic growth model which is of the form given by Equation (3)

$$\frac{dN}{dt}=rNF(N) \tag{3}$$

where $F(N)$ provides a model for environmental regulation. He indicated that, this function should satisfy $F(0)=1$ when the population grows exponentially with growth rate r and N is small, $F(k)=0$ indicating that the population stops growing at the carrying capacity, and $F(N)<0$ when $N>k$ thus the population decays when it is larger than the carrying capacity. The simplest function, $F(N)$ satisfying these conditions is linear and was given by $F(N)=1-N/K$. The resulting model is the well-known logistic equation given as Equation (4)

$$\frac{dN}{dt}=rN\left(1-\frac{N}{k}\right) \quad (4)$$

where $(1-N/k)$ represents the fractional deficiency of the current size from the saturation level, k. This is an important model for many processes besides bounded population growth.

2.3 Verhulst Logistic Growth Scaled by the 'Delaying' Factor $\left[1+c(N/K)\right]^{-1}$

The other regulation model to the Malthus exponential growth model is by Smith (1963). Smith also reported that the Verhulst logistic growth equation did not fit experimental data satisfactorily due to problems associated with time lags. According to Smith, the major problem in applying the logistic growth equation to data concerns an accurate portrayal of the portion of the limiting factor as yet unutilized given by $1-(N/k)$. He then argued that for a food-limited population, the term $1-(N/k)$ should be replaced with a term representing the proportion of the rate of food supply currently unutilized by the population. Thus. if F is the rate at which a population of size N uses food and T is the corresponding rate at saturation level, then the model can best be represented as Equation (5)

$$\frac{dN}{dt}=rN\left(1-\frac{F}{T}\right) \quad (5)$$

where $(F/T)>(N/K)$, since a growing population will use food faster than a saturated population. F must depend on N and dN/dt, and the simplest relationship was identified to be linear indicated as Equation (6)

$$F=aN+b\frac{dN}{dt},\ a>0,\ b>o \quad (6)$$

At saturation $F=T$, $N=k$, $\frac{dN}{dt}=0$, hence $T=ak$ and as a result the modified Verhulst logistic growth equation is given as Equation (7)

$$\frac{dN}{dt}=rN\left(\frac{1-\frac{N}{k}}{1+c\frac{N}{k}}\right) \quad (7)$$

where $c=rb/a$

2.4 Extensions of the Logistic Growth Model

In a survey paper, Buis (1991) revisited several previous logistic growth derived functions that have been introduced and outlined some of their respective properties. The generalised logistic growth model was deduced based on the three postulates of the kinetic theory of growth. The three postulates are stated as follows:

P1: The rate of change of size is jointly proportional to a monotonically increasing function ϕ_1 of the distance between the origin and the size, and to a monotonically increasing function ϕ_2 of the distance between size and ultimate size. This is represented mathematically as Equation (8)

$$\frac{dN}{dt}\ \alpha\ \phi_1\left[\delta_n(0,N)\right]\phi_2\left[\delta_n(N,k)\right] \quad (8)$$

P2: The monotonically increasing functions ϕ_1 and ϕ_2 are power functions where $\theta_1,\theta_2>0$. These are represented by Equations (9) and (10)

$$\varphi_1[\bullet]=[\bullet]^{\theta_1} \quad (9)$$

$$\varphi_2[\bullet]=[\bullet]^{\theta_2} \quad (10)$$

P3: The exponents θ_1 and θ_2 obey the constraints represented by Equations (11) and (12)

$$\theta_1=1-np \quad (11)$$

$$\theta_2=n+np \quad (12)$$

where $n>0,\ -1\le p\le 1/n,\ \theta_1+\theta_2>n+1$

Based on these postulates of the kinetic theory of growth, the generalized logistic function is defined as Equation (13):

$$\frac{dN}{dt}=rN^{\alpha}\left[1-\left(\frac{N}{k}\right)^{\beta}\right]^{\gamma} \quad (13)$$

where α,β and γ are positive real numbers. The emphasis is mostly on positive values for these parameters, as negative exponents do not always provide a biologically plausible model. Other growth models that follow come under extensions of generalised logistic growth model. Three main features that can be made out of the generalized logistic growth model are as follows:

(i) $\lim_{t\to\infty} it\ N(t)=k$, the population will ultimately reach its carrying capacity.

