Analytical Solution for Study of Damped of Clamped and Simply Supported Right Triangular Plate under Elastic Foundation

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Abstract: ***This study investigates the analytical solution for damped vibrations analysis of clamped, simply supported right triangular plates using the separation of variable method. The governing equation is solved for various boundary conditions applicable to the right triangular plate. An approximate frequency equation is obtained using the Rayleigh Ritz technique with a two-term deflection function generated through Gram Schmidt orthogonalization. The calculations include determining the frequency parameter, time period, deflection, and logarithmic decrement at different points for two vibration modes based on variations in parameters such as taper constant, aspect ratio, damping parameters and elastic foundation***

Keywords: Vibration, elastic foundation, Rayleigh-Ritz technique, orthogonal functions Edge condition

1. Introduction

It has been noted recently that because vibration plays such a major part in all applied sciences fields, vibration research is continuously gaining enormous relevance in modern science. In several technical domains, including aircraft, missile, navy ship, and telephone technologies, among others, plates of diverse geometries are employed as essential structural components. These plates offer the benefit of reduced weight and size due to the suitable variation in plate thickness. In addition, they are substantially more vibration-efficient than plates with a uniform thickness. So, researchers are interested in the vibration properties of plates with different thicknesses. Leissa [1] provided a thorough analysis of the literature up until 1985 on the linear vibration of isotropic and anisotropic plates with different geometries in his monograph and a number of review papers. Subsequently, numerous researchers have studied rectangular plates with uniform or non-uniform thickness. The study of vibration is becoming more and more important as space technology develops quickly. Gupta and Lal [2] used the quintic spline technique to examine the transverse vibrations of a rectangular plate with exponentially variable thickness sitting on an elastic basis. Since free vibrations are ideal and impractical, all vibrations in actuality are damped vibrations; so, no vibration can be imagined to exist without damping. Recent papers by DJO'Boy [3] and Alisjahbana and Wangsadinata [4] tackled a real world problem of structural dynamic for rigid road way pavement considering traffic load both the type of problem considered damping of flexural vibration. A study using classical plate theory is presented that addresses damped vibrations of a homogeneous isotropic rectangular plate sitting on an elastic basis and with a thickness that varies linearly along one direction, meeting the demands of modern science. Numerous numerical methods have been used to analyze the modes and frequencies for various structure which includes the Frobenious method [5], the finite difference method (FDM) [6], the simple polynomial approximation (SPA) [7], Galerkin's method (GA) [8,9], the Rayleigh-Ritz method (RRM) [10–12], the finite element method (FEM) [13], and the Chebyshev collocation method (CCM) [14]. Because each step of the numerical approaches introduces rounding and truncation errors, they require tiny interval sizes to produce results up to the necessary precision. The investigating strategy presented by Gorman [15, 16] established the placing of the construction structure for fully free vibration of the three corners. He also researched vibration analysis using various boundary conditions on the appropriate triangular plate. Christensen [17] used the grid work method to examine 45^0 right triangular cantilevered plates' vibrations. Using the Rayleigh-Ritz technique, Bhatt [18] determined the various parameters for the vibrations of polynomial plates in order to address the question about triangular plates. To investigate the free vibration of rectangular plates with regard to the manipulation of number shapes, Liew et al. [19] suggested the orthogonal plate function. The investigation only looked at natural frequencies. In Lam et al.'s study [20], natural frequencies of isotropic and orthotropic triangular plates were examined and analyzed. In order to establish an analytical solution of damped vibration and elastic vibration of an isotropic right triangular plate with a varied combination of edge conditions, the present problem expands on the preliminary work of several scholars. With respect for the Rayleigh Ritz method and the following parameters: taper constant, damping parameters, and elastic foundation, the deflection function, frequencies, and time period have been derived for the first two modes using 2D orthogonal plate functions.

2. Mathematical Model for the Problem

We obtain mathematical model for equation of isotropic right triangular plate with variable thickness by introducing K as the damping parameter and K_f as the foundation parameter assume that the damping forces are proportional to velocity, then model equation given by Leissa [1] is transformed and equation of motion is given as

$$\tilde{d}\left(\frac{\partial^4\tilde{w}}{\partial x^4}+2\frac{\partial^4\tilde{w}}{\partial x^2\partial y^2}+\frac{\partial^4\tilde{w}}{\partial y^4}\right)+2\frac{\partial\tilde{d}}{\partial x}\left(\frac{\partial^3\tilde{w}}{\partial x^3}+\frac{\partial^3\tilde{w}}{\partial x\partial y^2}\right)+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3\tilde{w}}{\partial y^3}+\frac{\partial^3\tilde{w}}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2\tilde{w}}{\partial x^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\tilde{w}}{\partial y^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\tilde{w}}{\partial x\partial y}+K\frac{\partial\tilde{w}}{\partial t}+\rho h\frac{\partial^2\tilde{w}}{\partial t^2}+K_f\tilde{w}=0 \quad (1)$$

