# A Two - Warehouse Fuzzy Inventory Model of Deteriorating Items under Exponentially Increasing Demand

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**Abstract:** ***In this paper, our study is to develop an inventory policy for deteriorating items dealing with two-warehouse system owned and rented under fuzzy environment. The constant rate of deteriorating items and exponentially increasing demand pattern are considered in the present model. The owned warehouse (OW) of finite capacity and rented warehouse (RW) of large capacity of items are assumed here. Normally retailer wants to sell the inventory in RW first, inventory of RW reaches zero level due to deterioration and demand, while that of OW is depleted due to deterioration only. Both crisp and fuzzy models are developed in the proposed study. The signed distance method (SD) is used to defuzzify the total system cost of the model. Suitable illustrations for both crisp model and fuzzy model are furnished and lastly a sensitivity analysis of the optimum solution towards changes the fuzzy cost parameters are given along with few pictorial presentations discussing the observation on the outcomes of the system solution.***

**Keywords:** Inventory, fuzzy, deteriorating, owned warehouse, rented warehouse, exponential demand

**Subject classification:** AMS Classification No. 90B05

## 1. Introduction

Regularly, inventory models are mostly developed with single warehouse space. But it is quite impossible to set up big shops or showrooms or market complex at the heart of a city due to scarcity of space and also for very high rents. So the retailers usually are planning for two storage capacities, one in the market place named as owned warehouse (OW) with finite capacity of storage, other is placed near by the market with a large capacity named as rented warehouse(RW). The retailers control the function of these two warehouses as per customers' demand.

The facets of managing stock are inventory management and warehouse management. Inventory management gives a high-level view, while warehouse management focuses on the details of the movement of stock providing many information like sales trends, profit margins, system costs, product movement activities, product storage and information transfer. Inventory theory under two-warehouse includes storage of raw materials, WIP and finished goods seasonal and in transit, efficient distribution of goods during adverse time, price stabilization during time of scarcity, grading, presentation of perishable goods like fruits, meat etc. In this context, I must mention few authors like Pakkala [1], Lee [2], Singh [3], Mandal P [4], Sheikh S R [5], Lee S S [6] etc who developed several research models in this regard.

In many inventory models, uncertainty is occurring due to fuzziness and it is the closed possible approach to reality. Zadeh Lotfi A [7] introduced the concept of fuzzy set theory in inventory modelling. Zadeh L A and Bellman R E [8] have developed an inventory model on fuzzy environment. Later researchers like Kacpryzk [9], Zimmerman [10], Kao and Hsu [11], De and Rawat [12], Sujit [13 ], Yadav[14] , Biswaranjan [15] are mentioned a few. Consequently, many inventory models with major parameters as fuzzy under two-warehouse system are discussed by Yadav A S [16], Indrajitsingha S K [17], Malik A K [18] etc.

In the competitive market, the demand of some product may increase due to the consumer's preference on some eye-catching product. Therefore, the demand of the product at the time of its growth and the phase of declination may be approached by continuous-time-dependent function. These continuous-time-dependent functions may be a function of exponential or ramp type, linear type, quadratic, cubic in nature. Ritchie [19] discussed the solution of a linear increasing time-dependent demand, which is obtained by Donaldson [20]. Silver and Meal [21] developed a model for deterministic time-varying demand, which also gives an approximate solution procedure termed as Silver-Meal Heuristic.Ramp type demand pattern is considered by Biswaranjan [22], Exponential demand has been developed byM. Dhivya Lakshmi [23] and many other researchers.

In this paper, we first consider a crisp two-warehouse inventory model with constant rate of deteriorating items under exponentially increasing demand. Thereafter, we develop the corresponding fuzzy model. The signed distance method (SD) is used for defuzzification in the present model. Two examples are illustrated for both crisp and fuzzy models. Sensitivity analysis of the optimal solution under fuzzy cost parameters (deterioration cost, storage cost for RW and OW) is furnished and lastly some pictorial presentations are given and discussing the observation on the outcomes of the system solution.

