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Significant Results on Softset Theory

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Abstract: ***Soft set theory is an important mathematical tool introduced by Prof. D. Molodstov in 1999 which deals with uncertainties. In this paper we will some important results on soft set theory which is based on some operations such as union, intersection of soft sets.***

Keywords: Soft set

1. Introduction

Soft set theory [2] is a mathematical tool introduced by D. Molodstov in 1999 which deals with uncertainities. This theory has large applications in solving many problems in the areas of economics, engineering, social science etc. Therefore the work on soft set theory is progressing rapidly. K.V.Babitha and J.J.Sunil [4] have given many operations on soft sets and functions such as union, intersection, equivalence relations and partitions on soft sets etc. Inspired by this work, we have given many results on these operations. Attribute reduction using multi soft sets and its applications is discussed in 3 and application of normal parameter reduction of soft sets in decision making is given in [5]. In 2014, R. Thumbakara and B. George [9] has introduced a concept of soft graph, more work on soft graph can be found in [5]–[11].In the present paper, we have given many important results on the operations of soft set theory.

2. Preliminaries

Definition 2.1. Soft set [2]

Let U be an universe and E be a set of parameters. Let P(U) denote the power setof U and A be a non-empty subset of E. A pair (F,A) is called a soft set over U, where F is a mapping given by F : A → P(V).In other words, a soft set over U is a parameterized family of subsets of the universeU. For ϵ ∈ A, F(ϵ) may be considered as the set of ϵ-elements of the soft set (F,A)or as the set of ϵ-approximate elements of the soft set.

Definition2.2.Soft subset [2]

For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B) if

1) $A \subset B$
2) For all $\epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations.

We write$(F, A) \subset (G, B)$.

Also, (F, A) is said to be soft super set of (G, B) , if (G, B) is soft subset of (F, A).

Definition2.3.Union of Soft Sets [2]

A union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H,C),

Where $C=A \cup B$, and $\forall e \in C$.

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

Wewrite$(F,A) \cup (G,B)=(H,C)$.

Definition 2.4. Intersection of Soft Sets [2]

An intersection of two softsets (F, A) and (G, B) over a common universe U is the softest (H, C), Where $C= A \cap B$, and $\forall e \in C$, $H(e)=F(e) \cap G(e)$.

We write $(F, A) \cap (G, B)= (H, C)$.

3. Results on Softset Theory

Theorem 3.1: If (F, A) and (G, B) are any two softsets over a common universe then $(F, A) \subseteq (F, A) \cup (G, B)$ and $(G, B) \subseteq (F, A) \cup (G, B)$.

Proof: As (F, A) and (G, B) are any two softsets, then by definition of union of two softsets
$(F, A) \cup (G, B) = H\ (C)$
Where $C= A \cup B$, and $\forall e \in C$.

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

As $C= A \cup B \Rightarrow A \subseteq C$ and $B \subseteq C$

Also $F\ (e) \subseteq H\ (e)$ and $G\ (e) \subseteq H\ (e)$

So by the definition of soft subset

$(F, A) \subseteq (H, A \cup B)$ and $(G, B) \subseteq (H, A \cup B)$

i.e. $(F,A) \subseteq (F,A) \cup (G,B)$ and $(G,B) \subseteq (F,A) \cup (G,B)$.

Theorem 3.2: If (F, A) and (G, B) are any two softsets over a common universe U then $(F, A) \cap (G, B) \subseteq (F, A)$ and $(F, A) \cap (G, B) \subseteq (G, B)$.
Proof: As (F, A) and (G, B) are any two soft sets, then by definition of intersection of two softsets

$(F, A) \cap (G, B)=(H,C)$

Where $C= A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$.

