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Manufacturer-Retailer Integrated System for Multi-Item Considering Two-Level Trade Credit, Volume Flexible, and Selling Price Dependent Demand

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Abstract: ***In this paper, an integrated manufacturer-retailer inventory model is developed considering trade credit policy. Integrated system designs inventory policy for multi-item. Demand rate is considered as the non-linear function of selling price. Concept of volume flexibility is adopted by the manufacturer to tune with the customer's demand. Two-level trade credit policy is adopted in which the manufacturer offers trade credit period to retailer and the retailer provides trade credit period to his/her customers. This paper deals with the problem to determine the optimal values of production rate and production run time so that the total profit of the integrated system is maximum. Proposed model is illustrated with the help of numerical example and sensitivity analysis.***

Keywords: Manufacturer-retailer inventory model; volume flexibility; two-level trade credit; ideal time; selling price-dependent demand

1. Introduction

In the current scenario, research on competition within and among integrated systems is one the burning fields. Numerous inventory problems in the field of integrated systems were brought up by researchers. However, in recent times, there has been a greater focus on how an integrated system's structure affects its members and the system's ability to compete. Many researchers focused on the inclusion of producer, and retailers in integrated systems to design the optimal inventory policy. Most of them suggested that integrated approach gives better solution to the inventory problems. Most of the researchers obtained the optimal solutions for integrated system considering single item.

The present work enriches the literature by considering a situation that can be frequently observed in practice: A manufacturer produces multi-item with the help of acquired raw materials and fulfill the demand of the retailer. Retailer used these products to satisfy the demand of the customers. This is very common in automobiles industries such Eicher Motors Ltd., Bajaj Auto Ltd., Tata Motors Ltd., TVS Motor Company Ltd., etc. in India. In these industries, inventory control is indeed a major issue in supply chain (SC), i.e. an approach that addresses SC issues under an integrated perspective.

1.1 Structure of Study

Current study organized as follows: in the subsequent section literature review is presented to identity the research gap. Further, to formulate the mathematical model, required assumptions and notations is presented in section 2. Mathematical formulation of proposed model is incorporate in the section 3. Section 4 compromises numerical analysis to illustrate the proposed model. Concluding remarks along with future extension is presented in section 5.

1.2 Literature Review

Studies in the past have typically determined the optimal inventory policy for the producer, and retailer separately. Practically, their objectives are conflicting to each other. To remove this conflict, researchers suggested to form the chain which is termed as integrated system. For researchers, the study of integrated systems in inventory is still relatively new. Integration system inventory management has gained a lot of attention as the business market becomes more competitive. First integrated inventory model was presented by **Goyal** (**1976**) considering joint total cost for the buyer and the supplier. Integrated model for vendor and buyer was developed by **Banerjee (1986)** considering finite production rate. **Ahmed et al.** (**2008**) developed a coordinated system considering vendor and buyer considering production interruptions for restoring of the quality of the production process. **Pal et al. (2012)** developed an integrated system considering imperfect production process, items reliability and reworking of defective items. Order size of raw materials and production rate are taken as decision variables. Supply chain management is designed by **Shastri et al. (2014)** considering two-level trade credit. They assumed that supplier offer trade credit period (M) to the retailer which is longer than the trade credit period offered by the retailer to the customer. They also consider that the demand of the customer depends on the selling price of items. In the field of integrated inventory system many researchers such as **Yadav et al. (2016), Yadav and Swami (2018), Motla et al. (2021), Kuraie et al. (2021), Kuraie et al. (2022), Yadav et al. (2022), Handa et al. (2023), Mandal et al. (2024)** enriched the literature with very interesting results.

Volume flexibility enables manufacturers to suitably tailor production capacity in accordance with the required demand, so mitigating the economically severe implications of a demand-supply mismatch. **Sethi and Sethi (1990)** described different types of flexibility associated with manufacturing system. **Sana et al. (2007)** presented an EPQ model considered volume flexibility. They considered unit production cost as a function of production rate. They suggested that for optimal solution production manager is strictly and thoroughly disciplined about feeding updates into the system. **Roy and Maiti (2010)** presented inventory model considering volume flexibility, deterioration, time varying demand, and shortages. With the help of numerical help, they contradict the popular myth associated with the manufacturing system according to this total cost of the manufacturing system is minimum when per unit production cost is minimum. **Panda et al. (2017)** suggested that flexibility helps for better coordination between demand and excessive stocks. **Kamna et al. (2021)** presented a sustainable production inventory model and suggested that inclusion of volume agility in the model have increases the efficiency of manufacturing system. Many researchers such as **Wang et al. (2021), Yadav et al. (2022), Ma et al. (2022), Kar et al. (2023), Singh et al. (2024)** obtained very fruitful results with the inclusion of volume flexibility in the manufacturing model.