## 2. Notations and Assumptions

The present inventory model is developed under the following notations and assumptions:

**Notations:**

(i) R(t) : Demand rate.
(ii) OW : Owned warehouse
(iii) RW : Rented warehouse
(iv) $N_r(t)$ : Stock amount in RW at time t.
(v) $N_o(t)$ : Stock amount in OW at time t.
(vi) $W_0$ : Storage capacity of OW.
(vii) x: The deterioration rate in RW where $0 \le x < 1$
(viii) y : The deterioration rate in OW where $0 \le y < 1$
(ix) T : The fixed length of each production cycle.
(x) $d_o$ : Ordering cost per order
(xi) $d_c$ : Deterioration cost per unit item in RW/OW.
(xii) $h_r$ : The storage cost per unit item in RW.
(xiii) $h_o$ : The storage cost per unit item in OW.
(xiv) TAC: The total average cost in the system.
(xv) $\tilde{d}_c$ : The fuzzy deterioration cost per unit item in RW/OW.
(xvi) $\tilde{h}_r$ : The fuzzy storage cost per unit item in RW.
(xvii) $\tilde{h}_o$ : The fuzzy storage cost per unit item in OW.
(xviii) $\widetilde{TAC}$ :The fuzzy total average cost of the system per unit time.

**Assumptions**

1) Lead time is zero.
2) Replenishment rate is infinite but size is finite.
3) The time horizon is finite.
4) There is no repair of deteriorated items occurring during the cycle.
5) The demand rate is a time dependent exponentially increasing function
   R(t) = $e^{\lambda t}$, $\lambda > 0$
6) The storage cost per unit in RW is more than that of OW.
7) Items are kept in OW first.
8) The priority has given to RW for first consumption.

## 3. Model development and Solution

The proposed model deals with a two-warehouse inventory model. For RW, the inventory level $N_r(t)$ reaches at zero level at time t = $t_1$. During period (0, $t_1$), the demand of the customer fulfils from RW, and in between some items deteriorates in OW in same period having inventory level $N_{o1}(t)$. After RW empty, the customers' demand fulfils by OW during the period ($t_1$, T) having inventory level $N_{o2}(t)$. The initial inventory for OW is $W_o$. A pictorial presentation of the proposed two-warehouse inventory model is given in Fig. 1.

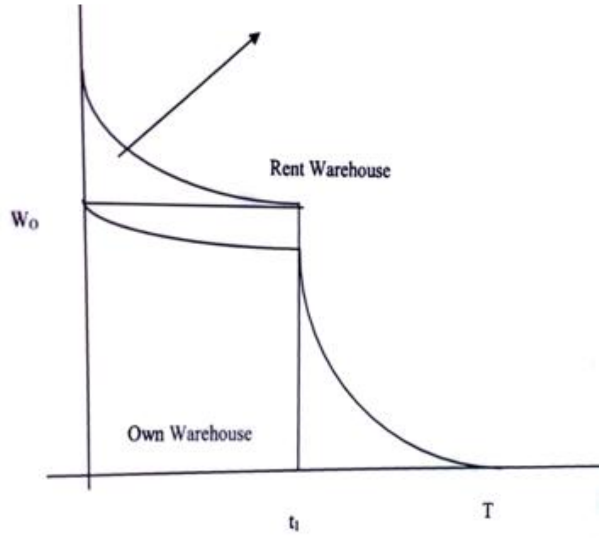


**Figure 1:** The proposed two-warehouse inventory model

**Crisp Model**

The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dN_r(t)}{dt} + xN_r(t) = -e^{\lambda t}, 0 \le t \le t_1 \quad (3.1)$$

$$\frac{dN_{o1}(t)}{dt} + yN_{o1}(t) = 0, 0 \le t \le t_1 \quad (3.2)$$

And
$$\frac{dN_{o2}(t)}{dt} + yN_{o1}(t) = -e^{\lambda t}, t_1 \le t \le T \quad (3.3)$$