For solution of equation (1) the following form, which is taken in the form of products of two functions is assumed as

$$\tilde{w}(x,y,t)=\tilde{W}(x,y)\,\tilde{T}(t) \quad (2)$$

Substituting the value from (2) in (1) we get the following transformed equations i.e. (3) and (4)

$$\frac{\partial^2\tilde{T}}{\partial t^2}+\frac{K}{\rho h}\frac{\partial\tilde{T}}{\partial t}+\left(\frac{K_f}{\rho h}\right)\tilde{T}=0 \quad (3)$$

and

$$\tilde{d}\left(\frac{\partial^4\tilde{w}}{\partial x^4}+2\frac{\partial^4\tilde{w}}{\partial x^2\partial y^2}+\frac{\partial^4\tilde{w}}{\partial y^4}\right)+2\frac{\partial\tilde{d}}{\partial x}\left(\frac{\partial^3W}{\partial x^3}+\frac{\partial^3W}{\partial x\partial y^2}\right)+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3W}{\partial y^3}+\frac{\partial^3W}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2W}{\partial x^2}+\upsilon\frac{\partial^2W}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\tilde{w}}{\partial y^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\tilde{w}}{\partial x\partial y}-\rho h\mathcal{P}^2\tilde{w}=0 \quad (4)$$

Thus Equation (3) and (4) are the required differential equation of motion for isotropic plate and time function of free and damped vibration having variable thickness respectively. We shall now separately solve above two equation.

Time function of vibration of plates

The time function is given by equation (3) and its solution is as

$$\tilde{T}(t)=e^{\alpha t}\{\tilde{c_1}\cos\beta t+\tilde{c_2}\sin\beta t\} \quad (5)$$

Where $\alpha=-\frac{K}{2\rho h}$ and $\beta=P\sqrt{1-\frac{\alpha^2}{P^2}+\frac{K_f}{\rho hP^2}}=P_0$, here P is natural frequency, P_0 is angular frequency and $\tilde{c_1}$, $\tilde{c_2}$ are arbitrary but fixed which are calculated from the primary restriction of the plate.

For the present study we have as

$$\tilde{T}=1 \text{ and } \dot{\tilde{T}}=0 \text{ at } t=0 \quad (6)$$

Using equation (6) in equation (5), one obtains

$$\tilde{c_1}=1 \text{ and } \tilde{c_2}=-\frac{\alpha}{\beta} \quad (7)$$

$$\text{And } \tilde{T}(t)=e^{\alpha t}\{\cos\beta t-\frac{\alpha}{\beta}\sin\beta t\} \quad (8)$$

For damped transverse vibration of the triangular plate amplitude ($\tilde{w}$) can be expressed as

$$\tilde{w}(x,y,t)=\tilde{W}(x,y)\left[e^{\alpha t}\left\{\cos\beta t-\frac{\alpha}{\beta}\sin\beta t\right\}\right] \quad (9)$$

Substituting the above value from (9) in (4) and equating the coefficient sine and cosine term we establish the equation for deflection function as

$$\tilde{d}\left(\frac{\partial^4\tilde{W}}{\partial x^4}+2\frac{\partial^4\tilde{W}}{\partial x^2\partial y^2}+\frac{\partial^4\tilde{W}}{\partial y^4}\right)+2\frac{\partial\tilde{d}}{\partial x}\left(\frac{\partial^3\tilde{W}}{\partial x^3}+\frac{\partial^3\tilde{W}}{\partial x\partial y^2}\right)+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3\tilde{W}}{\partial y^3}+\frac{\partial^3\tilde{W}}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2\tilde{W}}{\partial x^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\tilde{W}}{\partial y^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\tilde{W}}{\partial x\partial y}-\frac{k^2\tilde{W}}{4\rho h}-\rho hP^2\tilde{W}+K_f\tilde{w}=0 \quad (10)$$

Assuming that the thickness variation of the plate in x direction, the flexural rigidity of the plate $\tilde{d}$ is written as (assuming possion's ratio υ is constant)

$$\tilde{d}=\frac{Eh_0^3}{12(1-\upsilon^2)}\left(1+\beta^*\frac{x}{a}\right)^3,\quad \tilde{d}=\tilde{d}_0\left(1+\beta^*\frac{x}{a}\right)^3 \quad \text{Where } \tilde{d}_0=\frac{Eh_0^3}{12(1-\upsilon^2)} \quad (11)$$

substituting (11) in (10) and introducing non dimensional variables X′=x/a and Y′=y/b and simplifying one gets

$$a^4(1+\beta^*X')^4\left(\frac{\partial^4\tilde{W}}{\partial X'^4}+2\frac{\partial^4\tilde{W}}{\partial X'^2\partial Y'^2}+\frac{\partial^4\tilde{W}}{\partial Y'^4}\right)+2a^4(1+\beta^*X')\frac{\partial}{\partial X'}(1+\beta^*X')^3\left(\frac{\partial^3\tilde{W}}{\partial X'^3}+\frac{\partial^3\tilde{W}}{\partial X'\partial Y'^2}\right)+2a^4(1+\beta^*X')\frac{\partial(1+\beta^*X')^3}{\partial Y'}\left(\frac{\partial^3\tilde{W}}{\partial Y'^3}+\frac{\partial^3\tilde{W}}{\partial Y'\partial X'^2}\right)+a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial X'^2}\left(\frac{\partial^2\tilde{W}}{\partial X'^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial Y'^2}\right)+a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial Y'^2}\left(\frac{\partial^2\tilde{W}}{\partial Y'^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial X'^2}\right)+2(1-\upsilon)a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial X'\partial Y'}\frac{\partial^2\tilde{W}}{\partial X'\partial Y'}-\frac{k^2a^4\tilde{W}}{4\rho h_0\,\tilde{d}0}-\frac{\rho h_0a^4P^2}{\tilde{d}0}(1+\beta^*X')^2\tilde{W}+\frac{K_f(1+\beta^*X')a^4\tilde{W}}{\tilde{d}0}=0 \quad (12)$$

