

Comprehensive Cost Study for an Inventory Model with Quadratic Demand

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Abstract: *In this inventory model, we developed an inventory system having the time varying demand. The deterioration of items is not considered in this inventory model. The rate of inventory is a combined effect of time and demand. Shortages are also not considered in this model. Holding cost, ordering cost are major effective costs in this model. My aim in this model to study of the total cost to minimize with reference of parameters used in this model, that is major objective of any industry for establishment of business. For validation of the model, the numerical example is also illustrated in the end of study.*

Keywords: Inventory, demand, holding cost, ordering cost

1. Introduction

In the real-life business setup is very difficult and long lasting, market competition is at very high level so Business strategy is very important to establishment. Inventory management is difficult task, so in any industry or investing area effective management have to a clear vision, strategic leadership, and open communication, guiding businesses towards growth even in challenging scenarios. By prioritizing allocation of items, customer satisfaction, and talent development, optimizes efficient management and innovation. Adaptability and sound financial practices ensure resilience, while a culture of continuous improvement drives ongoing success. With these principles, businesses can navigate uncertainties, capitalize on opportunities, and achieve remarkable development, even amidst dynamic environments.

The inventory level affects a business's economy and the market value of business. While storing goods is challenging, optimizing inventory for sales and profit. Market demand for items is very changing by uses and behaviorally, but strategic measures can satisfy demand, showcasing advantages across disciplines. Product display and approach play lead roles in ignite customer interest. Notably, not everyone requires every product, and products have limited lifespan. Certain goods like medicine and electronics longer lifespan, whereas deteriorated items such as fruits and vegetables have shorter lifespans. Managing deteriorated inventory demands careful attention to prevent deterioration and should take optimal use. Effective inventory management involves balancing supply and demand dynamics, anticipating market trends, and minimizing wastage. Moreover, inventory strategies to develop market preferences and market conditions is essential for sustained success. Industry should deploy high effective inventory management systems and use innovative approaches to succeed demand, maximize profitability, and maintain competitiveness in dynamic market environments. Many of academics have developed various inventory models that either include deterioration or not. Deterioration plays an important role in production or storage system. Any item's demand can change by simultaneously by time and inventory presentation.

In early stages an EOQ inventory model with weibull distribution deterioration is published by Covert and Philip [14]. Goyal [17] has established an economic order quantity model with condition on payment delay. Goswami and Chaudhuri [1] have given an EOQ inventory model in which they considered deterioration and shortages of items. In the model they also study the effect of linear type demand. Ordering policies of deteriorating items is given by aggarwal and jaggi [18], they considered permissible delay in payments. Hariga [10] has given an optimal economic order quantity inventory models for deteriorating items, he also considers the time dependent demand. Jamal, Sarkaer and Wang [2] have developed an optimum ordering inventory system with deterioration of items, they also study the effect of shortage in the ordering level and permissible delay in payments is considered in this model. Teng, Chang, Dye and Hung [7] have studied an inventory policy and established the cost model for deteriorating items with time varying demand and partial backlogging. There is a note on an EOQ model for items, Dye [5] analysis an inventory system having the deterioration follow the weibull distributed. Shortages of inventory is also considered. Demand follow the power pattern law. Soni and Shah [6] has published an optimal order inventory policy for stock dependent demand under payment scheme type as progressive. Mishra and Mishra [12] developed the inventory model to evaluate the price for an EOQ model for deteriorating items under perfect competition. Sakaguchi [20] has given an inventory model for an inventory system with time varying demand rate. Singh and shrivastava [24] studied an economic order quantity inventory model for perishable items with stock dependent selling rate and permissible delay in payment and partial backlogging. Singh, Singh and Dutt.[25] have developed an EOQ inventory model for deteriorated items, the demand of items follow power demand pattern. Partial backordering is also considered in this model. A deteriorating inventory model with time dependent demand is established by mishra and singh [26]. They also study the effect of partial backlogging. An optimum ordering inventory model is published by sharma and Preeti [21], in this inventory model they consider random deterioration and stock dependent demand rate and shortages. They study the price and profit due to different parameter. Hsueh [4] has published an article on inventory control model with

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consideration of remanufacturing and product life cycle. A comprehensive extension of optimal ordering policy for stock dependent demand is written by Teng, Krommyda, Skouri and Lou[8] under progressive payment scheme. Rajeswari and Vanjikkodi[15] have developed an inventory model having the deteriorating of items. They study the effect of power pattern demand and partial backlogging. An inventory model is published by Rajeswari and Vanjikkodi[16] in which they use deterioration as two parameter weibull distribution and partial backlogging is also allowed in the model. An economic order quantity inventory model is given by Sarkar[3], they study the effects of payment delay and time dependent demand. An economic order quantity inventory model is presented by Teng, Min and Pan[9] their study is on effect of trade credit financing for non-decreasing demand. Sharma and Vijay [23] have published an EOQ inventory model for deteriorating items with price dependent demand with shortages under trade credit. Mishra and Singh[19] published an research article on economic order quantity inventory model for queued customer behavior with partial backlogging, power pattern demand and quadratic deterioration. Sharma and Vijay [22] have developed a replenishment policy for deteriorating inventory system with power demand and backlogging. Ghasemi and Damghani[13] have given a robust simulation-optimization approach for pre-disaster multi-period location-allocation-inventory planning. Tripathy, Sharma and Sharma[11] has published an article on an EOQ inventory model, in this inventory model they study the effect of deterioration with constant demand under progressive financial trade credit facility.