The boundary conditions are

$$N_r(t_1) = 0, N_{o1}(0) = W_o, N_{o2}(T) = 0 \quad (3.4)$$

The solutions of the equations (3.1), (3.2) and (3.3) using (3.4) are given by the following

$$N_r(t) = \frac{1}{\lambda + x} e^{\lambda t}\{e^{(\lambda + x)(t_1 - t)} - 1\}, 0 \le t \le t_1, \quad (3.5)$$

$$N_{o1}(t) = W_o e^{-yt}, 0 \le t \le t_1 \quad (3.6)$$

And $$N_{o2}(t) = \frac{1}{\lambda + y} e^{\lambda t}\{e^{(\lambda + y)(T - t)} - 1\}, t_1 \le t \le T \quad (3.7)$$

Since $N_{o1}(t_1) = N_{o2}(t_1)$, we get the following expression of storage capacity of OW using the equations (3.6) and (3.7)

$$W_o = \frac{1}{\lambda + y}\{e^{(\lambda + y)T} - e^{(\lambda + y)t_1}\} \quad (3.8)$$

Now, TAC (The total average cost in the system) consists of the following cost components:

1) The inventory ordering cost (IOC) = $d_o$ (3.9)
2) The inventory storage cost (ISC) in RW is

$$ISC_R = h_r\int_0^{t_1} N_r(t)dt = h_r\int_0^{t_1}\frac{1}{\lambda+x}e^{\lambda t}\{e^{(\lambda+x)(t_1-t)}-1\}dt$$

$$= \frac{h_r}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)] \quad (3.10)$$

1) The inventory storage cost (ISC) in OW is

$$ISC_O = h_o[\int_0^{t_1} N_{o1}(t)dt+\int_{t_1}^{T} N_{o2}(t)dt] = h_o[\int_0^{t_1} W_o e^{-yt}dt+\int_{t_1}^{T}\frac{1}{\lambda+y}e^{\lambda t}\{e^{(\lambda+y)(T-t)}-1\}dt]$$

Integrating and then putting the value of $W_0$ expressed in (3.8), we get the following

$$ISC_O = \frac{h_o}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})] \quad (3.11)$$

2) The inventory deterioration cost in RW is given by

$$IDC_R = d_c\int_0^{t_1} xN_r(t)dt = \frac{d_c x}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)] \quad (3.12)$$

3) The inventory deterioration cost in OW is given by

$$IDC_O = d_c[\int_0^{t_1} yN_{o1}(t)dt+\int_{t_1}^{T} yN_{o2}(t)dt]$$

$$= \frac{d_c y}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})] \quad (3.13)$$

Therefore the total average cost per unit time is

$$\text{TAC}\,(t_1) = \frac{1}{T}[IOC+ISC_R+ISC_O+IDC_R+IDC_O]$$

$$= \frac{1}{T}[d_o+\frac{h_r}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)]+\frac{h_o}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})]+$$
$$\frac{d_c x}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)]+\frac{d_c y}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})]] \quad (3.14)$$

For minimum, the necessary condition is $\frac{d\,TAC(t_1)}{dt_1}=0$

$$\text{Or, } \frac{h_r+d_c x}{x}(e^{(\lambda+x)t_1}-e^{\lambda t_1})-\frac{h_o+d_c y}{y}(e^{(\lambda+y)t_1}-e^{\lambda t_1})=0 \quad (3.15)$$

For minimum, the sufficient condition $\frac{d^2\,TAC(t_1)}{dt_1^{\,2}}>0$ would be satisfied.

Solving the equation (3.15), we get the optimal value of $t_1$ $=t_1^*$.

The optimal values of the storage capacity ($W_0$) and the total average cost (TAC) are obtained by putting $t_1=t_1^*$ from the expressions (3.8) and (3.14).

**Fuzzy Model**

In this section, we presented fuzzy inventory model through the signed distance method(SD). Due to global market scenario, the price of items fluctuating with storage costs and deterioration cost in RW and OW cannot be assumed fix. We consider these costs for both the warehouses as a triangular fuzzy numbers.