Replacing $\frac{4\rho h_0\,\tilde{d}0}{a^4}$ by $\frac{K_0{}^2}{\mu^2}$, $\frac{\rho h_0a^4P^2}{\tilde{d}0}$ by Λ^2 And E_f by $\frac{K_f a^4}{\tilde{d}0\Lambda^2}$

We shall now find the deflection function $\tilde{W}$ using orthogonal plate function.

Orthogonal plate function

Let the deflection function be assumed as

$$\tilde{W}(X',Y')=A_1\Phi_1+A_2\Phi_2 \quad (13)$$

where Φ_1 and Φ_2 are orthogonal plate function $\Phi_1(X',Y')$ is so chosen for triangular plate in our study such that it at least satisfy the geometrical boundary conditions of the plate and a better approximation and convergence is achieved if $\Phi_1(X',Y')$ it also satisfy the natural boundary conditions.

For present problem the required function in equation (13) is

$$\Phi_i(X',Y')=\prod_{k=1}^{3}\Theta_k(X',Y') \quad (14)$$

Apply the geometrical conditions we obtain Φ_1 which is given below table (1) and

$$\text{For } \Phi_2(X',Y')=f_2(X',Y')\Phi_1(X',Y')-a_{2,1}\Phi_1(X',Y') \quad (15)$$

$$\iint\Phi_i(X',Y')\Phi_j(X',Y')dX'dY'=\begin{cases}0, & if\ i\neq j\\ 1, & if\ i=j\end{cases}$$

(By gram Schmidt orthogonalization definition) (16)

Where the integration is carried out over the plate domain R, the coefficient

$$a_{mi}=\frac{\iint f_m(X',Y')\Phi_1(X',Y')\Phi_i(X',Y')dX'dY'}{\iint\Phi_i(X',Y')\Phi_i(X',Y')dX'dY'} \quad (17)$$

Where $f_m(X',Y')$ is the generating function

$$r=\lceil\sqrt{m-1}\,\rceil \quad (18)$$

$$t=(m-1)-r^2 \quad (19)$$

If t is even, then

$$s=t/2;\ 0\leq s\leq r \quad (20)$$

$$f_m(X',Y')=X'^{r}Y'^{s} \quad (21)$$

If t is odd, then

$$s=(t-1)/2;\ 0\leq s\leq r-1 \quad (22)$$

$$f_m\,(X', Y') = X'^r Y'^s \qquad (23)$$

Where [ ] in equation (18) denotes the greatest integer function and f_m (X', Y') is calculated by deciding the parameters

Table 1

Boundary condition	Generating functions	Plate functions
CSS	$f_1(X', Y')=1$	$\Phi_1=Y'^2(X'-1)(Y'-X')$
	$f_2(X', Y')=X'$	$\Phi_2=X'\Phi_1(X', Y')-a_{2,1}\Phi_1(X', Y')$
CCS	$f_1(X', Y')=1$	$\Phi_1=Y'(X'-1)^2(Y'-X')^2$
	$f_2(X', Y')=X'$	$\Phi_2=X'\Phi_1(X', Y')-a_{2,1}\Phi_1(X', Y')$

Method of Analysis

The plate geometry for thin triangular plate is given in fig. (a) and approximate solution is derived using Rayleigh's principle, which states that

$$\tilde{V}_{max} = \tilde{T}_{max} \qquad (24)$$

Here $\tilde{V}_{max}$ is max. Strain energy and $\tilde{T}_{max}$ is max. Kinetic energy

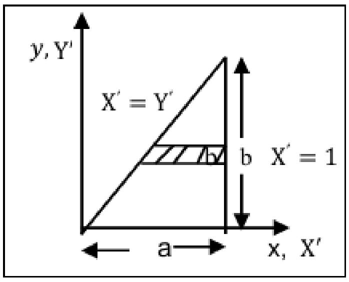

Figure (a): Geometry of right triangular plate

Equation of motion

The expression for K.E. (T) and S.E. (V) are

$$\tilde{T}_{max} = \frac{1}{2}ab\Lambda^2 \iint [((1+\beta^* X')^2 + D_f - E_f(1+\beta^* X')]\tilde{W}^2((X',, Y')dX'dY' \qquad (25)$$

$$\tilde{V}_{max} = \frac{1}{2}ab\iint\{(1+\beta^* X')^4\left(\frac{\partial^2\tilde{W}}{\partial X'^2}\right)^2 + 2\upsilon\alpha'^2(1+\beta^* X')^4\left(\frac{\partial^2\tilde{W}}{\partial X'^2}\frac{\partial^2\tilde{W}}{\partial Y'^2}\right) + \alpha'^4(1+\beta^* X')^4\left(\frac{\partial^2\tilde{W}}{\partial Y'^2}\right)^2 + 2(1-\upsilon)(1+\beta^* X')^4\alpha'^2\left(\frac{\partial^2\tilde{W}}{\partial X'\partial Y'}\right)^2\}dX'dY' \qquad (26)$$

