Vague Generalized β* - Closed Sets in Vague Topological Spaces

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Abstract: ***The aim of this paper to explore the notation of vague sets and define a new class of vague generalized β*-closed sets in topological spaces. we are characterizations several examples and produced some theorems are given to explained the concepts to in this paper.***

Keywords: Vague set, Vague topology, Vague generalized β*- closed set

1. Introduction

Topology is a mathematical study of the properties that are preserved through deformations, twisting and stretchings of objects. Topology began with the study of curves, surface band other objects in the plane. The objects of topology are often formally defined as topological spaces.

Topology was introduced by Johann Benedict Listing in the 19th century. Although it was not until the first decades of 20th century that the idea of Topological space was developed. By the middle of the 20th century, topology had become a major branch of Mathematics.

In 1970, Levine [3] initiated the study of generalized closed set The concept of fuzzy sets was introduced by Zadeh [2] who introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set as a mathematical tool to solve problems and vagueness in everyday life. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one and later At Anassov [1] generalized this idea to intuitionistic fuzzy sets. The theory of fuzzy topology was introduced by C. L. Chang [4] in 1968 several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. AtAnassov [1] introduced the degree of nonmembership / falsehood (F) in 1986 and defined the intuitionistic fuzzy set as a generalization of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehre [5] as an extension of fuzzy set theory in 1993

In this paper we introduce the concept of Vague generalized β* -closed sets and their properties are obtained. Also its relationship with other existing sets is compared and discussed with example.

2. Preliminaries

Definition 2.1([7]).
A vague set A in the universe of discourse U is characterized by two membership functions given by:
(i) A true membership function t_A :U → [0,1] and
(ii) A false membership function f_A:U → [0,1]

Where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x)+ f_A(x)\leq 1$. Thus, the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x),1-f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x)\leq\mu(x)\leq 1-f_A(x)$. The vague set A is written as A= {<x, $[t_A(x),1-f_A(x)]$>/uεU} where the interval $[t_A(x),1-f_A(x)]$ is called the vague value of x in A, Denoted by VA(x).

Definition 2.2[7]:
Let A and B be vague sets of the form A= {<x, $[t_A(x),1-f_A(x)]$>/xεX} and
B= {<x, $[t_B(x),1-f_B(x)]$>/xεX} then

a) A⊆B if and only if $t_A(x)\leq t_B(x)$ and $1-f_A(x)\leq 1-f_B(x)$ for all xεX
b) A=B if and only if A⊆B and B⊆A
c) A={<x,$f_A(x),1-t_A(x)$>/xεX}
d) A∩B= {<x, min($t_A(x)$, tB(x)), min($1-f_A(x),1-f_B(x)$)>/xεX}
e) A∪B= {<x, ($t_A(x)\vee t_B(x)$, ($1-f_A(x)\vee 1-f_B(x)$)>/xεX}

For the sake of simplicity, we shall use the notation A= {<x,[$t_A,1-f_A$]>} instead of A = {<x,[$t_A(x),1- f_A(x)$]>/xεX}.

Definition 2.3:[5]
A subset A of a topological space (X,τ) is called

a) A preclosed set if cl(int(A)) ⊆A
b) A semi-closed set if int(cl(A)) ⊆A
c) A regular closed set if A =cl(int(A))
d) A α-closed set if cl(int(cl(A)))⊆A
e) (V) A closed set if cl(A)=A

Definition 2.4:
A subset A of a topological space (X,τ) is called

1) A generalized closed set (briefly g- closed) if cl(A)⊆U whenever A⊆U and U is an open set in X
2) A generalized closed set (briefly sg- closed) if scl(A) ⊆ U whenever A⊆U and U is semi-open set in X
3) A generalized semi-closed set (briefly gs-closed) if scl(A) ⊆U whenever A⊆U and U is open set in X
4) A generalized semi pre closed set (briefly gsp-closed) if spcl(A) ⊆U whenever A⊆U and U is open set in X
5) A generalized pre closed set (briefly gp-closed) if pcl(A) ⊆U whenever A⊆U and U is open set in X
6) A generalized α-closed set (briefly gα-closed) if αcl(A) ⊆U whenever A⊆U
7) and U is α-open set in X

8) A α-generalized closed set (briefly αg-closed) αcl(A) ⊆ U whenever A⊆ U and U is open set in X.

Definition 2.5:[6]
A vague topology (VT in short) on X is a family τ of vague sets in X satisfying the following axioms.
(i) 0,1 ∈ τ
(ii) G1 ∩ G2 ∈ τ, for any G1, G2 ∈ τ
(iii) ∪ Gi ∈ τ for any family {Gi/ i∈J} ⊆ τ

In this case the pair (X,τ) is called a Vague topological space (VTS in short) and any vague set in τ is known as a Vague open set (VOS in short) in X.

The complement A^c of a vague open set A in a Vague topological space (X,τ) is called a vague closed set (VCS in short) in X.

