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Deviation for Numbers in An Arithmetic Progression

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Abstract: ***This was designed in order to develop a formula in the determination of the Mean and Standard Deviation of Numbers in Arithmetic Sequence. Purpose: This study provides a formula for calculating the mean and standard deviation of a set of numbers in an arithmetic sequence. This will minimize the time spent in using the traditional method and thus provide an effective and efficient ways. There is also a brief summary in this paper on how to determine the standard deviation and variance though it is limited to certain sequences. Value: The importance of this study to student, teachers, lecturer, research among others cannot be overemphasized. It provides a method that has stood the test of times and greatly enhanced speed and accuracy in the performance of task. Methods: Random data sets were generated from multiple activities on which the calculations were based. Key findings: The formula for determining the standard deviation is only applicable to numbers in an Arithmetic Progression 1,2,3,4 and 5. Conclusion: This is method of calculating the mean for Arithmetic Sequence is effective and efficient, though the standard deviation and variance can only hold for specific cases.***

Keywords: Formula, Mean, Arithmetic Sequence, Standard Deviation, Variance

1. Introduction

The Mean is an average and in the same category at the Mode and Median, though they are collectively known as Measures of Location or Measures of Central Tendencies. Though it is very easy to calculate, it is distorted by extreme values. The mean for a sample is $\bar{\mathbf{x}}$ pronounced as x bar or bar x, and the mean for the population is **μ pronounced** as mew.

The Standard Deviation is a Measure of Dispersion or Spread in a dataset. It measures how a set of numbers deviate from the Mean. A standard deviation of zero means the observed set of numbers are the same, a low standard deviation value illustrates that the data are clustered around the mean and a high standard deviation value shows that the data are far away from the mean.
It is also worth noting that when two or more set of data have the same mean, in order to determine the most efficient data set, you consider the data set with the lowest standard deviation as it is deemed more efficient.

Preliminaries:
a) In an Arithmetic Sequence the difference between one term and the next is a constant.

In other words, we just add some value each time ... on to infinity.

In General we can write an arithmetic sequence like this:
{a, a+d, a+2d, a+3d, ... }

where:
a is the first term, and
d is the difference between the terms (called the **"common difference"**)
T_n= the nth term
n=1, 2,3,4……
And we can make the rule: $T_n = a + d(n-1)$

b) Sample Mean Formula
The sample mean formula is the following:

$$\bar{\mathbf{x}} = \frac{\sum x_n}{n}$$

Where:
$\bar{\mathbf{x}}$ = is the sample average of variable x.
$\sum \mathbf{x_n}$ = sum of n values.
n = number of values in the sample

Recalled: The sum of series $\sum r = \frac{n(n+1)}{2}$ ……… Eq 1

In order to get the mean r-bar, we divide through by n.
Mean = $\frac{(n+1)}{2}$

In this case n = the last term and 1= First term

Note: Therefore, to get the mean of a set of numbers in an Arithmetic sequence, take the sum of the first number and the last, then divide by 2.

Example 1: Find the mean and standard of 1, 2, 3, 4 and 5.

Using the Traditional Method: Mean = $\frac{1+2+3+4+5}{5}$

Mean = $\frac{15}{5}$

Mean= 3

On the other hand, using the Formula developed from sum of series for r, the mean can thus be calculated as:

Mean = $\frac{First\ term+last\ term}{2}$

Mean = $\frac{1+5}{2}$

Mean = 3

Example 2: Find the mean of 10, 20, 30, 40, 50,……100.
Solution:

Mean = $\frac{10+20+30+40+50+\cdots 100}{10}$

$= \frac{550}{10}$

= 55

Using the rule developed the mean becomes:

Mean = $\frac{First\ term + last\ term}{2}$

Mean = $\frac{10+100}{2}$

Mean= 55

Also $\sum r^2 = \frac{(n+1)(2n+1)}{6}$ ……………………… Eq 2

Standard Deviation:
$S^2 = \frac{\sum r2}{n}$ – sq $\{\frac{\sum r}{n}\}$ ………………………………… Eq 3

Substituting Eq1 and Eq2 in Eq 3 gives:

$S^2 = \frac{(n+1)(2n+1)}{6}$ – sq($\frac{n(n+1)}{2}$)

$s^2 = \frac{2(n+1)(2n+1)-3sq(n+1)}{6}$

$\mathbf{S^2 = \frac{(n+1)(n-1)}{12}}$

** Note the rule for standard deviation holds for n= 1,2,3,4 and 5

Standard Deviation
$S^2 = \frac{(n+1)(n-1)}{12}$

Based on the above, the last term n=5 and the first term =1

Substituting in S^2

$S^2 = \frac{(5+1)(5-1)}{12}$

$S^2 = \frac{24}{12}$, The variance is 2

S= sqr 2

S = 1.4142

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Author Profile

Emmanuel Thinka-Kamara was a member of the Chartered Governance Institute in the UK and a holder of a Master of Science Degree in Statistics. He is a former lecturer at Cuttington and University of Liberia respectively. He is presently the Manager of Academic Audit and Institutional affairs Directorate at the Tertiary Education Commission. The author is happily married to Tenneh Thinka-Kamara with two kids Emma and Ella.