On Some Topological, Reciprocal Indices and Polynomials of Line Graph of Dutch Windmill Graph

N. K. Raut

Ex. Head, Department of Physics, Sunderrao Solanke Mahavidyalaya Majalgaon, Dist. Beed (M.S.) India
Email: *rautnk87[at]gmail.com*

Abstract: ***First Zagreb polynomial, first multiplicative Zagreb index and reciprocal first Zagreb index of a graph G with vertex set V(G) and edge set E(G) are defined as $M_1(G,x) = \sum_{uv\in E(G)} x^{d_u+d_v}$, $M_1II(G) = \prod_{uv\in E(G)} d_u \times d_v$ and $RM_1(G) = \sum_{uv\in E(G)} \frac{1}{d_u+d_v}$ respectively [1-3]. In this paper fifth versions of (M_1, M_2, M_3, hyper M_1, hyper M_2)-Zagreb polynomials and multiplicative (M_1, M_2, hyper M_1, hyper M_2)-Zagreb indices and reciprocal (M_1, M_2, hyper M_1, hyper M_2)-Zagreb indices are investigated for line graph of Dutch windmill graph.***

Keywords: Degree, Dutch windmill graph, fifth Zagreb index, line graph, multiplicative index, reciprocal index, sum degree

1. Introduction

Let G be a finite, simple connected graph with vertex set V(G) and edge set E(G). The degree of a vertex u ∈ V(G) is denoted by d_u and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv. A molecular graph is presentation of the structural formula of a chemical compound in terms of graph theory whose vertices correspond to the atoms of compound and edges correspond to chemical bonds. Graph theory has found considerable use in chemistry, particularly in modeling chemical structures. Topological indices are designed basically by transforming a molecular graph into a number [4]. The fifth M-Zagreb index is a graph theoretic topological index that describes the degree of branching of a molecule. It is modification of the original Zagreb index which sums the degrees of all vertices in a molecule [5]. The correspondence between general Zagreb index and some other vertex degree-based topological indices with particular values of a and b were studied in [6]. The Nirmala index is reciprocal index of sum-connectivity index. The relations between topological indices and their reciprocals with some basic properties were discussed by I.Gutman et al.[7].The reduced-reverse degree versions of some topological indices for metal-organic frameworks were studied by V.Ravi [8].Computation of some reverse topological indices and reverse multiplicative topological indices for Zanamivir and Oseltamivir appear in [9].Inverse multiplicative second Zagreb index and inverse multiplicative first hyper Zagreb index for methyl cyclopentane were studied in [10]. Some reduced M-polynomials and topological indices were computed in [11].

A Dutch windmill graph denoted by D_n^m,m ≥ 1,n ≥ 3 is the graph obtained by making m copies of cycle graph C_n with a vertex in common. Dutch windmill graph has order (n-1)(m+1) and size mn [12-15]. If L(G) is line graph of a Dutch windmill graph D_n^m,then $V(D_n^m)^L$ = mn and $E(D_n^m)^L$ = $2m^2$+mn-2m [16-18].The first and second multiplicative Zagreb indices of double graph of Dutch windmill graph D_3^p were computed in [19].Fifth multiplicative Zagreb indices of molecular graph were studied by V.R.Kulli [20].Some S_u degree-based GA_5 index of Armchair polyhex nanotube were computed by M. R. Farahani [21].The $X_\alpha(G)$, $II_1^*(G)$, $II_{1,c}(G)$ and $II_2(G)$ multiplicative topological indices of silicate, chain silicate, hexagonal oxide and honeycomb networks were computed by J.B.Liu et al.[22].Some multiplicative topological indices of silicate networks were studied in [23].

