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Special Category of Assembly of Linear Combination of m-Gonal Numbers

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Abstract: ***In the article, linear combinations of two separate m −gonal numbers of unlike ranks grade square of an integer are determined by employing the solutions to the eminent Pythagorean equation $x^2 + y^2 = z^2$. Also, Python program for authorization of the statement of the problem with numerical values for each combination is exhibited.***

Keywords: figurate numbers, Pythagorean equation, integral solutions

1. Introduction

Non-negative integers that are classified as polygonal numbers are portrayed by regular polygons constructed from points that are evenly spaced and ordered geometrically. The rank of a polygonal numbers is the number of dots on a side of the outermost layer of the polygonal number. Numerous authors created works on the realationship between the polygonal numbers. Janaki G. and Radha R [1, 3, 4] provided some amazing information regarding the Pythagorean triples and Harshad numbers. Authors in [7, 8, 10] described the association between the polygonal numbers and Narayana sequence. Pandichelvi and others [2, 5, 6, 9] creatively establish the relation between figurative numbers. In this article, the solutions to the Pythagorean equation $x^2 + y^2 = z^2$ are analogized in order to determine the linear combinatons of two discrete m-gonal numbers of different ranks score square of an integer. Additionaly, a Python program for confirming the problem description with numberical values for every combinations is displayed.

2. Process of Exploration

In section 3.1 and 3.2, linear combinations of two distinct m −gonal numbers of different ranks afford square of an integer are determined.

2.1. Linear Combination of Nanogonal and Icositetragonal numbers as square number

Let $T_{9,a}$ and $T_{24,b}$ be the Nanogonal and Icositetragonal numbers of rank a and b respectively.

The general form these two numbers with sides nine and twenty four offers that

$$T_{9,a} = \frac{7a^2-5a}{2}, \quad T_{24,b} = 11b^2 - 10b$$

Assume that the linear combination $56T_{9,a} - 11\,T_{24,b}$ of the members in the pair $(T_{9,a}, T_{24,b})$ provides a square number.

Let us indicate the above proclamation that

$$56T_{9,a} - 11\,T_{24,b} = Y^2$$

This implies that

$$(11b-5)^2 - (14a-5)^2 = Y^2 \qquad (1)$$

Assign $X = 11b - 5, Z = 14a - 5$ (2)

Then, the equation (1) reduces to the succeeding the renowned equation so called Pythagorean equation

$$X^2 + Y^2 = Z^2 \qquad (3)$$

The generalized solutions to (3) are exemplified by

$X = 2rs,\ Y = r^2 - s^2,\ Z = r^2 + s^2$
where $r \neq s \neq 0$ (4)

Comparison of (2), (3) and (4), delivers the ranks of the numbers in the pair $(T_{9,a}, T_{24,b})$ comply with the hypothesis as mentioned below.

$$a = \frac{r^2+s^2+5}{14}, \quad b = \frac{2rs+5}{11}$$

Since the ranks of figurate numbers must be in integers, it is examined that the above said values of a and b are integers for the ensuing options of r and s.

$r = 154R + 7$ and $s = 154S + 24,\ R, S \in Z$

Hence, the ranks of m-gonal numbers under deliberation are assessed by

$$a = 1694R^2 + 1694S^2 + 154R + 528S + 45$$
$$b = 4312RS + 672R + 196S + 31$$

Diminutive examples for the parameters satisfying the supposition are tabulated below.

Table 3.1

R	S	a	b	$T_{9,a}$	$T_{24,b}$	$56T_{9,a} - 11T_{24,b}$
3	−2	21473	−24217	1613760369	6451336149	$19405883025 = 139305^2$
−1	1	3807	−4757	50716854	248967109	$101505625 = 10075^2$
0	1	2267	227	17981844	564549	$1000773225 = 31635^2$
1	0	1893	703	12537339	5429269	$642369025 = 25345^2$
1	1	4115	5211	59256000	298647621	$33212169 = 5763^2$
2	1	9351	10195	306020826	1143216325	$4561786681 = 67541^2$
3	3	32583	41443	3715700154	18892330309	$263575225 = 16235^2$

From the table given above, it is identified that $56T_{9,a} - 11\,T_{24,b}$ is square of an integer.