Solution and frequency equation

$$\frac{\partial}{\partial A_i}(\tilde{V}max - \tilde{T}max) = 0 \qquad (27)$$

Which leads to the governing Eigen value equation

$$\sum[K_{ij} - \Lambda^2 M_{ij}]c_i = 0 \qquad (28)$$

$K_{ij}=P_{ij}+\alpha'^4 Q_{ij} + \alpha'^2\,\upsilon\,(R_{ij}+S_{ij})+2(1-\upsilon)\alpha'^2 T_{ij}$

$M_{ij}=\iint[(1+\beta^* X')^2 + D_f - E_f(1+\beta^* X')]\Phi_i(X', Y')\Phi_j(X', Y')dX'dY'$,

$P_{ij}=\iint(1+\beta^* X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial X'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial X'}dX'dY'$,

$Q_{ij}=\iint(1+\beta^* X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial Y'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial Y'\partial Y'}dX'dY'$,

$R_{ij}=\iint(1+\beta^* X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial Y'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial X'}dX'dY'$,

$S_{ij}=\iint(1+\beta^* X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial X'}\frac{\partial^2\Phi_j(X',Y')}{\partial Y'\partial Y'}dX'dY'$,

$T_{ij}=\iint(1+\beta^* X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial Y'}dX'dY'$

$F_{ij}= K_{ij} - \Lambda^2 M_{ij}$, i,j=1,2 On simplifying (28) one gets

$$F_{i1}A_1 + F_{i2}A_2 = 0,\ i=1,2 \qquad (29)$$

Here F_{i1}, F_{i2} (i=1,2) contains various parameter used and frequency parameter.

For non zero solution, we must have from equation (29)

$$\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} = 0 \qquad (30)$$

By equation (30) we obtain a quadratic equation in Λ^2 and two values of Λ are calculated. After deciding the values of A_1 and A_2 we obtain $\tilde{W}$ from equation (29).

Taking A_1=1, we get $A_2=-\frac{b_{11}}{b_{12}}$ and so $\tilde{W}$ is

$$\tilde{W} = \Phi_1 + \left(-\frac{b_{11}}{b_{12}}\right)\Phi_2 \qquad (31)$$

Thus deflection $\tilde{w}$ may be expressed, by using equation (31) and (8) in equation (2), to give

$$\tilde{w}(x,y,t) = \left(\Phi_1 + \left(-\frac{b_{11}}{b_{12}}\right)\Phi_2\right)\left[e^{\alpha t}\left\{\cos\beta t - \frac{\alpha}{\beta}\sin\beta t\right\}\right] \qquad (32)$$

Time period for triangular plate vibration is

$$\tilde{K} = \frac{2\Pi}{P} \qquad (33)$$

Numerical evaluations

For the solving the problem, the following values of parameter were used [11]: $E=7.08\times10^{10}N/M^2$, $G=2.632\times10^{10}N/M^2$, $\rho=2.80\times10^3 kg/M^3$, υ=0.3, The diameter of the plate's middle section has been measured at h=0.01m.

3. Results and Discussion

For the two boundary conditions mentioned above, the first two frequency modes and the first two time period modes of vibration have been calculated in the current

Table 1: Values of frequency parameter of right triangular plate for different value of taper parameter (β^*), and aspect ratio (a/b=1/2)

1(a)

β^*	D_f=0.0, E_f=0.0			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	75.43	190.58	54.80	119.92
0.2	86.96	226.99	61.50	135.34
0.4	98.46	264.74	67.96	152.40
0.6	109.98	303.35	74.30	170.58
0.8	121.55	342.56	80.62	189.54

1(b)

β^*	D_f=0.4, E_f=0.0			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	63.75	161.07	46.31	101.35
0.2	75.99	198.45	53.57	117.79
0.4	88.15	237.28	60.51	135.73
0.6	100.29	277.01	67.29	154.69
0.8	112.47	317.32	74.00	174.37

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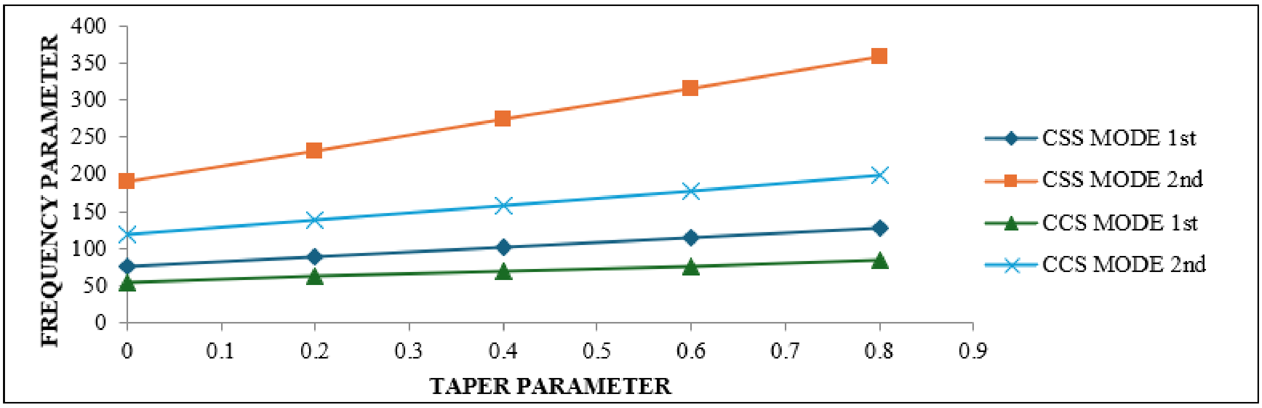
1(c)