The fifth M-Zagreb polynomials are defined as [24-29]

$$M_1G_5(G,x) = \sum_{uv\in E(G)} x^{S_u+S_v}. \quad (1)$$
$$M_2G_5(G,x) = \sum_{uv\in E(G)} x^{S_u\times S_v}. \quad (2)$$
$$M_3G_5(G,x) = \sum_{uv\in E(G)} x^{|S_u-S_v|}. \quad (3)$$
$$HM_1G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u+S_v)^2}. \quad (4)$$
$$HM_2G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u\times S_v)^2}. \quad (5)$$

The fifth multiplicative M-Zagreb indices are defined as

$$M_1G_5II(G) = \prod_{uv\in E(G)} S_u + S_v. \quad (6)$$
$$M_2G_5II(G) = \prod_{uv\in E(G)} S_u \times Sv. \quad (7)$$
$$HM_1G_5II(G) = \sum_{uv\in E(G)} (S_u + S_v)^2. \quad (8)$$
$$HM_2G_5II(G) = \sum_{uv\in E(G)} (S_u \times S_v)^2. \quad (9)$$

We introduce some reciprocal fifth M-Zagreb indices which can be defined as

$$RM_1G_5(G) = \sum_{uv\in E(G)} \frac{1}{S_u+S_v}. \quad (10)$$
$$RM_2G_5(G) = \sum_{uv\in E(G)} \frac{1}{S_u\times S_v}. \quad (11)$$
$$RHM_1G_5(G) = \sum_{uv\in E(G)} \frac{1}{(S_u+S_v)^2}. \quad (12)$$
$$RHM_1G_5(G) = \sum_{uv\in E(G)} \frac{1}{(S_u\times S_v)^2}. \quad (13)$$

In these equations S_u is the sum degree of all neighbours of vertex u in G or in other words
$S_u = \sum_{uv\in E(G)} d_v$ and similarly, for S_v.

All the symbols and notations used in this paper are standard and taken from books of graph theory [30-31]. In this paper we study:
$M_1G_5(G,x)$, $M_2G_5(G,x)$, $M_3G_5(G,x)$, $HM_1G_5(G,x)$, $HM_2G_5(G,x)$, $M_1G_5II(G)$, $M_2G_5II(G)$, $HM_1G_5II(G)$,

$HM_2G_5II(G)$, $RM_1G_5(G)$, $RM_2G_5(G)$, $RHM_1G_5(G)$ and $RHM_2G_5(G)$ for the line graph of Dutch windmill graph $(D_n^m)^L$.

2. Materials and Methods

Let vertex set and edge set of a graph G be V(G) and E(G) respectively and let the number of vertices and edges of G be n = |V(G)| and m = |E(G)| respectively. The edge connecting vertices u and v is denoted by uv. Dutch windmill graph D_n^m contains (n-2)m vertices of degree two and one vertex of degree 2m. We partition the edges of D_n^m into edges of the types $E_{(d_u,d_v)}$. Line graph L(G) of a graph G is a graph such that each vertex of L(G) represents an edge of G and two vertices in L(G) are adjacent if and only if their corresponding edges share a common vertex in G. The Dutch windmill graph D_n^m given in figure 1(a) and line graph of Dutch windmill graph $(D_n^m)^L$ with n = 4 in figure 1(b).

3. Results and Discussion

It is observed from line graph of Dutch windmill graph there are three edges (table 1) as $E_1 = |E_{(2+2m,2+2m)}|$, $E_2 = |E_{(2+2m,4m^2-2m+2)}|$ and $E_3 = |E_{(4m^2-2m+2,4m^2-2m+2)}|$ for S_u, S_v with frequency m, 2m and (2m-1)m respectively.

Fifth M-Zagreb polynomials

Theorem 1.1. Fifth M_1-Zagreb polynomial of line graph of Dutch windmill graph is
$mx^{4(1+m)} + 2mx^{4(m^2+1)} + (2m-1)\, mx^{4(2m^2-m+1)}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 for edge partition of E_1, E_2 and E_3 edges and equation (1), we have

$M_1G_5((D_n^m)^L, x) = \sum_{uv\in E(G)} x^{S_u+S_v}$

$= |E_1|x^{(2+2m)+(2+2m)} + |E_2|x^{(2+2m)+(4m^2-2m+2)} + |E_3|x^{(4m^2-2m+2)+(4m^2-2m+2)}$

$= mx^{4(1+m)} + 2mx^{4(m^2+1)} + (2m-1)mx^{4(2m^2-m+1)}$.

