International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

A New Study of Expansive Mappings and Pythagorean Theorem Relationship

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Abstract: ***This paper explores the relationship between expansive mappings and Pythagoras' theorem in metric space. It examines how expansion mappings keep points apart in their images and connects this concept to the Pythagorean theorem, which deals with side length ratios in right-angled triangles. The interplay between these mappings and the theorem is analyzed to reveal geometric and topological features of metric spaces. The study highlights how expansion mappings both preserve and alter distance relationships, directly linking them to geometric principles. This paper provides a comprehensive view of geometric mapping theory through abstract principles and specific examples.***

Keywords: Pythagorean right triangle theorem, Fixed point theorems, Expansive mapping, Metric space, Euclidean space, Coordinate plane.

1. Introduction

The Pythagorean theorem, first appearing in ancient Greece, is often attributed to the mathematician Pythagoras or his disciples. This theorem states that in a right-angled triangle, the square of the hypotenuse "the side opposite the right angle, equals the sum of the squares of the other two sides" (D. Veljan, 2000). Mathematically, it can be expressed as follows:

$$c^2 = a^2 + b^2 \qquad (1)$$

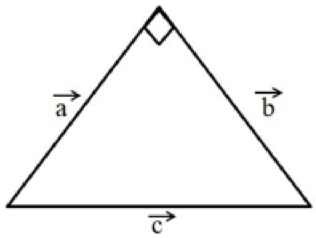

Figure 1: Pythagorean Right Triangle Theorem

The profound implications of the Pythagorean theorem in geometry and mathematics are matched by the historical significance of fixed- point theory, originating from mathematician David Brouwer's pioneering work in the early 20th century (R. P. Agarwal, 2020, A. L. Shields, 1964, R. P. Agarwal, 2020). In 1912, Brouwer's fixed point theorem asserted that any continuous function mapping a closed interval to itself must have at least one fixed point. This concept has become an essential tool across multiple mathematical disciplines, with applications in fields such as topology, economics, and computer science (S. Park, 1999, V. Brattka et al., 2019, V. Das, 2009). Wang et al. proved several fixed point theorems for expansive mappings (S. Z. Wang, 1984). While extensive research exists on the Pythagorean theorem, right-angled triangles, and expansive mappings within Euclidean space, their integration remains unexplored. we have studied the relationship between the contraction mappings and the Pythagorean theorem (S. A. Al-Salehi et al., 2024). This article investigates the relationship between the Pythagorean theorem and expansive mappings in Euclidean space R^2, particularly in self-mappings in metric spaces, to enhance understanding of these mathematical constructs and their inter-dependencies. This article explores the relationship between the Pythagorean theorem and expansive mappings in Euclidean space R^2, aiming to unify these concepts and provide new insights into expansive mappings and fixed points, with potential advancements in various fields.

2. Preliminaries

Definition 2.1 (P. Z. Daffer, 1992) Let a self-map $T: X \rightarrow X$ be defined on (X, d), such that $d(T(x), T(y)) \geq \lambda\, d(x, y)$, $\forall x, y \in X$ and $\lambda > 1$, then T is called an expansive mapping (or expansion).

Definition 2.2 (A. Azizi et al., 2014) Let (X, d) be a metric space and a mapping $T: B \rightarrow X$ be expansive, where $B \subseteq X$, such that $d(T(x), T(y)) \geq \lambda\, d(x, y)$, $\forall x, y \in B$ and $\lambda > 1$.

3. Main Results

First, we will start by explaining the idea of this study, as follows: This study is based on the existence of ($n \in \mathbb{N}$) number of points on the sides of a right triangle, connected by ($n \in \mathbb{N}$) number of straight lines. If the point x is fixed and the other points are movable, then when x coincides with T(x) this implies the existence of a fixed point. Similarly, if we extend the triangle as follows: for example, if we want to extend the triangle at the vertex x along a straight line using the function T(x),

Where $T(x)=F(x, k)=(x^2, k\,x)$. The triangle can also be similarly extended from the vertex y.