Definition 2.6:
Let (X,τ) be a VTS and A= {<x, [t_A,1-f_A]>} be vague set in X. Then the vague interior and a vague closure are defined by
1) Vint(A) = ∪ {G/G is an VOS in X and G⊆A}
2) Vcl(A) = ∩ {K/K is an VCS in X and A⊆K}

Definition 2.7:[9]
A vague set A of (X,τ), is said to be a,
1) A vague pre-closed set if Vcl(Vint(A)) ⊆A
2) A vague semi-closed set if Vint (Vcl(A)) ⊆A
3) A vague regular- closed set if A=Vcl(Vint(A))
4) A vague α closed set Vcl(Vint(Vcl(A))) ⊆A
5) A vague closed set if Vcl(A)=A

Definition 2.8:[8]
An vague set A in (X,τ), is said to be a,
1) Vague generalized closed set (briefly VGC) if Vcl(A)⊆ U whenever A⊆ U and U is Vague open set in X
2) Vague generalized semi-closed set (briefly VGSC) if Vscl(A) ⊆ U whenever A⊆ U and U is Vague open set in X
3) Vague generalized pre closed set (briefly VGPC) if Vpcl(A) ⊆ U whenever A⊆ U and U is Vague open set in X

Definition 2.9[9]
1) Vague semi closed set (VSCS) if Vint(Vcl(A))⊆ A,
2) Vague semi open set (VSOS) if A⊆Vcl(Vint(A)),
3) Vague β closed set (VβCS) if Vint(Vcl(Vint(A)))⊆ A,
4) Vague β open set (VβOS) if A⊆Vcl(Vint(Vcl(A))),
5) Vague α closed set (VαCS) if Vcl(Vint(Vcl(A)))⊆ A
6) Vague α open set (VαOS) if A⊆Vint(Vcl(Vint(A))),
7) Vague regular open set (VROS) if A= Vint(Vcl(A)),
8) Vague regular closed set(VRCS) if A= Vcl(Vint(A)).

Definition 2.10:
Let A be VS of a VTS of (X,τ),then the Vague semi interior of A (Vsint(A)) and Vague semi closure of A(Vscl(A)) are defined by,
1) Vsint(A) =∪{G/ G is a VSOS in X and G ⊆A}
2) Vscl(A) =∩{K/ K is a VSCS in X and A ⊆K}

3. Result

Let A be VS of a VTS (X,τ)then
1) Vscl(A) = A∪Vint(Vcl(A)),
2) Vsint(A) = A∩Vcl(Vint(A)),

Definition 2.11:[9]
Let A be VS of a VTS (X,ι), then the Vague alpha interior of A (Vαint(A)) and Vague alpha closure of A (Vαcl(A)) are defined by
1) Vαint(A) = ∪{G/ G is a VαOS in X and G ⊆A}
2) Vcl(αA) = ∩{K/ K is a VαCS in X and A ⊆K}

Definition 2.12
Let A be VS of a VTS,of (X,τ),n
(1). Vαcl(A) = A∪Vcl(Vint(Vcl(A))),
(2)).Vαint(A) = A∩Vint (Vcl(VintA))),

Definition 2.13:[9]
A VS of a VTS , is of (X,τ),said to be Vague generalized closed set(VGS)
If Vcl(A) A⊆ U whenever A ⊆ U and U is VOS in X.

Definition 2.14:[9]
A VS of a VTS of (X,τ),is said to be Vague generalized semi closed set(VGSCS)
If Vscl(A)⊆ U whenever A⊆ U and U is VOS in X.

Definition 2.15: [9]
A VS of a VTS (X,τ),), is said to be Vague alpha generalized closed set(VαGCS)
If Vαcl(A)⊆ U whenever A⊆ U and U is VOS in X.

Definition 2.16:[6]
Let (X,τ) be a VTS and A= {<x, [t_A,1-f_A]>} be vague set in X. Then the
Vagueβ int(A)and Vague βcl(A) are defined by
1) Vβint(A) = ∪ {G/G is an VβOS in X and G⊆A}
2) Vβcl(A) = ∩ {K/K is an VβCS in X and A⊆K}

4. Vague Generalized β* − Closed Sets

Definition 3.1:
A VS A is said to be vague generalized β*-closed set (VGβ*CS) in (X,τ) if Vβcl(A) ⊆ U whenever
A ⊆ U and U is VβOS in X. The family of all VGβ*CSs of a VTS(X,τ) is denoted by VGβ*C(X).

Example:3.2
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.4,0.7] [0.4,0.6] >}
A = {x < [0.3,0.7] [0.4,0.5] >}
U^c = {x < [0.3,0.6] [0.4,0.6] >}
is VGβ*CS of X.

Theorem 3.3:
Every VGCS is VGβ*CS but not conversely.

Proof:
Let A be VGCS in X and let A ⊆ U and U be VOS in (X,τ). Since Vβcl(A) ⊆ Vcl(A) and by hypothesis, Vβcl(A) ⊆ U. Therefore, A is VGβ*CS in X.