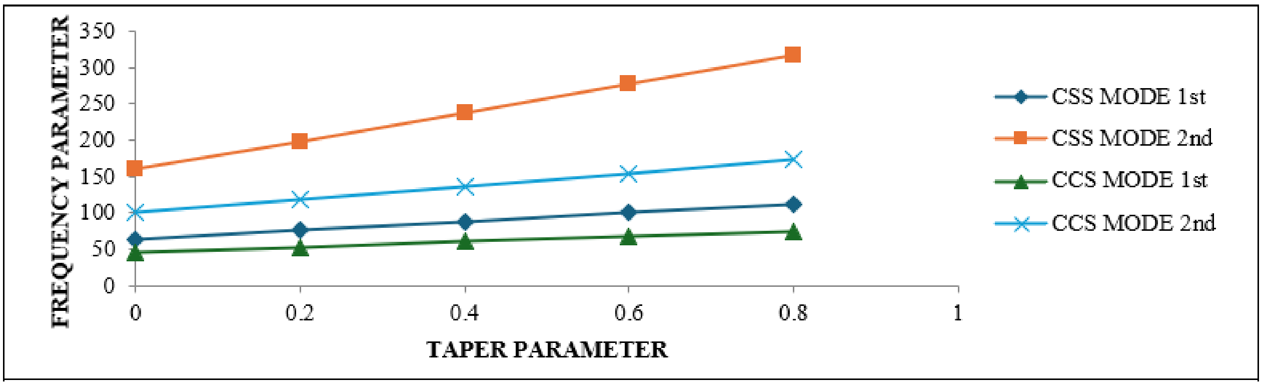
β^*	D_f=0.4, E_f=0.4			
	CSS		CCS	
	MODE1[st]	MODE2[nd]	MODE1[st]	MODE2[nd]
0.0	75.43	190.58	54.80	119.92
0.2	88.86	231.99	62.72	137.94
0.4	101.93	274.13	70.16	157.30
0.6	114.76	316.58	77.32	177.51
0.8	127.44	359.14	84.32	198.26

paper for various values of the foundation parameter E_f(0.0,0.2,0.4,0.6,0.8), damping parameter D_f(0.0,0.2,0.4,0.6,0.8), and taper parameter(β^*) (0.0,0.2,0.4,0.6,0.8) for Poisson ratios of υ =0.3, plate thickness h=0.01, and aspect ratio a/b=1/2. Additionally, for the same set of values of other parameters, it has been noted that the frequencies for CSS plates are greater than the frequencies for CCS plates.

For the fixed values of damping parameter Dk=0.0, 0.4 and foundation parameter Ef=0.0, 0.4 for the first two modes of vibration of CSS and CCS plate, Tables 1a-1c and Figures 1a-1c show the behavior of frequency parameter with the increasing value of taper constant. It is observed that the frequency parameter decreases with the increasing values of taper parameter



1(a)



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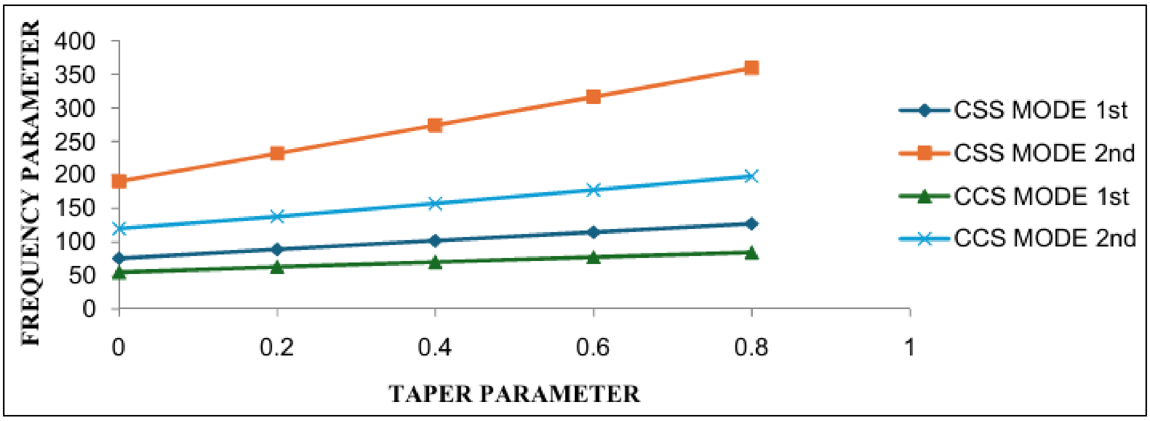


1(c)

Figure 1: Frequency parameter for clamped, simply supported plates:
1(a) modes for D_f=0.0, E_f=0.0,1 (b) modes for D_f=0.4,E_f=0.0,(c) modes for D_f=0.4,E_f=0.4

The inference of the damping parameter D_f on the frequency parameter for two values of the foundation parameter E_f=0.0 and 0.4, respectively, for the fixed value of the taper parameter β^*=0.4 is given in Tables 2a-2b and Figures 2a and 2b. When the damping parameter D_f is increased, it is seen that the frequency parameter lowers. For CSS plates, this decrease occurs at a faster pace than for CCS plates, even when all other plate parameters remain constant.