(ii) the relative growth rate dN/Ndt attains its maximum value at. N^* given by Equation (14)

$$N^*=\left(1+\frac{\beta\gamma}{\alpha-1}\right)^{-\left(1/\beta\right)}k \quad (14)$$

provided N^* is real and greater than N_0, otherwise it declines non-linearly reaching its minimum zero value at $N=k$. The maximum relative growth rate is given by Equation (15)

$$\left(\frac{1}{N}\frac{dN}{dt}\right)_{\max}=rk^{\alpha-1}\left(\frac{\alpha-1}{\alpha-1+\beta\gamma}\right)^{(\alpha-1)/\beta}\left(\frac{\beta\gamma}{\alpha-1+\beta\gamma}\right)^{\gamma} \quad (15)$$

Important limit values of N^* are

$$\lim_{\alpha\to0} it\ N^*=0$$

$$\lim_{\beta\to0} it\ N^*=e^{\gamma/(1-\alpha)}$$

$\lim_{\gamma \to 0} it\ N^{*} = k$

(iii) The population at the inflection point (where growth rate is maximum), is given by Equation (16)

$$N_{\inf} = \left(1 + \frac{\beta\gamma}{\alpha}\right)^{-\left(1/\beta\right)} k \ > \ N^{\square} \tag{16}$$

Few examples of growth equations that are derived from the generalised logistic growth model are considered below.

2.5 Richard's Growth Equation

Richard extended the growth equation developed by Von Bertalanffy to fit empirical plant data (Richards, 1959). In Richard's suggestion, he came up with Equation (17) which is also a special case of the Bernoulli differential equation:

$$\frac{dN}{dt} = rN\left[1 - \left(\frac{N}{k}\right)^{\beta}\right] \tag{17}$$

This has a solution given by Equation (18)

$$N(t) = \frac{kN_0}{\left[N_0^{\beta} + \left(k^{\beta} - N_0^{\beta}\right)e^{\beta rt}\right]^{1/\beta}} \tag{18}$$

Here, the inflection $N_{\inf}$ occurs at a value given by Equation (19)

$$N_{\inf} = \left(\frac{1}{1+\beta}\right)^{1/\beta} k \tag{19}$$

Richard's form is readily deduced from generalised logistic function (3.13) with $\alpha = \gamma = 1$. For $\beta = 1$, Equation (3.22) trivially reduces to the Verhulst logistic growth equation (3.4) and similarly exhibits the same inflexible inflection point value.

3. Model Formulation

The purpose of Harvesting model is to manage the forest resource so that the value of the harvest from the forest is determined such that, the forests is made to attain its stable state without going into extinction. This is possible if certain maximum allowable cut does not exceed the calculated proportion to be harvested within every one step period.

Model Assumptions:

The model development of this paper was based on the following assumptions:

1) A tree may advance at most one size class during the unit period from t to $t+1$.
2) Regeneration can occur only when trees are thinned from the forest.
3) The enumeration X(t) takes place right before harvesting.

The generalization of the process is as indicated in figure 1 where the population is counted before harvesting.

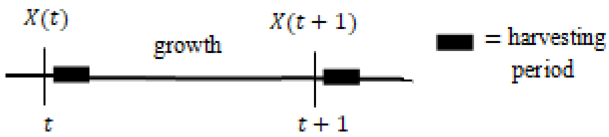

Figure 1: The population is counted before the harvest.

4) After growth period and after harvesting, the forest is returned to its prior state. This is represented asEquation (20)
5)

$$X(t+1) = X(t) \tag{20}$$

Whenever an environment cannot sustain an infinite population of a species, population growth will slow down as the population size increases. This study models the effect of these environmental factors with a carrying capacity that limits growth, leading to the application of a logistic equation for population growth of the form given as Equation (21)

$$P_{t+1} - P_t = rP_t\left(1 - \frac{P_t}{K}\right) \tag{21}$$

where r is the low-density growth rate and K is the carrying capacity. If the evolution of a population is following such logistic growth and r isn't too large, then the population size will evolve to a steady state of K even without harvesting. Ingeneral, for natural populations, a population following logistic growth doesn't require harvesting to maintain a steady population.

In most cases, the goal of harvesting a natural population is not population control, but simply to yield of a substantial harvest from the population. In such cases, some of the possible questions one may ask include: what harvest strategy will provide the maximum long-term yield, what is the maximum allowable harvest that will still retain the population, and what are the stable equilibrium sizes of the population under harvesting? In this study, we investigate these questions using the logistic model where the harvest is proportional to the population size, i.e., with the model where h_t is the fraction of the population harvested at time step t and represented as Equation (22).

$$P_{t+1} - P_t = rP_t\left(1 - \frac{P_t}{K}\right) - h_t P_t \tag{22}$$

4. Simulated illustrations for HarvestingLevels

As an illustrative example of the effects of harvesting,the study begins with the assumption of Equations (23) and (24)

$$P_0 = 1000 \tag{23}$$

$$P_{t+1} - P_t = 0.2P_t\left(1 - \frac{P_t}{K}\right) - hP_t \tag{24}$$

of a population with an initial value of 1000 trees,a low-density growth rate of 0.2 per time interval, with carrying capacity of 1000 trees and assume a constant harvesting rate h for all times. The goal is to determine what fraction, h may be harvested while still retaining the population.