As $C= A \cap B \Rightarrow A \cap B \subseteq A$ and $A \cap B \subseteq B$

i.e. $C \subseteq A$ and $C \subseteq B$

Also $F(e) \cap G(e) \subseteq F(e)$ and $F(e) \cap G(e) \subseteq G(e)$
i. e. $H(e) \subseteq F(e)$ and $H(e) \subseteq G(e)$

So by the definition of soft subset

$(H, A\cap B) \subseteq (F, A)$ and $(H, A\cap B) \subseteq (G,B)$
i.e.$(F, A) \cap (G, B) \subseteq (F,A)$ and $(F,A)\cap(G,B)\subseteq(G,B)$.

Theorem 3.3: If (F, A) and (G, B) are any two softsets over a common universe and $(F,A) \subseteq (G,B)$ then $(F, A) \cap (G, B) = (F, A)$.

Proof: By definition of intersection of two softsets
$(F, A) \cap (G, B) = (H, C)$

Where $C= A \cap B$, and $\forall e \in C$, $H(e)=F(e)\cap G(e)$.

From result 2 we know that $(H, A\cap B) \subseteq (F, A)$...(1)

As given that $(F, A) \subseteq (G, B) \Rightarrow A \subseteq B$

Then $A \subseteq A\cap B$
$\Longrightarrow F(e) \subseteq H(e) \forall e \in A$

So $(F, A) \subseteq (H, A\cap B)$...(2)
From (1) and (2)

$(F, A) \cap (G, B) = (F, A)$.

Theorem 3.4: If (F, A) and (G, B) are any two softsets over a common universe U and $(F, A) \subseteq (G, B)$ then $(F, A) \cup (G, B) = (G, B)$.

Proof: By definition of union of two softsets

$(F, A) \cup (G, B) = H(C)$ Where $C= A \cup B$, and $\forall e \in C$.

$$H(e) = \begin{cases} F(e) & if\ e \in A-B \\ G(e) & if\ e \in B-A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

But as $(F, A) \subseteq (G,B) \Longrightarrow A \subseteq B$

$$\Rightarrow H(e) = \begin{cases} G(e) & if\ e \in B-A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

$$\Rightarrow H(e) = \begin{cases} G(e) & if\ e \in B-A \\ G(e) & if\ e \in A \cap B \end{cases}$$

i.e. $H(e)=F(e);\forall e\in A\cup B$
So$(F,A)\cup(G,B)\subseteq(G,B)$...(3)
andfromresult1,$(G,B)\subseteq(F,A)\cup(G,B)$...(4)From(3)and(4)
)
$(F,A)\cup(G,B)=(G,B)$.

Theorem 3.5: If (F, A) and (G, B) are any two softsets over a common universe then $(F, A) \cup (G, B)= (G, B) \cup (F, A)$

Proof: As (F, A) and (G, B) are any two softsets, then by definition of union of two softsets

$(F, A) \cup (G, B)= (H, C)$ Where $C= A \cup B$, and $\forall e \in$ C

$$H(e) = \begin{cases} F(e) & if\ e \in A-B \\ G(e) & if\ e \in B-A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

$(G, B) \cup (F, A) = (S, D)$ Where $D= B\cup A$, and $\forall e\in D$.

$$S(e) = \begin{cases} G(e) & if\ e \in B-A \\ F(e) & if\ e \in A-B \\ G(e) \cup F(e) & if\ e \in B \cap A \end{cases}$$

As $A\cup B= B\cup A$ and $H(e)= S(e)$; $\forall e \in A\cup B$

then by the definition of equality of twos softsets, $(F,A)\cup(G,B)=(G,B)\cup(F,A)$.

Theorem 3.6: If (F, A) and (G, B) are any two softsets over a common universe U then $(F, A) \cap (G, B)= (G, B) \cap (F, A)$.

Proof: As (F, A) and (G, B) are any two soft sets, then by definition of intersection of two softsets
$(F, A) \cap (G, B) = (H, C)$

Where $C= A \cap B$, and $\forall e \in C$, $H(e)=F(e)\cap G(e)$.

and
$(G, B) \cap (F, A) = (S, D)$

Where $D= B \cap A$, and $\forall e \in D$, $S(e) = F(e) \cap G(e)$. as $A \cap B = B \cap A \Rightarrow H(e) = S(e)$; $\forall e \in A \cap B$then by the definition of equality of two soft sets, $(F, A) \cap (G,B)= (G,B) \cap (F, A)$.