Now, if we draw a right-angled triangle $\triangle(T(x), m, T(y))$ at $\angle m$ on a coordinate plane in the Euclidean space R^2 such that

T (x), T (y), m ∈ X are the vertices of △(T (x), m, T (y)), see Figure 2. Let x be on [mT (x)] and y be on [mT (y)] .

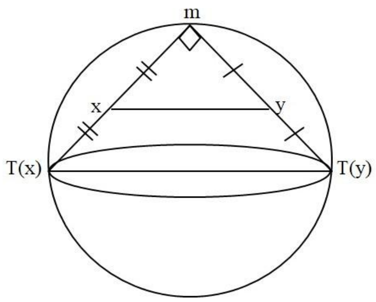

Figure 2: Expansive mapping with Pythagorean theorem

We draw the line xy. The triangle △(T (x), m, T (y)) satisfies the Pythagorean theorem:

$|T(y)-T(x)|^2 = |m-T(x)|^2 + |m-T(y)|^2$. Now, if we denote the side T (y)T (x) by d (T (x), T (y)) = |T (y) − T (x)|, such that d (T (x), T (y)) is the distance between T (x) and T (y), with T (x) = (T (x1), T (x2)) and T (y) = (T (y1), T (y2)).

Therefore

$$d(T(x), T(y)) = |T(y) - T(x)| = \sqrt{(T(y1) - T(x1))^2 + (T(y2) - T(x2))^2}. \quad (2)$$

Similarly, d (x, y) = |y − x|, and if x = (x1, x2) and y = (y1, y2), therefore,

$$d(x, y) = |y - x| = \sqrt{(y1 - x1)^2 + (y2 - x2)^2} \quad (3)$$

We can also denote the square of the distances $d(T(x), T(y))^2$ and $d(x, y)^2$ as follows:

$$d(T(x), T(y))^2 = |T(y) - T(x)|^2 = (T(y1) - T(x1))^2 + (T(y2) - T(x2))^2 \quad (4)$$

$$\text{And } d(x, y)^2 = |y - x|^2 = (y1-x1)^2 + (y2-x2)^2 \quad (5)$$

Let T: B → X be a mapping, where B ⊆X. If △(T (x), m, T (y)) satisfies the Pythagorean right triangle theorem at ∠m, then

$$d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2. \quad (6)$$

Note: From Figure (3), we get △(T (x), m, T (y)) also satisfies the Pythagorean properties as follows:

$$\frac{d(T(x),x)}{d(T(x),m)} = \frac{d(T(y),y)}{d(T(y),m)} = \frac{d(x,y)}{d(T(x),T(y))} \quad (7)$$

Theorem 3.1 Suppose that △(T (x), m, T (y)) is a right triangle at ∠m, in Euclidean space R^2. A mapping T: B → X, where B ⊆X is defined on a metric space (X, d). Let x, y be on [mT(x)] and [mT(y)] respectively. If △(T (x), m, T (y)) satisfies the Pythagorean right triangle theorem, then T satisfies the condition of an expansive mapping:

$d(T(x), T(y)) \geq \lambda\, d(x, y), \forall x, y \in B, \forall \lambda > 1.$

Proof. There are several cases, but we will only explain three cases to clarify our ideas.

Case (1): When d(T (x), x) = d(x, m) and d(T (y), y) = d(y, m). Since △(T (x), m, T (y)) satisfies the Pythagorean right triangle theorem, we have:

$$d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2 \quad (8)$$

but

$$d(T(x), m) = d(T(x), x) + d(x, m) \quad (9)$$

and

$$d(T(y), m) = d(T(y), y) + d(y, m). \quad (10)$$

If we put Equations (9) and (10) into Equation (8), we have $d(T(x), T(y))^2 = [d(T(x), x) + d(x, m)]^2 + [d(T(y), y) + d(y, m)]^2 = d(T(x), x)^2 + 2d(T(x), x)d(x, m) + d(x, m)^2 +$

$$d(T(y), y)^2 + 2d(T(y), y)d(y, m) + d(y, m)^2. \quad (11)$$

If we but $d(x, m)^2 + d(y, m)^2 = d(x, y)^2$ in Equation (11), we have

$$d(T(x), T(y))^2 = d(T(x), x)^2 + 2d(T(x), x)d(x, m) + d(T(y), y)^2 + 2d(T(y), y)d(y, m) + d(x, y)^2. \quad (12)$$