Theorem 1.2. Fifth M_2-Zagreb polynomial of line graph of Dutch windmill graph is
$mx^{(2+2m)^2} + 2mx^{(2+2m)\,(4m^2-2m+2)} + (2m-1)mx^{(4m^2-2m+2)^2}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 for edge partition of E_1, E_2 and E_3 edges and equation (2), we have

$M_2G_5((D_n^m)^L, x) = \sum_{uv\in E(G)} x^{S_u\times S_v}$

$= |E_1|x^{(2+2m)\times(2+2m)} + |E_2|x^{(2+2m)\times(4m^2-2m+2)} + |E_3|x^{(4m^2-2m+2)\times(4m^2-2m+2)}$

$= mx^{(2+2m)^2} + 2mx^{(2+2m)\times(4m^2-2m+2)} + (2m-1)mx^{(4m^2-2m+2)^2}$.

Theorem 1.3. Hyper fifth M_1-Zagreb polynomial of line graph of Dutch windmill graph is $mx^{[2(2+2m)]^2} + 2mx^{(4m^2+4)^2} + (2m-1)\, mx^{[2(4m^2-2m+2)]^2}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (4), we have

$HM_1G_5((D_n^m)^L,x) = \sum_{uv\in E(G)} x^{(S_u+S_v)^2}$

$= |E_1|x^{[(2+2m)+(2+2m)]^2} + |E_2|x^{[(2+2m)+(4m^2-2m+2)]^2} + |E_3|x^{[(4m^2-2m+2)+(4m^2-2m+2)]^2}$

$= mx^{[2(2+2m)]^2} + 2mx^{(4m^2+4)^2} + (2m-1)mx^{[2(4m^2-2m+2)]^2}$.

Theorem 1.4. Hyper fifth M_2-Zagreb polynomial of line graph of Dutch windmill graph is $mx^{(2+2m)^4} + 2mx^{[(2+2m)(4m^2-2m+2)]^2} + (2m-1)mx^{(4m^2-2m+2)^4}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (5), we have

$HM_2G_5((D_n^m)^L,x) = \sum_{uv\in E(G)} x^{(S_u\times S_v)^2}$

$= |E_1|x^{[(2+2m)\times(2+2m)]^2} + |E_2|x^{[(2+2m)\times(4m^2-2m+2)]^2} + |E_3|x^{[(4m^2-2m+2)\times(4m^2-2m+2)]^2}$

$= mx^{(2+2m)^4} + 2mx^{[(2+2m)\times(4m^2-2m+2)]^2} + (2m-1)mx^{(4m^2-2m+2)^4}$.

Theorem 1.5. Fifth M_3-Zagreb polynomial of line graph of Dutch windmill graph is $2m^2 + 2m\, x^{4m(1-m)}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (3), we have

$M_3G_5((D_n^m)^L,x) = \sum_{uv\in E(G)} x^{|S_u-S_v|}$

$= |E_1|x^{|(2+2m)-(2+2m)|} + |E_2|x^{|(2+2m)-(4m^2-2m+2)|} + |E_3|x^{|(4m^2-2m+2)-(4m^2-2m+2)|}$

$= 2m^2 + 2mx^{4m(1-m)}$.

Fifth multiplicative M-Zagreb indices

Theorem 2.1. Fifth multiplicative M_1-Zagreb index of line graph of Dutch windmill graph is
$(4+4m)^m \times (4m^2+4)^{2m} \times [2(4m^2-2m+2)]^{(2m-1)m}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (6), we have

$M_1G_5II((D_n^m)^L) = \prod_{uv\in E(G)} S_u + S_v$

$= [(2+2m)+(2+2m)]^m \times [(2+2m)+(4m^2-2m+2)]^{2m} \times [(4m^2-2m+2)+(4m^2-2m+2)]^{(2m-1)m}$

$= (4+4m)^m \times (4m^2+4)^{2m} \times [2(4m^2-2m+2)]^{(2m-1)m}$.