Traditionally in inventory models, it is assumed that retailer have to settle the account with supplier as soon as items are received by them. But practically, supplier provides certain period to retailer to settle the account. This period is termed as trade credit. **Goyal (1985)** was the first researcher who presented EOQ model considering permissible delay in payment. **Min et al. (2010)** presented EOQ model for deteriorating items with two-level delay in payment and stock-dependent demand. Numerically, they observed that sales quantity can be increases with the help of trade credit. **Chung et al. (2014)** designed EPQ inventory model for deteriorating items considering two-levels of trade credit. In this, supplier offers trade credit to retailer and retailer offers full trade credit to his/her customers. An EPQ inventory model was developed by **Khanna et al. (2017)** and discussed the combined effect imperfect quality items, rework process, faulty inspection, and two-levels trade credit. They considered retailer's trade credit period and order quantity as decision variables to obtain the optimal solution. In this direction, many researchers such as **Mashud et al. (2019), Roy et al. (2020), Wang et al. (2022), Mahato and Mahata (2022), Handa et al. (2023), Shah et al. (2024)** presented different inventory models considering two-levels trade credit.

Traditionally, most of the researchers considered static demand rate while developing EOQ models. But practically this is not true especially in the countries where economy is price sensitive. Therefore, demand rate is price sensitive as it has negative impact on the demand rate. **Yang et al. (2009)** presented an inventory model considering price-sensitive demand for non-instantaneous deteriorating items. They obtained the optimal values of selling price, duration of positive inventory, and replenishment cycle time so that total profit of the system is maximum. **Maihami and Kamalabadi (2012)** presented an inventory model to determine the optimal values of selling price, replenishment schedule, and order quantity so that total profit of the system is maximum. Numerically, they observed that revenue of the system cannot be optimize in absence of sale price. **Shaikh et al. (2017)** presented an inventory model considering price-and-stock dependent demand, inflation, and shortages. They observed that stock level decreases as the markup rate increases. **Saha and Sen (2019)** presented an inventory model for deteriorating item considering selling price-and-time dependent and constant holding cost. They observed that total average inventory cost decreases as the markup value, inflation, and holding cost. Many researchers such as **Dey et al. (2019), Rahman et al. (2020), Khan et al. (2020), Ruidas et al. (2021), Kumar et al. (2023), Yadav et al. (2023), Sharma et al. (2024)** developed an inventory model considering selling price dependent demand and provided very interesting results.

Due to increase or decrease of the selling price, demand changes and hence the profit of the system changes. Trade credit provided by the manufacturer and the retailers makes the bond of the integrated system stronger. From the literature review, it is observed that trade credit, volume flexibility, production rate dependent production cost, and selling price dependent demand rate have been extensively addressed. However, to the best of our knowledge, so far, no integrated system has been developed by considering trade credit, volume flexibility, and selling price dependent demand rate. Hence, modeling without above factors, a decision-making scenario can be distorted and can mislead a supply chain strategy. To remedy this drawback, in this paper an integrated system has been developed consisting of producer and retailer by incorporating these phenomena. Here, we obtained the optimal production rate and production period that maximized the total profit of the integrated system. The whole combination is unique and very much practical.

2. Assumptions and Notations

2.1 Assumptions

To develop the integrated inventory system following assumptions are made:

1) Integrated system consists single manufacturer and single retailer to deal the customers.
2) Nowadays, integrated system prefers to deal in multi-item because of cut throat competition. Therefore, in the current study multi-items are considered. (Sami et al., 2023)
3) In the current study, lead time is negligible.
4) Manufacturer prefers to tune the manufacturing rate according to the demand of the customer. Therefore, concept of volume flexibility is considered in the model to fulfill this requirement of manufacturer. (Singhal and Singh, 2015)
5) Manufacturing cost of unit item depends on the manufacturing rate of product.
6) Cost of Idle times at manufacturer end is also assumed.
7) In developing countries, selling price plays major role on the demand of the product. Therefore, in current study demand function is considered as a non-linear function of selling price. (Kumar et al., 2023)
8) Shortages are not allowed.
9) Trade credit is one of the means to form long term bonding among the different players of integrated system. Therefore, in the current study it is considered that manufacture gives a credit period to the retailer. Further, to attract more customers retailer provides trade credit to his/her customers. (Singh et al., 2021)