Table 2: Values of frequency parameter of right triangular plate for different value of damping parameter parameter (D_f), and aspect ratio (a/b=1/2)

2(a)

D_f	β^*=0.4, E_f=0.0			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0	98.46	264.74	67.96	152.4
0.2	92.88	249.88	63.91	143.33
0.4	88.15	237.28	60.51	135.73
0.6	84.08	226.43	57.6	129.22
0.8	80.53	216.95	55.07	123.58

2(b)

D_f	β^*=0.4, E_f=0.4			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	118.89	319.34	82.59	185.19
0.2	109.43	294.13	75.62	169.53
0.4	101.93	274.13	70.16	157.30
0.6	95.79	257.75	65.73	147.41
0.8	90.62	244.01	62.05	139.18

Figures 3a and 3b, as well as Tables 3a and 3b, illustrate the relationship between frequency parameter and foundation parameter Ef for two distinct damping parameter values (Dk = 0.0 and 0.4) and a fixed taper parameter value of 0.4.

Table 3: Values of frequency parameter of right triangular plate for different value of elastic foundation and aspect ratio (a/b=1/2)

3(a)

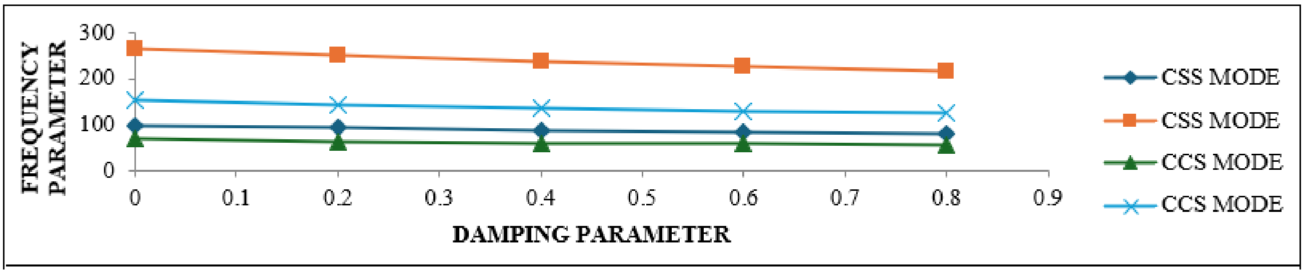
E_f	β^*=0.4, D_f=0.0			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0	98.46	264.74	67.96	152.4
0.2	107.24	288.22	74.21	166.4
0.4	118.89	319.34	82.59	185.19
0.6	135.4	363.48	94.63	212.29
0.8	161.47	433.48	114.11	256.68

3(b)

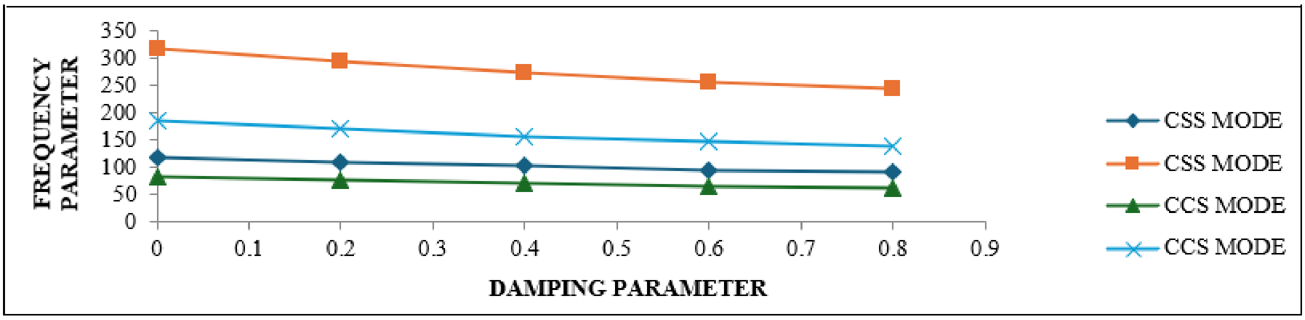
E_f	β^* =0.4, D_f=0.4			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	88.15	237.28	60.51	135.73
0.2	94.29	253.72	64.80	145.33
0.4	101.93	274.13	70.16	157.30
0.6	111.77	300.44	77.11	172.83
0.8	125.16	336.17	86.64	194.14

It is observed that, for both CSS and CCS plates, the frequency parameter steadily rises with the rising value of the foundation parameter E_f, regardless of the other plate parameters' values.