2.2 Linear Combination of various m-gonal numbers as square numbers

In this section, the ranks of countable number of m −gonal numbers with various sides such that their linear combination results a square number by applying the cognate process outlined in section 3.1 are investigated.

Table 3.2 (a) and Table 3.2 (b) possess selections of few m-gonal numbers of unlike ranks together with their linear combinations, the potential values of X, Y and Z, the choices of r and s and a corresponding values ranks of the m −gonal as per the notations offered in section 3.1.

Table 3.2 (a)

S. No	m −gonal numbers with different ranks	Linear Combination of m-gonal numbers	X and Z	r, s
1	$T_{28,c}$ −Icosioctagonal number of rank c $T_{16,d}$ −Hexadecagonal number of rank d	$13T_{28,c} - 28\,T_{16,d}$	$X = 14d - 6$ $Z = 13c - 6$	$r = 182R + 2$ $s = 182S + 9$
2	$T_{18,e}$ −Octadecagonal number of rank e $T_{32,f}$ −Triacontadigonal number of rank f	$32T_{18,e} - 15\,T_{32,f}$	$X = 15f - 7$ $Z = 16e - 7$	$r = 240R + 4$ $s = 240S + 61$
3	$T_{36,g}$ −Triacontadigonal number of rank g $T_{20,h}$ −Icosigonal number of rank h	$17T_{36,g} - 36\,T_{20,h}$	$X = 18h - 8$ $Z = 17g - 8$	$r = 612R + 1$ $s = 612S + 5$

Table 3.2 (b)

S. No	m −gonal numbers in column (2) of Table 3.2 (a)	Ranks of m −gonal numbers
1	$T_{28,c}$ −Icosioctagonal number of rank c and $T_{16,d}$ −Hexadecagonal number of rank d	$c = 2548R^2 + 56R + 2548S^2 + 252S + 7$ $d = 4732RS + 234R + 52S + 3$
2	$T_{18,e}$ −Octadecagonal number of rank e and $T_{32,f}$ −Triacontadigonal number of rank f	$e = 3600R^2 + 120R + 3600S^2 + 1830S + 234$ $f = 7680RS + 1952R + 128S + 33$
3	$T_{36,g}$ − Triacontadigonal number of rank g and $T_{20,h}$ − Icosigonal number of rank h	$g = 22032R^2 + 72R + 22032S^2 + 360S + 2$ $h = 41616RS + 340R + 68S + 1$

Numerical examples for the statement of the problem corresponding to each of row demonstrated in tables 3.2 (a) and 3.2(b) are displayed in separate tables.

The m − gonal numbers with dissimilar ranks and their linear combination in the first row of tables 3.2 (a) and 3.2(b) are listed in Table 3.3.

Table 3.3

R	S	c	d	$T_{16,d}$	$T_{28,c}$	$13T_{28,c} - 28\,T_{16,d}$
3	−2	32795	−27791	5406544513	13981262785	$30373169841 = 174279^2$
−1	1	5299	−4911	168854913	364968625	$16654561 = 4081^2$
0	1	2807	55	20845	102396553	$1330571529 = 36477^2$
1	0	2611	237	391761	88593841	$1140750625 = 33775^2$
1	1	5411	5021	176442961	380561041	$6890625 = 2625^2$
2	1	13111	9987	698121261	2234520841	$9501375625 = 97475^2$
2	2	21007	19503	2662452045	5736570553	$26759929 = 5173^2$
2	3	33999	29019	5894542413	15026708025	$30300016761 = 174069^2$
3	1	25907	14953	1565055745	8724933553	$69602575329 = 263823^2$
3	2	33803	29201	5968713601	14853950881	$25977380625 = 161175^2$
3	3	46795	43449	13214448513	28466474785	$59613841 = 7721^2$

The m − gonal numbers with unlike ranks and their linear combination in the second row of tables 3.2 (a) and 3.2(b) are registered in Table 3.4.