From Equation (12), we get either

$$2d(T(x), x)d(x, m) + 2d(T(y), y)d(y, m) = \frac{1}{2} d(T(x), T(y))^2 \quad (13)$$

and

$$d(T(x), x)^2 + d(T(y), y)^2 = \frac{1}{4} d(T(x), T(y))^2, \quad (14)$$

or

$$2d(T(x), x)d(x, m) + 2d(T(y), y)d(y, m) = 2d(x, y)^2 \quad (15)$$

and

$$d(T(x), x)2 + d(T(y), y)2 = d(x, y)^2. \quad (16)$$

Now there are two cases:

Case (1-a): By substituting Equations (13) and (14) into Equation (12), we get:

$d(T(x), T(y))^2 = \frac{1}{2} d(T(x), T(y))^2 + \frac{1}{4} d(T(x), T(y))^2 + d(x, y)^2 = \frac{1}{4} d(T(x), T(y))^2 + d(x, y)^2$. (17)

$\Rightarrow d(x, y) = \frac{1}{2} d(T(x), T(y))$ or $d(T(x), T(y)) = 2d(x, y)$. (18)

Case (1-b): By substituting Equations (15) and (16) into Equation (12), we get

$d(T(x), T(y))^2 = 2d(x, y)^2 + d(x, y)^2 + d(x, y)^2 = 4d(x, y)^2$. (19)

$$\Rightarrow d(x, y) = \frac{1}{2} d(T(x), T(y)) \text{ or } d(T(x), T(y)) = 2d(x, y). \quad (20)$$

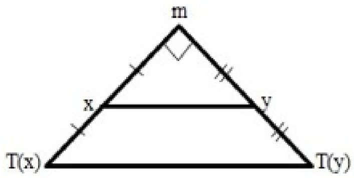

Figure 3: Pythagorean Right Triangle Theorem for Expansive Mapping

Thus, Equations (18) and (20) are equal when d(T (x), x) = d(x, m) and d(T (y), y) = d(y, m). To verify the validity of Case (1), see Example 3.3.

Case (2): When d(T (x), x) = $\frac{1}{2}$ d(x, m) and d(T (y), y) = $\frac{1}{2}$ d(y, m).

Since △(T (x), m, T (y)) satisfies the Pythagorean right triangle theorem, by the following similar processing in Case (1), we get

$$d(T (x), T (y))^2 = d(T (x), x)^2 + 2d(T (x), x)\, d(x, m) + d(T (y), y)^2 + 2d(T (y), y)d(y, m) + d(x, y)^2. \quad (21)$$

Now, either

$$2d(T (x), x)\, d(x, m) + 2d(T (y), y)\, d(y, m) = \frac{4}{9} d(T (x), T (y))^2 \quad (22)$$

and

$$d(T (x), x)^2 + d(T (y), y)^2 = \frac{1}{9} d(T (x), T (y))^2, \quad (23)$$

or

$$2d(T (x), x)\, d(x, m) + 2d(T (y), y)\, d(y, m) = d(x, y)^2, \quad (24)$$

and

$$d(T (x), x)^2 + d(T (y), y)^2 = \frac{1}{4} d(x, y)^2. \quad (25)$$

There are two cases:

Case (2-a): By substituting Equations (22) and (23) into Equation (21), we get

$$d(T (x), T (y))^2 = \frac{1}{9} d(T (x), T (y))^2 + \frac{4}{9} d(T (x), T (y))^2 + d(x, y)^2 = \frac{5}{9} d(T (x), T (y))^2 + d(x, y)^2 \quad (26)$$

$$\Rightarrow d(x, y) = \frac{2}{3} d(T (x), T (y)) \text{ or } d(T (x), T (y)) = \frac{3}{2} d(x, y) = 1.5\, d(x, y). \quad (27)$$

Case (2-b): By substituting Equations (24) and (25) into Equation (21), we get

$$d(T (x), T (y))^2 = d(x, y)^2 + \frac{1}{4} d(x, y)^2 + d(x, y)^2 = \frac{9}{4} d(x, y)2\ . \quad (28)$$

$$\Rightarrow d(x, y) = \frac{2}{3} d(T (x), T (y)) \text{ or } d(T (x), T (y)) = \frac{3}{2} d(x, y) = 1.5\, d(x, y). \quad (29)$$

Thus, Equations (27) and (29) are equal when d(T (x), x) = $\frac{1}{2}$ d(x, m) and d(T (y), y) = $\frac{1}{2}$ d(y, m).

To verify the validity of Case (2), see Example 3.4.

Case (3): When d(T (x), x) = $\frac{1}{3}$ d(x, m) and d(T (y), y) = $\frac{1}{3}$ d(y, m) by following a similar process as in Case (2), we also get two sub-cases, and we obtain:

$$d(x, y) = \frac{3}{4} d(T (x), T (y)) \ \ [\text{check}]. \quad (30)$$

or

$$d(T (x), T (y)) = \frac{4}{3} d(x, y) = 1.33333333d(x, y) \quad (31)$$

To verify the validity of Case (3), see Example 3.5. Now, if we repeat the previous cases by choosing various proportions for both d(T (x), x) and d(T (y), y), we get the condition of expansive mapping:

$d(T (x), T (y)) \geq \lambda\, d(x, y),\ \forall\, x, y \in B,\ \lambda > 1.$

We obtain this condition from all the previous cases. However, if T (x)→x and T (y)→y, then we get a small value for λ, which will be slightly more than 1. Conversely, if x→m and y→m we get a large value for λ, which will be much greater than 1. As for the fixed point, it exists, and its value depends on the function that connects the values T (x) and x. We will explain this in the next example.

Theorem 3.2 Suppose that △(T (x), m, T (y)) is a right triangle at the ∠m in Euclidean space R^2 on any sphere, where T (x), m and T (y) are the vertices of the triangle. If a mapping T: B→X, where B ⊆X defined on (X, d) and the triangle satisfies the Pythagorean right triangle theorem as $d(T (x), T (y))^2 = d(T (x), m)^2 + d(T (y), m)^2$. If x lies between T (x) and m and if y lies between T (y) and m, then T satisfies the condition of expansive meaning d(T (x), T (y)) ≥ λ d(x, y), for all x, y ∈ B, λ > 1.

Furthermore, T has a unique fixed point.

Proof. To prove the theorem, we must follow these steps:

Step 1: Similar to the previous theorem, we can prove that T satisfies the condition of expansive mapping, i.e., d(T (x), T (y) ≥ λ d(x, y) for all x, y ∈ B, λ > 1.

Step 2: The presence of a fixed point is related to the function that connects T (x) with x. We will discuss this in the following examples after this proof.

Step 3: Now, we prove that T has a unique fixed point. Suppose there are two fixed-points x and y of T.
i.e., T (x) = x and T (y) = y. We will use the property of expansive mapping to prove that x = y.

Using the expansive mapping property, d(T (x), T (y)) ≥ λ d(x, y). Since both x and y are fixed points, we have d(T (x), T (y)) = d(x, y). Combining these inequalities, we get d(x, y) ≥ λ d(x, y).

Now, since λ > 1 and d(x, y) > 0, we can divide both sides by d(x, y), we obtain (1 ≥ λ), which is a contradiction since λ > 1. Thus, our assumption that there are two distinct fixed points is incorrect. Hence, T has a unique fixed point.