Example:3.4
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.4,0.7] [0.4,0.6] >}
A = {x < [0.3,0.7] [0.4,0.5] >}
U^c = {x < [0.3,0.6] [0.4,0.6] >}
is VGβ*CS in X but not VGCS in X.

Theorem:3.5
Every VαCS is VGβ*CS but not conversely.

Proof:
Let A be VαCS in X and let A⊆ U and U be VOS in X. By hypothesis, Vcl(V int (V cl(A))) ⊆ A. Since A ⊆ V cl(A), Vint (V cl (Vint(A))) ⊆ A. Hence Vβcl(A) ⊆ A⊆ U. Therefore A is VGβ*CS in X.

Example 3.6:
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.6,0.4] [0.4,0.6] >}
A = {x < [0.1,0.4] [0.4,0.6] >}
U^c = {x < [0.6,0.4] [0.4,0.6] >}
is VGβ*CS in X but not VαCS in X.

Theorem 3.7:
Every VRCS is VGβ*CS but not conversely.

Proof:
Let A be a VRCS in X. By definition, A = Vcl(V int(A)). This implies Vcl(A) = Vcl(Vint(A)). Therefore Vcl(A) = A. That is A is VCS in X. By Therefore A is VGβ*CS in X.

Example 3.8:
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.2,0.3] [0.3,0.7] >}
A = {x < [0.2,0.3] [0.3,0.7] >}
U^c = {x < [0.7,0.8] [0.3,0.7] >}
is VGβ*CS in X but not VRCS in X.

Theorem 3.9:
Every VαGCS is VGβ*CS but not conversely.

Proof:
Let A be VαGCS in X and let A⊆U and U be VOS in (X,τ). A ∪Vcl(Vint(Vcl(A))) ⊆ U.

This implies Vcl(Vint(Vcl(A))) ⊆ U and Vint(Vcl(Vint(A)) ⊆ U.

Thus V$\beta cl(A)$ =A ∪Vint (Vcl(Vint(A)) ⊆ U. Therefore A is VGβ*CS in X.

Example 3.10:
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.7,0.8] [0.7,0.8] >}
A = {x < [0.4,0.2] [0.4,0.2] >}
U^c = {x < [0.2,0.3] [0.2,0.3] >}
is VGβ*CS in X but not VαGCS in X.

Theorem 3.11:
Every Vβ*CS is VSCS but not conversely.

Proof:
Let A be Vβ*CS in X. Then V int (Vcl (Vint(A))) ⊆ A. Since A ⊆ V cl(A), Vint (Vcl(A)) ⊆ A . Hence A is VSCS in X.

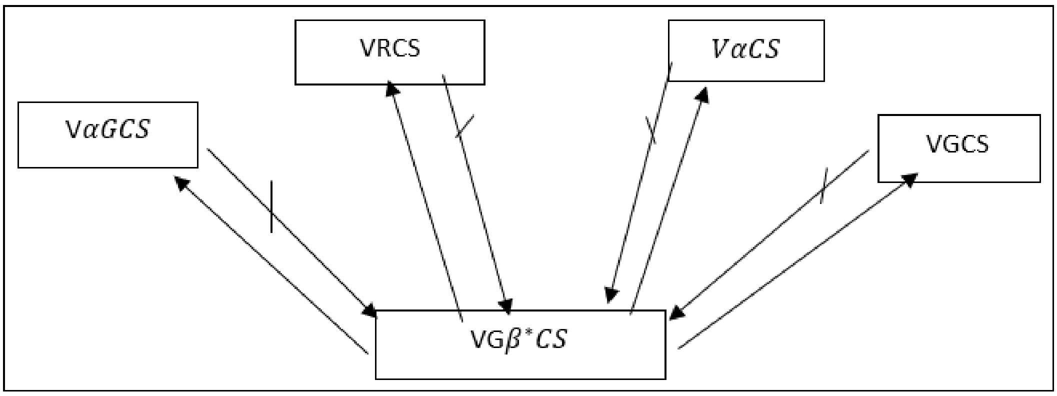
Example 3.12:
Let X = {a,b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.4,0.7] [0.4,0.6] >}
A = {x < [0.3,0.7] [0.4,0.5] >}
U^c = {x < [0.3,0.6] [0.4,0.6] >}
is VSCS in X but not Vβ*CS in X.

Theorem 3.13:
Every Vβ*CS is VGCS but not conversely.

Proof:
Let A be Vβ*CS in X. Then Vint(V cl(Vint(A))) ⊆A .Since A ⊆V cl(A),
Vint (Vcl(A) ⊆ A. Here A is vague semi closed sets in X.

Example 3.14:
Let X = {a, b} and τ = (0,G,1) is a vague topology on X, where
U = {x < [0.4,0.7] [0.4,0.6] >}
A = {x < [0.3,0.7] [0.4,0.5] >}
U^c = {x < [0.3,0.6] [0.4,0.6] >}
is VGCS but not Vβ*CS in X.

5. Diagrammatic Representation



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