It is found that the increase rate of frequency parameter for CSS plate is higher than that for CCS plate for two modes

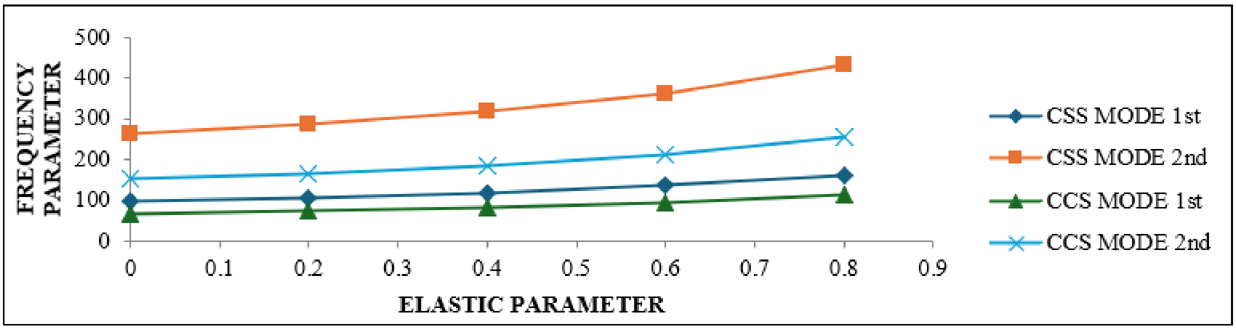


2(a)

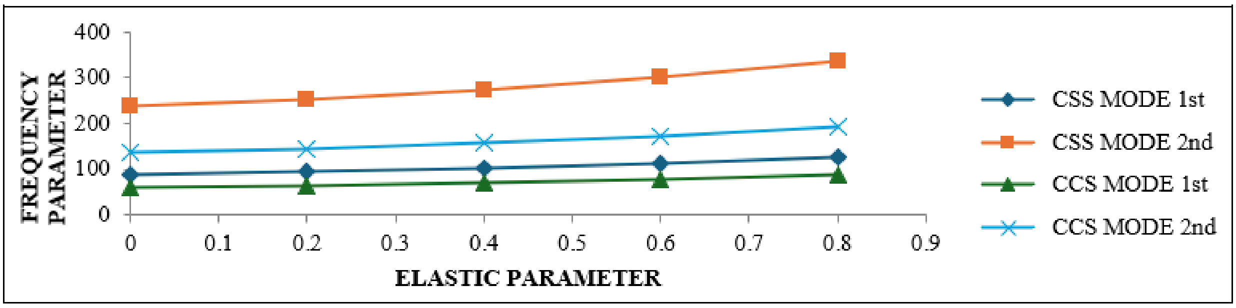


2(b)

Figure 2: Frequency parameter for clamped, simply supported plates:
2(a) modes for β^*=0.4, E_f=0.0, 2 (b) modes for β^*=0.4, E_f=0.4;



3(a)



3(b)

Figure 3: Frequency parameter for clamped, simply supported plates:
3(a) modes for β^*=0.4, D_f=0.0, 3 (b) modes for β^*=0.4, D_f=0.4;

For two distinct values of the damping parameter D_f = 0.0, 0.4, and for the fixed value of the taper parameter β^* = 0.4,

Table 4: Values of Time Period ($K\times10^{-5}$) of right triangular plate for different value of elastic foundation and aspect ratio a/b=1/2

4(a)

E_f	β^* =0.4, D_f=0.0			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	419.36	155.96	607.52	270.93
0.2	385.00	143.26	556.33	248.12
0.4	347.28	129.29	499.93	222.96
0.6	304.93	113.59	436.33	194.49
0.8	255.71	95.25	361.82	160.86

4(b)

E_f	β^* =0.4, D_f=0.4			
	CSS		CCS	
	MODE1st	MODE2nd	MODE1st	MODE2nd
0.0	468.36	174.01	682.28	304.21
0.2	437.87	162.73	637.10	284.11
0.4	405.08	150.62	588.47	262.48
0.6	369.40	137.43	535.44	238.89
0.8	329.89	122.82	476.56	212.68

Tables 4a and 4b and Figures 4a and 4b illustrate the decrease in the time period parameter as the elastic foundation parameter E_f increases. It is observed that, regardless of the value of the other plate parameters, the time period consistently gets shorter with an increase in the foundation parameter E_f for the CSS and CCS plates.

Table 5 and table 6 contains the deflection function values (i.e. the amplitude of the vibration modes)for the first two vibration modes with aspect ratios a/b = 1/2, respectively, for distinct values of X' and Y'.The following parameter are used to calculate $\widetilde{w}$ are β^*=0.4,K/K_0=0.4 ,time $\widetilde{T}$ =0$\widetilde{K}$, time $\widetilde{T}$ =5$\widetilde{K}$ for CSS and CCS plates .which values in table 5 and table 6 is bold indicate deflection at $\widetilde{T}$ =5$\widetilde{K}$

Table 5: Deflection ($\widetilde{w}$) of a CSS right triangular plate for different value of $X'$and Y',a constant aspect ratio (a/b=1/2) and β^*=0.4, K/K_0=0.4, E_f=0.4 and Time $\widetilde{T}$= 0$\widetilde{K}$ AND 5$\widetilde{K}$

