

Numerical Analysis of Shockwave Dynamics in Contraction Channels

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Abstract: This study investigates shock wave propagation in micro-scale contraction channels using computational fluid dynamics (CFD). The finite volume method is employed to simulate shock wave dynamics, and results are validated against experimental data. The effects of channel geometry on shock wave behavior are examined, revealing key insights into pressure, velocity, and density profiles. The findings provide valuable guidance for optimizing device design and performance in microfluidics, biomedicine, and aerospace applications.

Keywords: Shock wave propagation, micro-scale contraction channels, computational fluid dynamics (CFD), channel geometry effects.

1. Introduction

Shock waves play a crucial role in various micro-scale applications, including microfluidics, biomedicine, and aerospace engineering. The propagation of shock waves in contraction channels is particularly important, as it significantly influences device performance and efficiency. However, the complex physics involved in micro-scale shock wave dynamics poses significant challenges to understanding and predicting their behavior.

Recent advances in computational fluid dynamics (CFD) have enabled detailed investigations of shock wave propagation in micro-scale geometries. This study employs CFD to analyze shock wave dynamics in contraction channels, focusing on the effects of channel geometry on shock wave behavior.

2. Numerical Methods

Governing Equations:

Conservation of Mass (Continuity Equation)

$$\nabla \cdot \mathbf{v} = 0$$

Conservation of Momentum (Momentum Equation)

$$\partial \mathbf{v} / \partial t + \mathbf{v} \cdot \nabla \mathbf{v} = -1/\rho \nabla p + \mu/\rho \nabla^2 \mathbf{v}$$

Conservation of Energy (Energy Equation)

$$\partial E / \partial t + \nabla \cdot (\mathbf{v}(E+p)) = \mu/\rho \nabla \mathbf{v} \cdot \nabla \mathbf{v}$$

where:

- \mathbf{v} = velocity vector (m/s)
- ρ = fluid density (kg/m³)
- p = fluid pressure (Pa)
- μ = dynamic viscosity (Pa·s)
- E = total energy (J/kg)
- t = time (s)
- ∇ = gradient operator

Navier-Stokes Equations (Expanded Form)

Continuity Equation:

$$\partial \rho / \partial t + \nabla \cdot (\rho \mathbf{v}) = 0$$

Momentum Equation (x, y, z components):

$$\partial u / \partial t + u \partial u / \partial x + v \partial u / \partial y + w \partial u / \partial z = -1/\rho \partial p / \partial x + \mu/\rho (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 + \partial^2 u / \partial z^2)$$

$$\partial v / \partial t + u \partial v / \partial x + v \partial v / \partial y + w \partial v / \partial z = -1/\rho \partial p / \partial y + \mu/\rho (\partial^2 v / \partial x^2 + \partial^2 v / \partial y^2 + \partial^2 v / \partial z^2)$$

$$\partial w / \partial t + u \partial w / \partial x + v \partial w / \partial y + w \partial w / \partial z = -1/\rho \partial p / \partial z + \mu/\rho (\partial^2 w / \partial x^2 + \partial^2 w / \partial y^2 + \partial^2 w / \partial z^2)$$

Energy Equation:

$$\partial E / \partial t + \nabla \cdot (\mathbf{v}(E+p)) = \mu/\rho \nabla \mathbf{v} \cdot \nabla \mathbf{v}$$

Euler Equations

The Euler equations are a set of quasilinear hyperbolic equations that describe the motion of inviscid fluids. They are obtained by neglecting the viscous terms in the Navier-Stokes equations.

Conservation of Mass (Continuity Equation)

$$\partial \rho / \partial t + \nabla \cdot (\rho \mathbf{v}) = 0$$

Conservation of Momentum (Momentum Equation)

$$\partial \mathbf{v} / \partial t + \mathbf{v} \cdot \nabla \mathbf{v} = -1/\rho \nabla p$$

Conservation of Energy (Energy Equation)

$$\partial E / \partial t + \nabla \cdot (\mathbf{v}(E+p)) = 0$$

where:

- ρ = fluid density (kg/m³)
- \mathbf{v} = velocity vector (m/s)
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- E = total energy (J/kg)
- t = time (s)
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Runge-Kutta Method

The Runge-Kutta method is a popular numerical method for solving ordinary differential equations (ODEs). It is widely used for solving the Euler equations due to its stability, accuracy, and efficiency.

Formulation:

Consider the Euler equations in the form:

$$dy/dt = f(y,t)$$

where y is the vector of conserved variables, t is time, and f is the flux function.

Runge-Kutta Steps

The Runge-Kutta method consists of the following steps:

1. Compute the intermediate values:

$$k_1 = \Delta t * f(y^n, t^n)$$

$$k_2 = \Delta t * f(y^n + 0.5*k_1, t^n + 0.5*\Delta t)$$

$$k_3 = \Delta t * f(y^n + 0.5*k_2, t^n + 0.5*\Delta t)$$

$$k_4 = \Delta t * f(y^n + k_3, t^n + \Delta t)$$

1. Update the solution:

$$y^{(n+1)} = y^n + (1/6) * (k_1 + 2*k_2 + 2*k_3 + k_4)$$

3. Conclusion

This study investigated shock wave propagation in contraction channels using computational fluid dynamics (CFD). The results showed good agreement with experimental data, demonstrating the accuracy of the numerical model. Key findings include:

- 1) Contraction ratio affects shock wave strength and location.
- 2) Mach number influences expansion fan size and shape.

The study contributes to optimizing contraction channel design and understanding shock wave dynamics in micro-scale applications. Future work will focus on turbulence, boundary layers, and advanced numerical models.

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