2.2 Notations: The following notations have been used to develop the proposed model:

Manufacturer's Notations

P_i : Manufacturing rate for i-item (decision variable)
t_1^i : Production run time (decision variable)
$I_{P1}^i(t)$: Inventory level of manufacturer during production run time for i-item
$I_{P2}^i(t)$: Inventory level of manufacturer during non-production run time for i-item
A_p^i : Manufacturer's set up cost for i-item
h_p^i : Manufacturer's carrying cost per unit per unit time for i-item
I_p^i : Cost per unit idle time of manufacturer for i-item
C_p^i : Manufacturing cost per unit item for i-item $\left(= \delta_p^i + \frac{L^i}{P_i} + \vartheta^i P_i\right)$
δ_p^i : Fixed cost per unit of finished item for i- item
$\frac{L^i}{P_i}$: The labor/energy cost which is equally distributed over production size P_i for i-item
$\vartheta^i P_i$: The tool/die cost per unit finished product which is proportional to the size of manufacturing rate for i-item
s_p^i : Selling price per unit item of manufacturer for i-item

Retailer's Notations

D_c : Customer's demand rate i-item $\left(= a_1^i e^{-b_1^i s_r^i}, a_1^i, b_1^i > 0\right)$
D_r : Retailer's demand rate i-item $\left(= a_2^i e^{-b_2^i s_p^i}, a_1^i, b_1^i > 0\right)$
A_r : Retailer's ordering cost i-item
h_r^i : Retailer's carrying cost for i-item
s_r^i : Selling price per unit item of retailer for i-item
T_i : Cycle length of retailer for i-item
M_i : Trade credit period offered by manufacturer to the retailer for i-item
N_i : Trade credit period offered by retailer to the customer for i-item
I_e^i : Interest earn for i-item
I_p^i : Interest paid for i-item

3. Formulation of Mathematical Model for Integrated System

Two players manufacturer and retailer are considered to form the integrated system. Integrated system deals in multi-item to satisfy the demand of the customers. Systematic, mathematical formulation of different players is presented in subsequent section.

3.1 Mathematical Model for Manufacturer's Inventory System: -

Here, manufacturer produces multi-item to meet the demand of the customers. Manufacturing process for i-item start at t=0 with manufacturing rate P_i. During manufacturing run time t_1^i, inventory level increases after adjusting the demand at retailer end. Stored items during the period [0, t_1^i] are used to fulfill the demand during [t_1^i, K_1T]. Mathematically, inventory level for i-item at any time 't' can be represented as follows:

$$\frac{dI_{P1}^i(t)}{dt} = P_i - a_1^i e^{-b_1^i s_p^i}, \qquad 0 \le t \le t_1^i \qquad (1)$$

with $I_{P1}^i(0)$=0.

Under the given condition, solution of above equation is as follows:

$$I_{P1}^i(t) = \left(P_i - a_1^i e^{-b_1^i s_p^i}\right) t, \qquad 0 \le t \le t_1^i \qquad (2)$$

Inventory level of manufacturer at t=t_1^i is

$$I_{P1}^i(t_1^i) = \left(P_i - a_1^i e^{-b_1^i s_p^i}\right) t_1^i, \qquad 0 \le t \le t_1^i \qquad (3)$$

During the non-manufacturing period, manufacturer's inventory level for i-item is as follows:

$$\frac{dI_{P2}^i(t)}{dt} = -a_1^i e^{-b_1^i s_p^i}, \qquad t_1^i \le t \le K_i T_i \qquad (4)$$

with $I_{P2}^i(K_i T_i)$=0.

Under the given condition, solution of above equation is as follows:

$$I_{P2}^i(t) = a_1^i e^{-b_1^i s_p^i}(K_i T_i - t), \qquad t_1^i \le t \le K_i T_i \qquad (5)$$

On applying the condition of continuity $I_{P1}^i(t_1^i) = I_{P2}^i(t_1^i)$, we get

$$\left(P_i - a_1^i e^{-b_1^i s_p^i}\right) t_1^i = a_1^i e^{-b_1^i s_p^i}(K_i T_i - t_1^i)$$