X'	0.2		0.4	
Y'	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.2	0 **0**	0 **0**	0.00719 **2.620× 10^{-10}**	-0.0974 **-3.547× 10^{-9}**
0.4	-0.04619 **-4.72× 10^{-10}**	0.85301 **8.73123× 10^{-9}**	0 **0**	0 **0**
0.6	-0.20789 **-2.1279× 10^{-9}**	3.83858 **3.92905× 10^{-8}**	-0.0647 **-2.358× 10^{-9}**	0.8770 **3.192× 10^{-8}**
0.8	-0.5543 **-5.6744× 10^{-9}**	10.2362 **1.04775× 10^{-7}**	-0.2303 **-8.38411× 10^{-9}**	3.11849 **1.13524× 10^{-7}**

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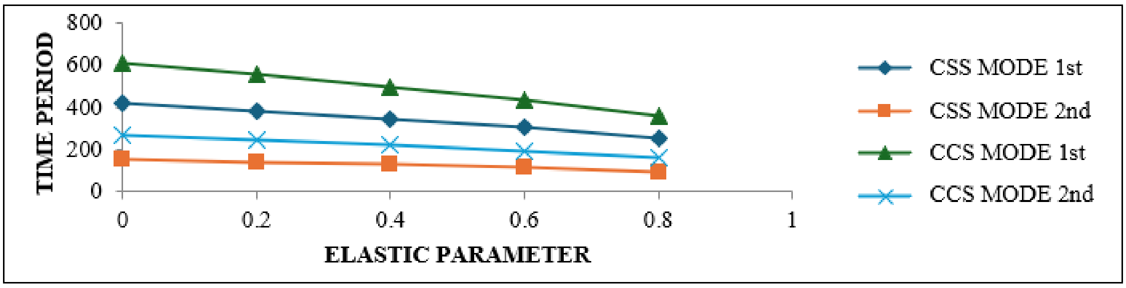
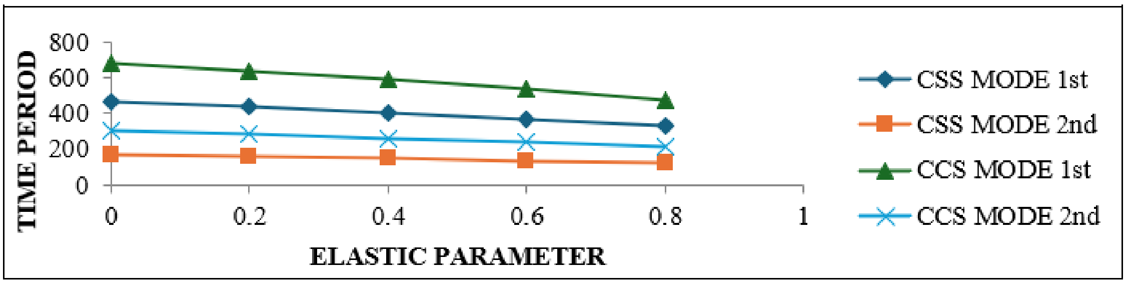


4(a)



4(b)

Figure 4: Time period for clamped, simply supported plates:
3(a) modes for $\beta^*=0.4$, $D_f=0.0$, 3 (b) modes for $\beta^*=0.4, D_f=0.4$;

Table 6: Deflection ($\widetilde{w}$) of a CCS right triangular plate for different value of X' and Y',a constant aspect ratio (a/b=1/2)and $\beta^*=0.4$, $K/K_0=0.4, E_f=0.4$ and Time $\widetilde{T}= 0\widetilde{K}$ and $5\widetilde{K}$

X′	0.2		0.4	
Y′	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.2	0 **0**	0 **0**	0.00333 **1.213×10^{-10}**	-0.2444 **-8.899×10^{-9}**
0.4	0.0133 **1.365×10^{-10}**	-1.685 **-1.725×10^{-8}**	0 **0**	0 **0**
0.6	0.0800 **8.193×10^{-10}**	-10.115 **-1.035×10^{-7}**	0.009996 **3.639×10^{-10}**	-0.7334 **-2.669×10^{-8}**
0.8	0.2401 **2.457×10^{-9}**	-30.346 **-3.106×10^{-7}**	0.053315 **1.94088×10^{-9}**	-3.9116 **-1.423×10^{-7}**

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Notation
a ,b the side of plate
$\tilde{a}_{ij}$ coefficient
$\tilde{c}_i$ coefficient
x,y Cartesian coordinate
h plate thickness at the point (x, y)
f(X′, Y′) generating function
E Young's modulus
$\tilde{d}$ =$Eh^3/12(1-v^2)$, flexural rigidity
ρ mass density per unit volume of the plate material
υ poisson'ratio
t time
$\widetilde{w}(x,y,t)$ deflection of the plate i.e. amplitude
$\widetilde{W}$(x,y) deflection function
$\tilde{T}$(t) time function
β^* taper constant in x- direction
$\tilde{K}$ time period
$\mathcal{P}$ Angular frequency
$\tilde{T}_{max}$ maximum kinetic energy
$\tilde{V}_{max}$ maximum strain energy