Example 3.3: Let (X, d) be a metric space and T: B → X be a mapping, where B ⊆X. Suppose that △(T (x), m, T (y)) is a right triangle at ∠m in Euclidean space R^2 and satisfies the Pythagorean right triangle theorem as
$d(T (x), T (y))^2 = d(T (x), m)^2 + d(T (y), m)^2$.

Let x lie between T (x) and m, and y lie between T (y) and m.
If x = (0, 4), y = (3, 8), m = (0, 8), T (x) = (0, 0), and
T (y) = (6, 8). Calculating the distances as follows:
$d(T (x), T (y))^2 = (0 - 6)^2 + (0 - 8)^2 = 36 + 64 = 100 \Rightarrow d(T (x), T (y)) = 10$;
$d(T (x), m)^2 = (0 - 0)^2 + (0 - 8)^2 = 0 + 64 = 64 \Rightarrow d(T (x), m) = 8$;
$d(T (y), m)^2 = (6 - 0)^2 + (8 - 8)^2 = 36 \Rightarrow d(T (y), m) = 6$;
d(x, y)2 = (0 − 3)2 + (4 − 8)2 =9 + 16 = 25 ⇒ d(x, y) = 5.

From the resulting values, we note that the Pythagorean right triangle theorem is indeed verified, i.e., $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2 \Rightarrow 100 = 64 + 36$. We need to show that $d(T(x), T(y)) \geq \lambda\, d(x, y)$, for all x, y∈ B, λ > 1. We choose $1 < \lambda \leq \sqrt{\frac{100}{25}} \leq \sqrt{4} \leq 2$. we have $d(T(x), T(y)) \geq \lambda\, d(x, y)$, for all x, y∈ B, λ > 1. Thus verifying the condition of expansive mapping. Since both T (x) and x are located on the y-axis, and T (x) is the point of intersection of the x-axis and y-axis, there exists a fixed point ν such that T (ν) = ν. Given that T (x) = (0, 0), we conclude that T (0, 0) = (0, 0), so ν = x = (0, 0) is a fixed point.

Now, we prove that the fixed point is unique. Assume there is another fixed point υ = (υ1, υ2) such that T(υ1, υ2) = (υ1, υ2). Given that T is an expansive mapping, having any fixed point other than (0, 0) would violate the expansive mapping property because any other fixed point would imply a mapping that does not expand the distance sufficiently to satisfy the condition $d(T(x), T(y)) \geq \lambda\, d(x, y)$. Therefore, the unique fixed point is x = (0, 0).

Example 3.4 Let (X, d) be a metric space and T: B→X be a mapping, where B ⊆X. Suppose △(T (x), m, T (y)) is a right triangle at ∠m in Euclidean space R^2 and satisfies the Pythagorean right triangle theorem as $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2$. Let $d(T(x), x) = \frac{1}{2} d(x, m)$ and $d(T(y), y) = \frac{1}{2} d(y, m)$. suppose that T (x) = (0, 0), T (y) = (9, 9), m = (0, 9), x = (0, 3), and y = (6, 9). Using the Pythagorean theorem for △(T (x), m, T (y)) as $d(T(x), T(y))^2 = d(T(x), m)^2 +$
$d(T(y), m)^2$. We calculate the distances as follows:

$d(T(x), x)^2 = (0-0)^2 + (0-3)^2 = 9 \Rightarrow d(T(x), x) = 3 = d(T(y), y)$;
$d(x, m)^2 = (0-0)^2 + (3-9)^2 = 36 \Rightarrow d(x, m) = 6 = d(y, m)$;
$d(T(x), T(y))^2 = (0-9)^2 + (0-9)^2 = 81 + 81 = 162 \Rightarrow d(T(x), T(y)) = \sqrt{162} = 9\sqrt{2}$;
$d(T(x), m)^2 = (0-0)^2 + (0-9)^2 = 0 + 81 = 81 \Rightarrow d(T(x), m) = 9$;
$d(T(y), m)^2 = (9-0)^2 + (9-9)^2 = 81 \Rightarrow d(T(y), m) = 9$;
$d(x, y)^2 = (0-6)^2 + (3-9)^2 = 36 + 36 = 72 \Rightarrow d(x, y) = 6\sqrt{2}$.