Table 3.4

R	S	e	f	$T_{18,e}$	$T_{32,f}$	$32T_{18,e} - 15\,T_{32,f}$
3	−2	43734	−40447	15300995910	24539963393	$121532418225 = 348615^2$
−1	1	9144	−9471	668837880	1345630209	$1218359025 = 34905^2$
0	1	5664	161	256607520	386561	$8205642225 = 90585^2$
1	0	3954	1985	125045250	59075585	$3115314225 = 55815^2$
1	1	9384	9793	704409960	1438405633	$965034225 = 31065^2$
2	1	20304	19425	3297877200	5659687425	$20636759025 = 143655^2$
2	2	32934	34913	8676956310	18283274753	$3413480625 = 58425^2$
2	4	52764	50401	22271948220	38103206401	$141154247025 = 375705^2$
3	1	38424	29057	11810961240	12664231937	$187987280625 = 433575^2$
3	2	51054	52225	20851729950	40911028225	$53589935025 = 231495^2$
3	3	70884	75393	40195835460	85260511233	$7359066225 = 85785^2$

The $m -$ gonal numbers with disparate ranks and their linear combination in the third row of tables 3.2 (a) and 3.2(b) are recorded in Table 3.5.

Table 3.5

R	S	g	h	$T_{36,g}$	$T_{20,h}$	$17T_{36,g} - 36\,T_{20,h} = Y^2$
3	−2	285914	−248811	1389691287108	557164213977	$3566840177664 = 1888608^2$
−1	1	44354	−41887	33443004708	15791022017	$54287424 = 7368^2$
0	1	0	1	22394	69	$8524992708 = 380688^2$
1	0	22106	341	8307125316	1043801	$141183553536 = 375744^2$
1	1	44498	42025	33660512100	15894569425	$24206400 = 4920^2$
2	1	110666	83981	208196609796	63474603401	$1254256644096 = 1119936^2$
2	2	177122	167281	533324615076	251845058401	$96353856 = 9816^2$
2	3	287642	250581	1406540040516	565115533401	$3567021486336 = 1888656^2$
3	1	220898	125937	829527214500	142740144225	$8963317454400 = 299380^2$
3	2	287354	250853	1403724864708	566343041657	$3474973200384 = 1864128^2$
3	3	397874	375769	2691156871908	1270818066097	$216442944 = 14712^2$

Python Program for the procedure to attain the ranks of $m-$gonal numbers is deliberated in the following as follows.

Python Program

```
import math
R = int(input('ENTER THE VALUE OF R'))
S = int(input('ENTER THE VALUE OF S'))
Section
= int(input('ENTER THE VALUE OF SECTION'))
if Section == 1:
  a = 1694 * R * R + 154 * R + 1694 * S * S + 528 * S
               + 45
  b = 4312 * R * S + 672 * R + 196 * S + 31
  T9 = (7 * a * a - 5 * a)/2
  T24 = 11 * b * b - 10 * b
  Y = (56 * T9 - 11 * T24)
  print('a = ', a, 'b = ', b)
  print('T9 = ', T9, 'T24 = ', T24 - 1)
  root = math.sqrt(Y)
  print('root = ', root)
  if int(root + 0.5) ** 2 == Y:
    print('Y = ', Y, "Y is a perfect square")
  else:
    print('Y = ', Y, "Y is not a perfect square")
elif Section == 2:
  c = 2548 * R * R + 56 * R + 2548 * S * S + 252 * S + 7
  d = 4732 * R * S + 234 * R + 52 * S + 3
  T28 = 13 * c * c - 12 * c
  T16 = 7 * d * d - 6 * d
  Y = (13 * T28 - 28 * T16)
  print('c = ', c, 'd = ', d)
  print('T28 = ', T28, 'T16 = ', T16)
  root = math.sqrt(Y)
  print('root = ', root)
  if int(root + 0.5) ** 2 == Y:
    print('Y = ', Y, "Y is a perfect square")
  else:
    print('Y = ', Y, "Y is not a perfect square")
elif Section == 3:
  e = 3600 * R * R + 120 * R + 3600 * S * S + 1830 * S
               + 234
  f = 7680 * R * S + 1952 * R + 128 * S + 33
  T18 = 8 * e * e - 7 * e
  T32 = 15 * f * f - 14 * f
  Y = (32 * T18 - 15 * T32)
  print('e = ', e, 'f = ', f)
  print('T18 = ', T18, 'T32 = ', T32)
  root = math.sqrt(Y)
  print('root = ', root)
  if int(root + 0.5) ** 2 == Y:
    print('Y = ', Y, "Y is a perfect square")
  else:
    print('Y = ', Y, "Y is not a perfect square")
elif Section == 4:
  g = 22032 * R * R + 72 * R + 22032 * S * S + 360 * S
               + 2
  h = 41616 * R * S + 340 * R + 68 * S + 1
  T36 = 17 * g * g - 16 * g
  T20 = 9 * h * h - 8 * h
  Y = (17 * T36 - 36 * T20)
  print('g = ', g, 'h = ', h)
  print('T36 = ', T36, 'T20 = ', T20)
  root = math.sqrt(Y)
  print('root = ', root)
```

```
if int(root + 0.5) ** 2 == Y:
  print('Y = ',Y,"Y is a perfect square")
else:
  print('Y = ',Y,"Y is not a perfect square")
```