Theorem 3.7: If (F, A), (G, B) and (I, D) are any three softsets over a common universe U then $(F, A)\cup [(G, B) \cup (I, D)] = [(F, A) \cup (G, B)] \cup (I, D)$ on $(A\cup B)\cap D$.

Proof: By definition of union of two softsets

$(G, B) \cup (I, D)= (H, P)$ Where $P=B \cup$ D, and $\forall e \in$ P.

$$H(e) = \begin{cases} G(e) & if\ e \in B-D \\ I(e) & if\ e \in D-B \\ G(e) \cup I(e) & if\ e \in B \cap D \end{cases}$$

$(F, A) \cup [(G, B) \cup (I, D)]=[S,A\cup(B\cup D)]$

And

$$S(e) = \begin{cases} F(e) & if\ e \in (A \cup B) - D \\ H(e) & if\ e \in (B \cup D) - A \\ F(e) \cup H(e) & if\ e \in A \cap (B \cup D) \end{cases}$$

And

$[(F,A) \cup (G,B)] \cup (I,D) = [T,(A \cup B) \cup D]$

And

$$T(e) = \begin{cases} K(e) & if\ e \in (A \cup B) - D \\ M(e) & if\ e \in D - (A \cup B) \\ K(e) \cup M(e) & if\ e \in (A \cup B) \cap D \end{cases}$$

And

So if $e \in A \cup (B \cap D)$ we have $S(e) = F(e) \cup H(e)$ and if $e \in (A \cup B) \cap D$ we have
$T(e) = K(e) \cup M(e)$

But, $A \cup (B \cap D) = (A \cup B) \cap D$ and $S(e) = T(e)$; $\forall e \in A \cup (B \cap D)$ hence, $(F, A) \cup [(G, B) \cup (I, D)] = [(F, A) \cup (G, B)] \cup (I, D)$ on $(A \cup B) \cap D$.

Theorem3.8: If (F, A), (G, B) and (I, D) are any three soft sets over a common universe U then $(F, A) \cap [(G, B) \cap (I, D)] = [(F, A) \cap (G,B)] \cap (I, D)$.

Proof: By definition of intersection of two softsets $(F, A) \cap [(G, B) \cap (I, D)] = [S, A \cap (B \cap D)]$
and

$S(e) = F(e) \cap H(e); \forall e \in A \cap (B \cap D)$

and $T(e) = Q(e) \cap I(e); \forall e \in (A \cap B) \cap D$

but, $A \cap (B \cap D) = (A \cap B) \cap D$ and $S(e) = T(e)$ on $A \cap (B \cap D)$ so
$(F, A) \cap [(G, B) \cap (I, D)] = [(F, A) \cap (G, B)] \cap (I, D)$.

Theorem 3.9.: If (F, A) be any softest over a common universe U then $(F, A) \cup (F, A) = (F, A)$ and $(F, A) \cap (F, A) = (F,A)$.

Proof: Consider $(F, A) \cup (F, A) = (H, A \cup A) = (H, A)$ Where, $H(e) = F(e)$; $\forall e \in A \cup A$
$\Rightarrow H(e) = F(e); \forall e \in A$

Similarly,
$(F, A) \cap (F, A) = (S, A \cap A) = (S,A)$

Where, $S(e) = F(e)$; $\forall e \in A \cap A$
$\Rightarrow S(e) = F(e)$; $\forall e \in A$

By definition of equality of two soft sets, $(H, A) = (F, A)$ and $(S, A) = (F, A)$

So,
$(F, A) \cup (F,A) = (F,A)$ and $(F,A) \cap (F,A) = (F,A)$.

4. Conclusion

In this paper we have given some important results on Soft set theory which relies on union, intersection of Soft sets.

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