To begin with, if the equilibrium population size is denoted as P_e, setting both $P_{t+1} = P_e$ and $P_t = P_e$, one therefore solves for P_e as Equations (25) to (31):

$$P_{t+1} - P_t = 0.2P_t\left(1 - \frac{P_t}{K}\right) - hP_t \quad (25)$$

$$P_e - P_e = 0.2P_e\left(1 - \frac{P_e}{1000}\right) - hP_e \quad (26)$$

$$0 = 0.2P_e\left(1 - \frac{P_e}{1000}\right) - hP_e \quad (27)$$

$$0 = P_e\left(0.2\left(1 - \frac{P_e}{1000}\right) - h\right) \quad (28)$$

The product is zero if either

$$P_e = 0 \text{ or } 0.2\left(1 - \frac{P_e}{1000}\right) - h = 0 \quad (29)$$

We can solve the second option for P_e:

$$\left(1 - \frac{P_e}{1000}\right) = \frac{h}{0.2} = 5h \quad (30)$$

$$1000(1 - 5h) = P_e \quad (31)$$

Therefore, the two possible equilibrium values are given as Equations (32) and (33)

$$P_e = 0 \quad (32)$$

$$1000(1 - 5h) = P_e \quad (33)$$

Since P_e is a population size, the equilibria make physical sense only if they are positive. Thus, if $1-5h$ is negative, then the second equilibrium is negative and doesn't make sense and therefore, the only realistic equilibrium is $P_e = 0$.
If the harvest is more than 20% of the population present at each time, then $h > 0.2$ and $1-5h$ is negative. Such a large harvest rate will cause the population to die out. The low-density growth rate (births minus natural deaths) is 20%, and this is the maximum growth rate. If the harvest rate exceeds the low-density growth rate, the population will dwindle away and disappear. In fact, even if the harvest rate is exactly 20%, then the second equilibrium is $P_e = 1000(1-5(0.2)) = 0$, and the population will still die out.
To simplify the further analysis, the equation is normalised by dividing equation (22) by the carrying capacity 1000.This normalizes the initial equation by dividing by the carrying capacity to obtain Equation (34):

$$\frac{P_{t+1}}{1000} - \frac{P_t}{1000} = 0.2\frac{P_t}{1000}\left(1 - \frac{P_t}{1000}\right) - h\frac{P_t}{1000} \quad (34)$$

letting $P_t = P_t/1000$, the dynamical system for P_t can be written as Equations (35) and (36)

$$P_0 = 1 \quad (35)$$

$$P_{t+1} - P_t = 0.2P_t(1 - P_t) - hP_t \quad (36)$$

This normalization just replaces the carrying capacity by the number 1. This normalization is done to show that the value of the carrying capacity doesn't play an important role in determining the dynamics. If the carrying capacity had been a number other than 1000, we could have divided by that number, and the end result would be the same equation.

The equilibria for P_t are the same as the equilibria of equation (36) for P_t, with just the carry capacity 1000 changed to 1. If one denotes the equilibria for P_t as P_e, the equilibria are obtained as Equation (37)

$$P_e = 0 \text{ and } P_e = 1 - 5h \quad (37)$$

As long as the fractional harvest h is set at a number less than 0.2, both equilibria 0 and $1-5h$ are valid. It was observed that the equilibrium $P_e = 0$ is unstable and the equilibrium $P_e = 1-5h$ is stable. Thus, as long as $0 \le h < 0.2$, then one can be confident that, the model predicts that the population won't die out.