$\tilde{d}_c=(d_{c1},d_{c2},d_{c3})$, $\tilde{h}_r=(h_{r1},h_{r2},h_{r3})$, $\tilde{h}_o=(h_{o1},h_{o2},h_{o3})$

Now transforming the Crisp model of the total average cost into the fuzzy environment and then defuzzify the fuzzy total average cost using Signed Distance Method(SD), we get

$$\widetilde{TAC}_{SD}(t_1)=\frac{1}{4T}\left|\begin{matrix}\widetilde{TAC}_{SD1}(t_1)\\ \widetilde{TAC}_{SD2}(t_1)\\ \widetilde{TAC}_{SD_3}(t_1)\end{matrix}\right\rangle$$

Where $\tilde{TAC}_{SD1}(t_1)=d_o+\frac{h_{r1}+d_{c1}x}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)]+\frac{h_{o1}+d_{c1}y}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})]$

$$\tilde{TAC}_{SD2}(t_1)=d_o+\frac{h_{r2}+d_{c2}x}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)]+\frac{h_{o2}+d_{c2}y}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})]$$

$\tilde{TAC}_{SD3}(t_1)=d_o+\frac{h_{r3}+d_{c3}x}{\lambda+x}[\frac{e^{\lambda t_1}}{x}(e^{xt_1}-1)-\frac{1}{\lambda}(e^{\lambda t_1}-1)]+\frac{h_{o3}+d_{c3}y}{\lambda+y}[\frac{1}{y}\{e^{(\lambda+y)T}-e^{\lambda T}-e^{(\lambda+y)t_1}+e^{\lambda t_1}\}+\frac{1}{\lambda}(e^{\lambda t_1}-e^{\lambda T})]$ And

$$\tilde{TAC}_{SD}(t_1)=\frac{1}{4T}[\tilde{TAC}_{SD1}(t_1)+2\tilde{TAC}_{SD2}(t_1)+\tilde{TAC}_{SD_3}(t_1)] \qquad (3.16)$$

To minimize the value of the fuzzy total average cost per unit time, the optimum value of $t_1$ can be obtained by solving the equation

$$\frac{d\,\tilde{TAC}_{SD}(t_1)}{dt_1}=0$$

Or, $$\frac{(h_{r1}+2h_{r2}+h_{r3})+x(d_{c1}+2d_{c2}+d_{c3})}{x}\{(e^{(\lambda+x)t_1}-e^{\lambda t_1})\}$$

$$-\frac{(h_{o1}+2h_{o2}+h_{o3})+y(d_{c1}+2d_{c2}+d_{c3})}{y}\{(e^{(\lambda+y)t_1}-e^{\lambda t_1})\}=0 \quad (3.17)$$

The sufficient condition for minimum value of $\tilde{TAC}_{SD}(t_1)$ is $\frac{d^2\,\tilde{TAC}_{SD}(t_1)}{dt_1^{\,2}}>0$.

## 4. Numerical Examples

To explain the outcomes of offered model, we used two numerical examples which one for crisp and the second for fuzzy.

**Crisp Model:**

The following are the parametric values associated with the model:

$d_o=500$ per order ; $h_r=\$0.75$ per unit ; $h_o=\$0.4$ per unit ; $d_c=\$0.5$ per unit ; x = 0.06 ; y = 0.07 ; $\lambda=0.1$ ; T = 1 year.

Solving the equation (3.15) using computer based optimization technique,we find the following optimum outcomes

$t_1^*$ = 0.019 years, $W_o^*$ = 6.837 units and $TAC^*$ = \$ 506.33

It is checked that this solution satisfies the sufficient condition for optimality.

**Fuzzy Model:**

Let us consider a fuzzy inventory system with the following cost parameters in appropriate units as

$d_o=500$ ; $\tilde{h}_r=(0.5,0.7,0.9)$ ; $\tilde{h}_o=(0.2,0.4,0.6)$ ; $\tilde{d}_c=(0.4,0.5,0.6)$ ; x = 0.06 ; y = 0.07 ; $\lambda=0.1$ ; T = 1 year.

Solving the equation (3.17) with the help of computer using the above values of parameters, we find the following optimum outputs

$t_1^*$ = 0.029year; $W_o^*$ = 6.767 units and $\tilde{TAC}^*$ = \$.524.836

It is checked that this solution satisfies the sufficient condition for optimality.