Many of inventory models have developed with constant demand pattern or linear pattern with of without. The length of cycle time is fixed or infinite. Now here, we developed an economic production quantity inventory model. In which deterioration of inventory is not allowed. The demand of items is taken as quadratic pattern. Replenishment policy of items is also considered in this model. After some time this demand also depends on inventory level. In this model we calculate the ordering cost, holding cost and total cost. These costs are the functions of different parameters. We also studied the effect of different parameters. We illustrated a numerical example to check optimization of this model.

2. Assumption and Notation

- C_0 = ordering cost per unit item.
- C_h = Holding cost per unit item per unit time.
- $D(t)$ = demand = $\begin{cases} at^2 + bt + c, & 0 < t < t_1 \\ td, & t_1 \leq t \leq T \end{cases}$.
- d = parameter for demand.
- T = Length of cycle time.
- Q = Initially inventory quantity.
- $I(t)$ = Inventory level at time t .
- $C(t_1, T)$ = Total cost per cycle time.
- T_D = total demand in cycle time.
- O_c = ordering cost per order per unit.
- H_c = holding cost in cycle time.
- C_T = total cost in cycle time.

3. Mathematical Model

We consider a continuous inventory system over an infinite time horizon with quadratic demand pattern. After time t_1 , this pattern reflected as linear. Let $I(t)$ be stock level at time t . Replenishment period t_1 is the time duration in which replenishment size is being added to inventory. Also sale, managing, deterioration and procurement starts form the time to start of inventory level. In a cycle time, when the production stop then the inventory level decrease very fast and become zero at time T . The mathematical model for such system is follows:

$$\frac{d}{dt}I(t) = -(at^2 + bt + c)d, \quad 0 \leq t \leq t_1 \quad \dots(1)$$

$$\frac{d}{dt}I(t) = -td + I(t), \quad t_1 \leq t \leq T \quad \dots(2)$$

The boundary conditions are $I(0) = Q, I(T) = 0$.

Solution of differential equation (1) is

$$I(t) = -\left(\frac{at^3}{3} + \frac{bt^2}{2} + ct\right)d + Q$$

Approximate Solution of differential equation (2) is

$$I(t) = \left(-\frac{d}{2}t^2 + c_2\right)(1 + t + t^2)$$

After using the boundary condition

$$I(t) = \frac{d}{2}(T^2 - t^2)(1 + t + t^2)$$

From the above two solutions, we find

$$Q = \frac{d}{6} \left\{ 3T^2(1 + t_1 + t_1^2) - t_1^3(3 - 2a) - 3t_1^2(1 - b) - 3t_1(t_1^3 - 2c) \right\}$$

The total demand in cycle time $[0, T]$ is given as $D =$

$$\int_0^T D(t)dt = \frac{aT^3}{3} + \frac{T^2}{2}(b + d) + cT - \frac{dt_1^2}{2}$$

Number of items in duration $[0, T]$

$$\begin{aligned} = \int_0^T I(t)dt &= \frac{d}{6} \left\{ 3T^3 \left(\frac{5-b}{3} + t_1 + t_1^2 + \frac{15-2a}{12}T + \frac{2}{5}T^2 \right) \right. \\ &\quad - Tt_1^3(3-2a) - 3Tt_1^2(1-b) + 3Tt_1(t_1^3 - 2c) \\ &\quad \left. - T^2 \left(-\frac{c}{2} - 3t_1 - \frac{3}{4}t_1^2 - t_1^3 \right) \right\} \end{aligned}$$

Now we calculate the different cost related to this inventory model

$$\text{Ordering cost} = QC_0 = C_0 \left\{ \frac{aT^3}{3} + \frac{T^2}{2}(b + d) + cT - \frac{dt_1^2}{2} \right\}$$

$$\begin{aligned} \text{Holding cost} = HC &= C_h \int_0^T I(t)dt \\ &= \frac{C_h d}{6} \left\{ 3T^3 \left(\frac{5-b}{3} + t_1 + t_1^2 + \frac{15-2a}{12}T + \frac{2}{5}T^2 \right) \right. \\ &\quad - Tt_1^3(3-2a) - 3Tt_1^2(1-b) + 3Tt_1(t_1^3 - 2c) \\ &\quad \left. - T^2 \left(-\frac{c}{2} - 3t_1 - \frac{3}{4}t_1^2 - t_1^3 \right) \right\} \end{aligned}$$