5. Results and Discussion

The objective of introducing harvesting component is to maximize the harvest without altering the stability of the forest stand or the population becoming tiny. The amount of the harvest will be h times the population size P_t. To maximize the harvest, we want both the population size P_t to be large and the harvest rate h to be large. Since increasing the harvest rate will decrease the population size, it's not so simple to see what the optimal harvest rate should be. However, $h = r$ is too large since the population will die out and the harvest will be zero. But, if $h = 0$, the harvest will also be zero. The question now dwells on what intermediate value of the fractional harvest h that will maximize the total harvest?
To optimize the harvest rate h, the study assume that harvesting has been done at the rate h for a long time so that the population has reached equilibrium. As long as $h < 0.2$, the population size will settle down to the stable equilibrium $P_e = 1-5h$. With the population size being fixed at P_e, then the total amount of harvest is given as Equation (38)

$$h \times P_e = h(1 - 5h). \quad (38)$$

The maximum harvest at equilibrium will be obtained at a harvest level that maximizes $h \times (1-5h)$. The study therefore determines the value of h for which $h(1-5h)$ is the largest. Let G be the total harvest as a function of harvest rate h. Then the total harvest can be represented as Equation (39)

$$G(h) = h(1 - 5h) \quad (39)$$

Calculating the critical points of G, one obtains Equation (40)

$$G^{/}(h) = 1 - 10h \quad (40)$$

At the critical points $G'(h)=0$ and this occurs when $h=0.1$

To determine local maximum or minimum, the study obtained $G''(h)$ given as Equation (41).

$$G''(h) = -10 < 0 \qquad (41)$$

Since G″(h) is less than zero, the critical point is a local maximum. Thus $h=0.1$ yields the maximum value of $h(1-5h)$ for any value of the harvesting rate h.

In this example, we had set $r=0.2$. The optimal 10% harvest strategy is exactly one-half the low-density growth rate. Furthermore, the equilibrium when $h=0.1$ is $P_e = 1-5h = 0.5$, which is exactly half the maximum supportable population

The study calculated this result for the special case when $r=0.2$, but it turns out that, this is a general property of logistic models.

From the illustrative example, it is evident that the value of the carrying capacity in the Logistic growth model doesn't play any important role in determining the dynamics of the forest stand. However, the extinction or otherwise of a forest stand depends on maximum allowable cut that must strictly be less than the low-density growth rate, thus $0 \le h < r$. As long as the fractional harvest h is set at a number less than 0.2, both equilibria 0 and $1-5h$ are valid. It was observed that the equilibrium $P_e = 0$ is unstable and the equilibrium $P_e = 1-5h$ is stable. As long as $0 \le h < 0.2$, then one can be confident that, the population won't die out.

6. Conclusion

The study used the Vehaulst's logistic growth model with a harvested proportion h to determine the dynamics of the forest stand.The Logistic growth model necessitated this study of modelling forest growth and its associated harvesting policy for determining future sequence in order not to compromise its benefits.it is evident that the value of the carrying capacity in the Logistic growth model doesn't play any important role in determining the dynamics of the forest stand. the extinction or otherwise of a forest stand depends on maximum allowable cut that must strictly be less than the low-density growth rate, thus $0 \le h < r$.

References

[1] Baldwin, J. P., Nye, P. H., Tinker, P. B., (1973), "Uptake of solutes by multiple root systems from soil III - a model for calculating the solute uptake by a randomly dispersed root system developing in a finite volume of soil", *Journal of Plant and Soil*, Vol 38, pp 621 - 635.

[2] Barber, S. A and Cushman, J. H., (1981), "Nitrogen uptake model for agronomic crops", pp.382-409 *In* I. K. Iskandar (ed.). Modeling waste water renovation-land treatment**:** Wiley-Interscience, New York.

[3] Buis, R., (1991), "On the generalization of the logistic law of growth", *Journal of Acta Biotheoretica*, Vol. 39, pp 185–190.

[4] Chen, F.F., (2007), "Sensitivity of goodness of fit indexes to lack of measurement invariance", *Structural equation modeling*, **Vol.** *14*, No 3, pp.464-504.

[5] Claassen, N. and Steingrobe, B., (1999), "Mechanistic simulation models for a better understanding of nutrient uptake from soil" p.327-367, In Z. Rengel (ed.). Mineral nutrition of crops: fundamental mechanisms and implications**.** Food Products Press, New York.

[6] Claassen, N., Syring, K. M., and Jungk, A., (1986), "Verification of a mathematical-model by simulating potassium uptake from soil", *Journal of Plant and Soil*, Vol. 95, pp 209-220.

[7] Claassen, N and Barber, S A., (1976), "Simulation model for nutrient uptake from soil by a growing plant root system", *Agronomy Journal*, No 68: pp 961-964.

[8] Claassen, N., Syring, K. M., Jungk, A., (1986), "Verification of a mathematical-model by simulating potassium uptake from soil", *Journal of Plant and Soil*, Vol. 95, pp 209-220.

[9] Comerford, N. B., Cropper, W. P. Jr., Hua, L., Smethurst, P. J. K., Van Rees, C. J., Jokela, E. J., Adams, F., (2006), "Soil Supply and Nutrient Demand (SSAND): a general nutrient uptake model and an example of its application to forest management", *Canadian Journal of Soil Science*, Vol. 86, pp 655-673.