Theorem 2.2. Fifth multiplicative M_2-Zagreb index of line graph of Dutch windmill graph is
$(2+2m)^{2m} \times (8m^3+4m^2+4)^{2m} \times (4m^2-2m+2)^{2(2m-1)m}$.

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n=4 as given in figure 1(b). Using table 1 and equation (7), we have

$M_2G_5II((D_n^m)^L) = \prod_{uv\in E(G)} S_u \times S_v$

$=[(2+2m)\times(2+2m)]^{m}\times[(2+2m)\times(4m^2-2m+2)]^{2m}\times[(4m^2-2m+2)\times(4m^2-2m+2)]^{(2m-1)m}$
$=(2+2m)^{2m}\times(8m^3+4m^2+4)^{2m}\times(4m^2-2m+2)^{2(2m-1)m}$.

Theorem 2.3. Fifth hyper multiplicative M_1-Zagreb index of line graph of Dutch windmill graph is
$$[2(2+2m)]^{2m}\times(4m^2+4)^{4m}\times[2(4m^2-2m+2)]^{2(2m-1)m}.$$

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (8), we have
$HM_1G_5II((D_n^m)^L)=\prod_{uv\in E(G)}(S_u+S_v)^2$
$=[(2+2m)+(2+2m)]^{2m}\times[(2+2m)+(4m^2-2m+2)]^{4m}\times[(4m^2-2m+2)+(4m^2-2m+2)]^{2(2m-1)m}$
$=[2(2+2m)]^{2m}\times(4m^2+4)^{4m}\times[2(4m^2-2m+2)]^{2(2m-1)m}$.

Theorem 2.4. Fifth hyper multiplicative M_2-Zagreb index of line graph of Dutch windmill graph is
$$(2+2m)^{4m}\times[(2+2m)(4m^2-2m+2)]^{4m}\times(4m^2-2m+2)^{4(2m-1)m}.$$

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (9), we have
$HM_2G_5II((D_n^m)^L)=\prod_{uv\in E(G)}(S_u\times S_v)^2$
$=[(2+2m)\times(2+2m)]^{2m}\times[(2+2m)\times(4m^2-2m+2)]^{4m}\times[(4m^2-2m+2)\times(4m^2-2m+2)]^{2(2m-1)m}$
$=(2+2m)^{4m}\times[(2+2m)(4m^2-2m+2)]^{4m}\times(4m^2-2m+2)^{4(2m-1)m}$.

Reciprocal fifth M-Zagreb indices

Theorem 3.1. Reciprocal fifth M_1-Zagreb index of line graph of Dutch windmill graph is
$$\frac{m}{4(1+m)}+\frac{m}{2(m^2+1)}+\frac{(2m-1)m}{2(4m^2-2m+2)}.$$

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (10), we have
$RM_1G_5((D_n^m)^L)=\sum_{uv\in E(G)}\frac{1}{S_u+S_v}$
$=|E_1|\frac{1}{(2+2m)+(2+2m)}+|E_2|\frac{1}{(2+2m)+(4m^2-2m+2)}+|E_3|\frac{1}{(4m^2-2m+2)+(4m^2-2m+2)}$
$=\frac{m}{4(1+m)}+\frac{m}{2(m^2+1)}+\frac{(2m-1)m}{2(4m^2-2m+2)}$.