Output for the values of *R* and *S*

```
ENTER THE VALUE OF R 3
ENTER THE VALUE OF S  − 2
ENTER THE VALUE OF SECTION 1
a = 21473 b = −24217
T9 = 1613760369.0 T24 = 6451336148
root = 139305.0
Y = 19405883025.0 Y is a perfect square
ENTER THE VALUE OF R  − 1
ENTER THE VALUE OF S 1
ENTER THE VALUE OF SECTION 1
a = 3807 b = −4757
T9 = 50716854.0 T24 = 248967108
root = 10075.0
Y = 101505625.0 Y is a perfect square
ENTER THE VALUE OF R 0
ENTER THE VALUE OF S 1
ENTER THE VALUE OF SECTION 2
c = 2807 d = 55
T28 = 102396553 T16 = 20845
root = 36477.0
Y = 1330571529 Y is a perfect square
ENTER THE VALUE OF R 1
ENTER THE VALUE OF S 0
ENTER THE VALUE OF SECTION 2
c = 2611 d = 237
T28 = 88593841 T16 = 391761
root = 33775.0
Y = 1140750625 Y is a perfect square
ENTER THE VALUE OF R 1
ENTER THE VALUE OF S 1
ENTER THE VALUE OF SECTION 3
e = 9384 f = 9793
T18 = 704409960 T32 = 1438405633
root = 31065.0
Y = 965034225 Y is a perfect square
ENTER THE VALUE OF R 2
ENTER THE VALUE OF S 3
ENTER THE VALUE OF SECTION 3
e = 52764 f = 50401
T18 = 22271948220 T32 = 38103206401
root = 375705.0
Y = 141154247025 Y is a perfect square
ENTER THE VALUE OF R 3
ENTER THE VALUE OF S 2
ENTER THE VALUE OF SECTION 4
g = 287354 h = 250853
T36 = 1403724864708 T20 = 566343041657
root = 1864128.0
Y = 3474973200384 Y is a perfect square
ENTER THE VALUE OF R 3
ENTER THE VALUE OF S 3
ENTER THE VALUE OF SECTION 4
g = 397874 h = 375769
T36 = 2691156871908 T20 = 1270818066097
root = 14712.0
Y = 216442944 Y is a perfect square
```

3. Conclusion

In the article, linear combinations of two detached m −gonal numbers of distinct ranks gives a square number are projected through the Pythagorean equation $x^2 + y^2 = z^2$. Also, Python program for the endorsement of the statement of the problem with numerical values for each combination is presented. In this way, one can search various relations among figurate numbers and the results can be be confirmed by different algarithms.

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