[10] Corrado, R., Cherubini, A. M., Pennetta, C., (2014), "Early warning signals of desertification transitions in semiarid ecosystems", *Journal of Physical Review Education*, Vol. 90, pp. 062705-1 to 062705-11.

[11] Malthus, T. R., (1798), "An essay on the principle of population, and a summary via principle of population", Penguin, Haarmondworth, England.

[12] Nyarko, P. K., Dontwi, I. K., Ennin, C. C., Owusu-Ansah, E., Dontwi, J. (2010), "Sustainable Timber Production in the Furi and Wawahi Forest Reserves in Ghana. A Bio-Economic Resource Modeling Approach", *European Journal of Scientific Research*, Vol. 40, No. 3, pp352 – 361.

[13] Nye, P. H. and Spiers, J. A., (1964), "Simultaneous diffusion and mass flow to plant roots. p. 535-541 *In* (editors missing.) Transactions: 8th International Congress of Soil Science, Bucharest, Romania, 31 Aug. -9 Sep. 1964. Publishing House of the Academy of the Socialist Republic of Romania, Rompresfilatelia, Bucharest.

[14] Nye, P. H., and Marriott, F. H. C., (1969), "A theoretical study of the distribution of substances around roots resulting from simultaneous diffusion and mass flow", *Journal of Plant and Soil, Vol.* 30, pp 459-472.

[15] Nye, P. H. and Tinker, P. B., (1977), "Solute Movement in the Soil-Root System", Blackwell Scientific Publications, University of California Press, Berkeley.

[16] Richards, F.J.A., (1959), "A Flexible growth function for empirical use", *J. Exp. Bot.* Vol.10 (29), 290–300.

[17] Smethurst, P. J. and Comerford, N. B., (1993b), "Simulating nutrient uptake by single or competing

and contrasting root systems", *Journal of Soil Science Society of America*, Vol. 57 pp 1361-1367.

[18] Smethurst, P. J., Mendham, D. S., Battaglia, M. and Misra, R., (2004), "Simultaneous prediction of nitrogen and phosphorus dynamics in a *Eucalyptus nitens* plantation using linked CABALA and PCATS models", pp. 565-569 *In* N. M. G. Borralho (ed.) Eucalyptus in a changing world. Proceedings of an International conference of the WP2.08.03 on silviculture and improvement of eucalypts, Aveiro, Portugal, 11-15 October 2004. IUFRO, Aveiro, Portugal.

[19] Smith, F. E., (1963), "Population dynamics in Daphnia Magna and a new model for population growth", *Journal ofEcological Modelling*, Vol.44, No 4, pp 651-660.

[20] Sharpe, P. J. H., Walker, J., Wu, I. (1985), "Physiologically based continuous-time Markov approach of plant growth modelling in semi-arid woodlands", *Journal ofEcological Modelling*, Vol.29, No 15, pp 189 - 213.

[21] Vanclay, J.K., (1994), "Modelling Forest growth and yield: applications to mixed tropical forests",

[22] *School of Environmental Science and Management Papers*, pp.537.

[23] Verhulst, P.F., (1838), "Notice sur la loi que la population suit dans son accroissement", *Corresp. Math. Phys.*, **Vol.** 10, pp.113-126.

[24] Wu, J.C., Merlino, G. and Fausto, N., (1994), "Establishment and characterization differentiated, nontransformed hepatocyte cell lines derived from mice transgenic for transforming growth factor alpha", *Proceedings of the National Academy of Sciences*, **Vol** 91, No 2, pp.674-678.

[25] Wu, H., Rota, R., Morbidelli, M. and Varma, A., (1999), "Parametric sensitivity in fixed- bed catalytic reactors with reverse-flow operation", *Chemical engineering science*, **Vol.** *54*, No. 20, pp.4579-4588.

[26] Wu, H., Rykiel Jr. E. J., Hatton, T., Walker, j., (1994), "An integrated rate methodology (IRM) for multi-factor growth rate modelling", *Ecological Modelling*, Vol. 73 pp 97-116

[27] Wu, R., Yankaskas, J., Cheng, E., Knowles, M. and Boucher, R., (1985), "Growth and differentiation of human nasal epithelial cells in culture: serum-free, hormone-supplemented medium and proteoglycan synthesis", *American Review of Respiratory Disease*, **Vol.** 132, No. 2, pp.311-320.

[28] Yanai, R. D., (1994), "A steady-state model of nutrient uptake accounting for newly grown roots", *Journal of Soil Science Society of America*, Vol. 58, pp 1562-1571.