Theorem 3.2. Reciprocal fifth M_2-Zagreb index of line graph of Dutch windmill graph is
$$\frac{m}{(2+2m)^2}+\frac{2m}{(2+2m)(4m^2-2m+2)}+\frac{(2m-1)m}{(4m^2-2m+2)^2}.$$

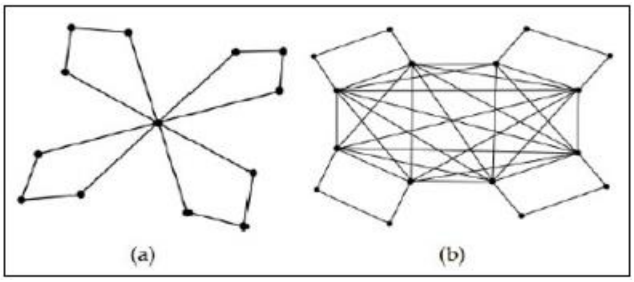
Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (11), we have
$RM_1G_5((D_n^m)^L)=\sum_{uv\in E(G)}\frac{1}{S_u+S_v}$
$=|E_1|\frac{1}{(2+2m)\times(2+2m)}+|E_2|\frac{1}{(2+2m)\times(4m^2-2m+2)}+|E_3|\frac{1}{(4m^2-2m+2)\times(4m^2-2m+2)}$
$=\frac{m}{(2+2m)^2}+\frac{2m}{(2+2m)(4m^2-2m+2)}+\frac{(2m-1)m}{(4m^2-2m+2)^2}$.

Theorem 3.3. Reciprocal hyper fifth M_1-Zagreb index of line graph of Dutch windmill graph is
$$\frac{m}{[2(2+2m)]^2}+\frac{2m}{[4(m^2+1)]^2}+\frac{(2m-1)m}{[2(4m^2-2m+2)]^2}.$$

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (12), we have
$RHM_1G_5((D_n^m)^L)=\sum_{uv\in E(G)}\frac{1}{(S_u+S_v)^2}$
$=|E_1|\frac{1}{[(2+2m)+(2+2m)]^2}+|E_2|\frac{1}{[(2+2m)+(4m^2-2m+2)]^2}+|E_3|\frac{1}{[(4m^2-2m+2)+(4m^2-2m+2)]^2}$
$=\frac{m}{[2(2+2m)]^2}+\frac{2m}{[4(m^2+1)]^2}+\frac{(2m-1)m}{[2(4m^2-2m+2)]^2}$.

Theorem 3.4. Reciprocal hyper fifth M_2-Zagreb index of line graph of Dutch windmill graph is
$$\frac{m}{(2+2m)^4}+\frac{2m}{[(2+2m)\times(4m^2-2m+2)]^2}+\frac{(2m-1)m}{(4m^2-2m+2)^4}.$$

Proof. Let $(D_n^m)^L$ be the line graph of Dutch windmill graph for n = 4 as given in figure 1(b). Using table 1 and equation (13), we have
$RHM_1G_5((D_n^m)^L)=\sum_{uv\in E(G)}\frac{1}{(S_u\times S_v)^2}$
$=|E_1|\frac{1}{[(2+2m)\times(2+2m)]^2}+|E_2|\frac{1}{[(2+2m)\times(4m^2-2m+2)]^2}+|E_3|\frac{1}{[(4m^2-2m+2)\times(4m^2-2m+2)]^2}$
$=\frac{m}{(2+2m)^4}+\frac{2m}{[(2+2m)\times(4m^2-2m+2)]^2}+\frac{(2m-1)m}{(4m^2-2m+2)^4}$.



D[P4] [C4] D[C4]

Figure1. (a): Dutch windmill graph D_4^4 **(b):** Line graph of Dutch windmill graph $(D_4^4)^L$

Table 1: Sum degree edge partition of line graph of Dutch windmill graph $(D_n^m)^L$.

(S_u, S_v): uv ∈E(G)	Number of edges
(2+2m,2+2m)	m
(2+2m,4m²-2m+2)	2m
(4m²-2m+2,4m²-2m+2)	(2m-1)m

4. Conclusion

In this paper we have obtained fifth M-Zagreb polynomials, fifth multiplicative M-Zagreb indices of $(D_n^m)^L$. Reciprocal fifth M-Zagreb indices are introduced and computed for line graph of Dutch windmill graph.

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