Thus, the Pythagorean right triangle theorem is satisfied, i.e., $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2$, therefore, 162 = 81+81. We need to show that $d(T(x), T(y)) \geq \lambda\, d(x, y)$, for all x, y ∈ B, λ > 1. Using the given points and calculations, choose $1 < \lambda \leq \sqrt{\frac{162}{72}} \leq \sqrt{\frac{9}{4}} = \frac{3}{2}$. So that $d(T(x), T(y)) \geq \lambda\, d(x, y)$, for all x, y ∈ B, λ > 1. Therefore, the condition of expansive mapping is satisfied. Similarly to the previous example, we can demonstrate the existence and uniqueness of the fixed point.

Example 3.5 Let (X, d) be a metric space and T: B→X be a mapping, where B ⊆X. Suppose △(T (x), m, T (y)) is a right triangle at ∠m in Euclidean space R^2 and satisfies the Pythagorean right triangle theorem as $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2$. Let $d(T(x), x) = \frac{1}{3} d(x, m)$ and $d(T(y), y) = \frac{1}{3} d(y, m)$. Suppose T (x) = (0, 0), T (y) = (8, 8), m = (0, 8), x = (0, 2), and y = (6, 8). Using the Pythagorean theorem for △(T (x), m, T (y)), we have $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2$. Calculating the distances as follows:
$d(T(x), T(y))^2 = (0-8)^2 + (0-8)^2 = 64 + 64 = 128 \Rightarrow d(T(x), T(y)) = 8\sqrt{2}$;
$d(T(x), m)^2 = (0-0)^2 + (0-8)^2 = 0 + 64 = 64 \Rightarrow d(T(x), m) = 8$;
$d(T(y), m)^2 = (8-0)^2 + (8-8)^2 = 64 \Rightarrow d(T(y), m) = 8$;
$d(x, y)^2 = (0-6)^2 + (2-8)^2 = 36 + 36 = 72 \Rightarrow d(x, y) = 6\sqrt{2}$.
Thus, the Pythagorean theorem is satisfied,
i.e., $d(T(x), T(y))^2 = d(T(x), m)^2 + d(T(y), m)^2$, therefore, 128 = 64 + 64. We need to show that
$d(T(x), T(y)) \geq \lambda\, d(x, y)$ for all x, y ∈ B, λ > 1. Using the given points, we choose
$1 < \lambda \leq \sqrt{\frac{128}{72}} \leq \sqrt{\frac{16}{9}} = \frac{3}{2}$. So that $d(T(x), T(y)) \geq \lambda\, d(x, y)$ for all x, y∈ B, λ > 1. Therefore, the condition of expansive mapping is satisfied.

Similarly to Example 3.3, we can demonstrate the existence and uniqueness of the fixed point.

4. Application

The application of expansive mappings and the Pythagorean theorem enhances image compression and reconstruction by preserving geometric relationships during dimensionality reduction, encoding, and reconstruction. This ensures efficient data storage and transmission with minimal distortion and high visual quality. By maintaining spatial integrity, this method achieves superior compression ratios and fewer artifacts compared to traditional techniques, making it a valuable approach for multimedia communication and storage.

5. Conclusion

We conclusion that the combining expansive mapping with the Pythagorean theorem provides a method for precise spatial adjustments in various fields. Expansive mapping allows for controlled enlargement of distances between points, while the Pythagorean theorem ensures that the geometric relationships between these points remain consistent. This approach ensures accuracy and balance when altering dimensions, making it useful not only in architectural planning but also in any scenario requiring structured spatial expansion. The method helps maintain integrity and order in layouts, ensuring that design and functional criteria are met even after modifications.

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