## 5. Sensitivity Analysis and Graphical presentation

We now study the effects of changes in the fuzzy cost parameters $\tilde{d}_c$ , $\tilde{h}_r$ and $\tilde{h}_o$ on theoptimal storage capacity of OW ($W_o^*$) and the optimal fuzzy total average cost ($\tilde{TAC}^*$) in the present fuzzy model. The sensitivity analysis is performed by changing each of the parameters by – 50%, –20%, +20% and +50%, taking one fuzzy cost parameter at a time and keeping remaining cost parameters unchanged. The results are furnished in table A.

**Table A:** Effect of changes in the fuzzy cost parameters on the model

| Changing parameter | % change in the system parameter | % change in | |
|---|---|---|---|
| | | $W_o^*$ | $TAC^*$ |
| $\tilde{d}_c$ | -50 | 0.36 | -0.16 |
| | -20 | 0.15 | -0.06 |
| | +20 | -0.15 | 0.06 |
| | +50 | -0.37 | 0.13 |
| $\tilde{h}_r$ | -50 | -8.49 | -6.01 |
| | -20 | -7.68 | -0.59 |
| | +20 | 7.05 | 0.20 |
| | +50 | 7.81 | 0.32 |
| $\tilde{h}_o$ | -50 | 2.99 | -1.95 |
| | -20 | 2.21 | -0.69 |
| | +20 | -5.30 | 0.38 |
| | +50 | -8.38 | 0.54 |