The total cost per unit time per unit item is given as

$$C_T = \frac{1}{T} (\text{Ordering cost} + \text{Holding cost})$$

$$= C_0 \left\{ \frac{aT^2}{3} + \frac{T}{2}(b+d) + c - \frac{dt_1^2}{2T} \right\} +$$

$$\frac{C_h d}{6} \left\{ 3T^2 \left(\frac{5-b}{3} + t_1 + t_1^2 + \frac{15-2a}{12} T + \frac{2}{5} T^2 \right) - t_1^3(3-2a) - 3t_1^2(1-b) + 3t_1(t_1^3 - 2c) - T \left(-\frac{c}{2} - 3t_1 - \frac{3}{4} t_1^2 - t_1^3 \right) \right\}$$

Now to minimize the total cost we take $\frac{\partial C_T}{\partial t_1} = 0 = \frac{\partial C_T}{\partial T}$.

And calculate stationary values of t_1 and T as t_1^*, T^* . At

the above calculated point we find $\frac{\partial^2 C_T}{\partial t_1^2}, \frac{\partial^2 C_T}{\partial T^2}, \frac{\partial^2 C_T}{\partial t_1 \partial T}$.

If $\frac{\partial^2 C_T}{\partial t_1^2} \times \frac{\partial^2 C_T}{\partial T^2} > \left(\frac{\partial^2 C_T}{\partial t_1 \partial T} \right)^2$ then the profit is maximum.

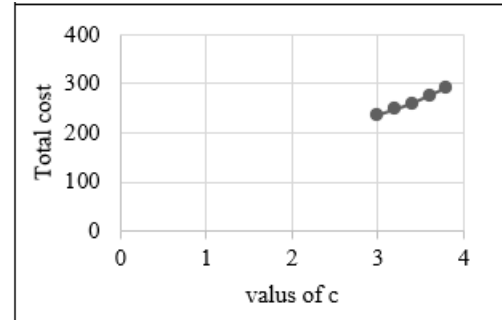
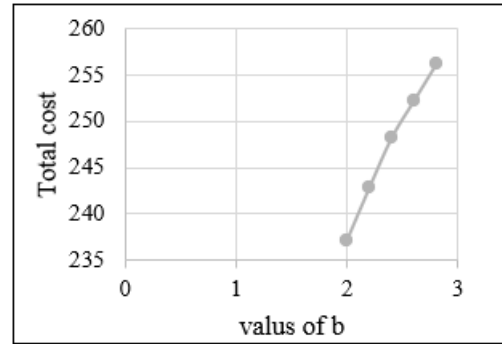
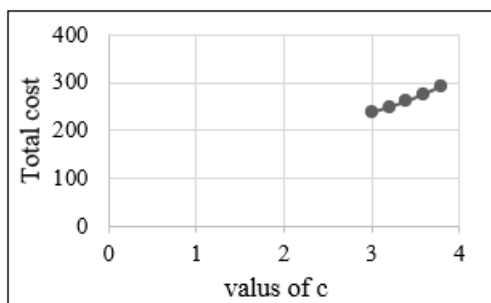
4. Numerical Example with Graph

Effect of different parameters on Total cost are given in following tables and graphs

a	b	c	t ₁	T	C _T
1	2	3	0.456	5.213	237.08
1.2	2	3	0.512	5.221	254.81
1.4	2	3	0.562	5.236	272.61
1.6	2	3	0.624	5.248	300.50
1.8	2	3	0.697	5.255	37.08

a	b	c	t ₁	T	C _T
1	2	3	0.456	5.213	237.08
1	2.2	3	0.461	5.215	242.78
1	2.4	3	0.464	5.219	248.21
1	2.6	3	0.467	5.221	252.24
1	2.8	3	0.470	5.222	256.23

a	b	c	t ₁	T	C _T
1	2	3	0.456	5.213	237.08
1	2	3.2	0.458	5.219	248.25
1	2	3.4	0.460	5.228	260.34
1	2	3.6	0.462	5.240	275.91
1	2	3.8	0.463	5.253	291.32



5. Conclusions and Remarks

In this research paper the inventory level of items is proportional to demand that is quadratic function of time and linear in time. It is obvious that rate of inventory directly depends on demand given by the users or market. Demand means customer satisfaction is must. From the above analysis the total cost affected by parameters a, b, c and the time t_1, T . Result of above calculation and study of graph say that the total cost increases as increasing values of a, b, c . These cost are at the minimum level by method of optimization. Based on the aforementioned data, we may enhance our inventory system and generate substantial total costs. These factors have much impact on all other costs and profit of industry. The profit of industry or production system is directly depends upon factors such as labor expenses, workforce capacity, machinery infrastructure, and government levies. This type of study may include more factors that can impact our inventory policy. These will be incorporated into our forthcoming studies.

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