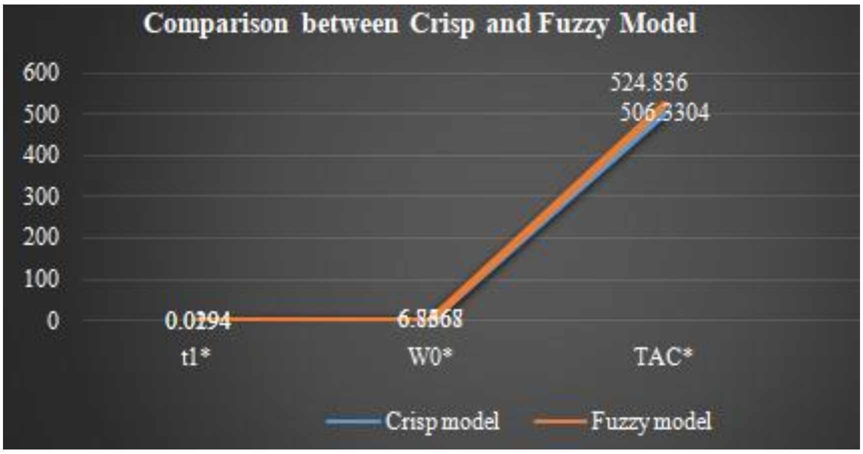


**Figure 2:** Comparison between crisp and fuzzy model

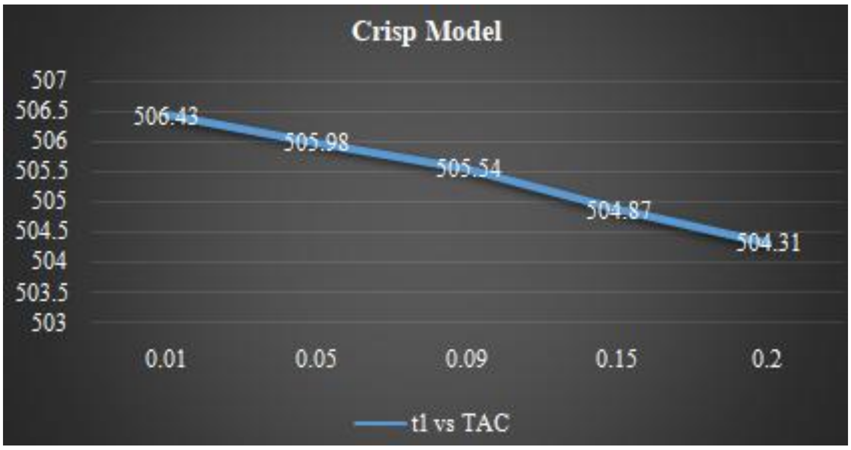


**Figure 3:** Variation of TAC (Total average cost) with $t_1$ (time) in Crisp Model

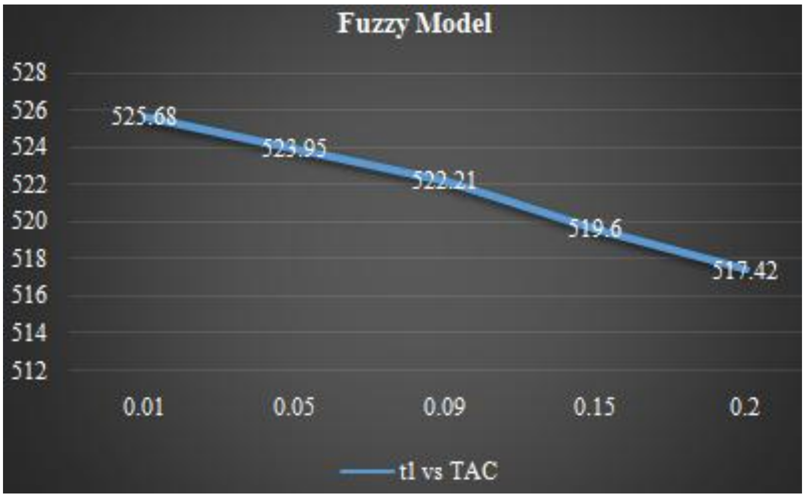


**Figure 4:** Variation of TAC (Total average cost) with $t_1$ (time) in Fuzzy Model

## 6. Observations

Analyzing the results of table A and graphical presentations, the following observations may be made:

1) The fuzzy total average cost ($\tilde{TAC}^*$) increases or decreases with the increase or decrease in the values of the fuzzy costparameters $\tilde{d}_c$, $\tilde{h}_r$ and $\tilde{h}_o$. The results obtained show that $\tilde{TAC}^*$ islow sensitive towards changes of the all costparameters.
2) The optimal storage capacity of OW ($W_o^*$) increases or decreases with the increase or decrease in the values of the fuzzy cost parameter $\tilde{h}_r$. On the other hand $W_o^*$ increases or decreases with the decrease or increase in the values of the cost parameters $\tilde{d}_c$ and $\tilde{h}_o$. The results obtained show that $W_o^*$ is moderate sensitive towards



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changes of $\tilde{h}_r$ and $\tilde{h}_o$, and less sensitive towards the changes of $\tilde{d}_c$.

3) Fig. 2 shows the comparison of $W_o^*$ and $\tilde{TAC}^*$ between crisp and fuzzy model. It is observed that both the models have about same optimum values and the optimum system cost is more minimum for crisp model in compare to fuzzy model.
4) Fig. 3 lists the variation of TAC (Total average cost) with $t_1$(time) in Crisp Model and it is observed that with the increase of $t_1$ (time), TAC (Total average cost) decreases.
5) Fig. 4 lists the variation of TAC (Total average cost) with $t_1$ (time) in Fuzzy Model and it is observed that an increase of $t_1$(time), results also an increase of TAC (Total average cost) of the fuzzy system.

## 7. Concluding Remarks

Many researchers ignored the assumption of deterioration of two-warehouse fuzzy inventory model. The present paper deals with a two-warehouse inventory model where the optimal inventory cost function is obtained using fuzzy approach under an uncertain environment. The demand function is assumed as a time dependent exponentially increasing function with constant rate of deteriorating items. Shortages are not considered in the present model. The inventory cost under fuzzy sense is optimized using a method of defuzzification, namely signed distance method(SD). Numerical illustrations and sensitivity analysis are performed along with some pictorial presentations furnished to observe the nature of variation of optimal TAC (Total average cost) with $t_1$(time) in the proposed fuzzy model. A future work will be further incorporated in the present model by considering shortages, inflation and time discounting, trade credit policy or profit based inventory model under this imprecise environment.

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