$$\Rightarrow T_i = \frac{P^i t_1^i}{K_i a_1^i e^{-b_1^i s_p^i}} \qquad (6)$$

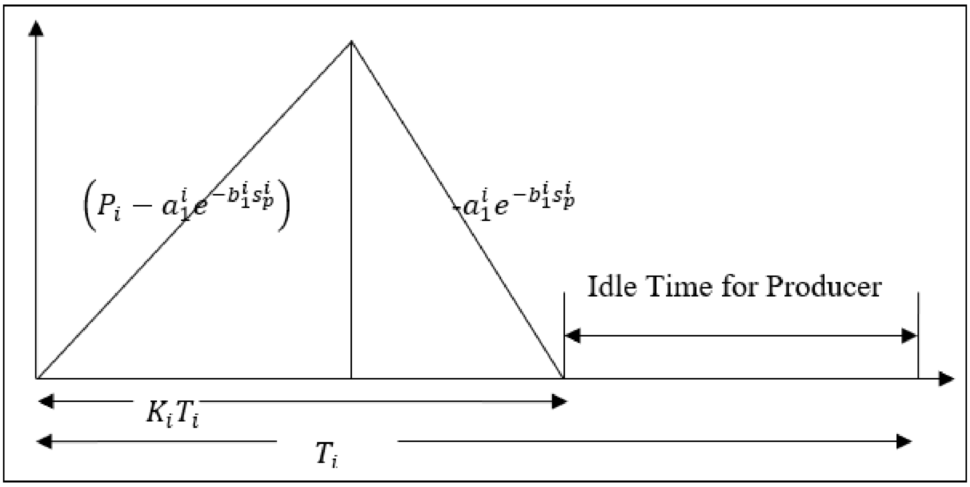

Figure 1: Manufacturer's Inventory Level for i- item

Manufacturer's set up cost for i-item=A_p^i.

Manufacturing system remains ideal during $[K_iT_i, T_i]$. Therefore, the total cost of idle time for i-item = $I_P^i(1-K_i)T_i$.

Manufacturer's income from i-item =$s_p^i P_i t_1^i$.

Manufacturer's carrying cost for i-item =$h_p^i\left[\int_0^{t_1^i} I_{P1}^i(t)dt + \int_{t_1^i}^{K_iT_i} I_{P2}^i(t)dt\right]$

$= h_p^i\left[\frac{1}{2}\left(P_i - a_1^i e^{-b_1^i s_p^i}\right)\left(t_1^i\right)^2 + \frac{a_1^i e^{-b_1^i s_p^i}(K_iT_i - t_1^i)^2}{2}\right]$.

Manufacturing cost for i-item = $C_P a_1^i e^{-b_1^i s_p^i} K_iT_i$.

Manufacturer's average profit for i-item

$$AP_i = \frac{1}{T_i}\left[s_p^i a_1^i e^{-b_1^i s_p^i} K_iT_i - A_p^i - h_p^i\left(\frac{1}{2}\left(P_i - a_1^i e^{-b_1^i s_p^i}\right)\left(t_1^i\right)^2 + \frac{a_1^i e^{-b_1^i s_p^i}(K_iT_i - t_1^i)^2}{2}\right) - C_P a_1^i e^{-b_1^i s_p^i} K_iT_i - I_P^i(1-K_i)T_i\right].$$

Manufacturer's average profit is

$$\mathrm{AP} = \sum_{\mathrm{i}=1}^{\mathrm{km}} \frac{1}{T_i}\left[s_p^i a_1^i e^{-b_1^i s_p^i} K_iT_i - A_1^i - h_p^i\left(\frac{1}{2}\left(P_i - a_1^i e^{-b_1^i s_p^i}\right)\left(t_1^i\right)^2 + \frac{a_1^i e^{-b_1^i s_p^i}(K_iT_i - t_1^i)^2}{2}\right) - C_P a_1^i e^{-b_1^i s_p^i} K_iT_i - I_P^i(1-K_i)T_i\right] \quad (7)$$

3.2 Mathematical Model for Retailer Inventory System: -

Retailer's inventory level at any time 't' for i-item is $I_{r1}^i(t)$. Due to supply of product from the manufacturer, retailer's inventory level rises at the rate $a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}$ during the period $[0, K_iT_i]$. When supply from manufacturer stops, retailer's inventory level decline at the rate $a_2^i e^{-b_2^i s_r^i}$. The accumulated inventory for i-item during $[0, K_iT_i]$, depletes and reaches to zero level at $t = T_i$.

Mathematical expression to reflects the inventory level for i-item during the interval $[0, K_iT_i]$ is as follows:

$$\frac{dI_{r1}^i(t)}{dt} = a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}, \quad 0 \le t \le K_iT_i \quad (8)$$

with $I_{r1}^i(0) = 0$.

Mathematical expression to reflects the inventory level for i-item during the interval $[K_iT_i, T_i]$ is as follows:

$$\frac{dI_{r2}^i(t)}{dt} = -a_2^i e^{-b_2^i s_r^i}, \quad K_iT_i \le t \le T_i \quad (9)$$

with $I_{r2}^i(T_i) = 0$.

On solving above two equations under the conditions are as follows:

$$I_{r1}^i(t) = \left(a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}\right)t, \quad 0 \le t \le K_iT_i \quad (10)$$

$$I_{r2}^i(t) = a_1^i e^{-b_1^i s_p^i} K_iT_i - a_2^i e^{-b_2^i s_r^i} t, \quad K_iT_i \le t \le T_i \quad (11)$$

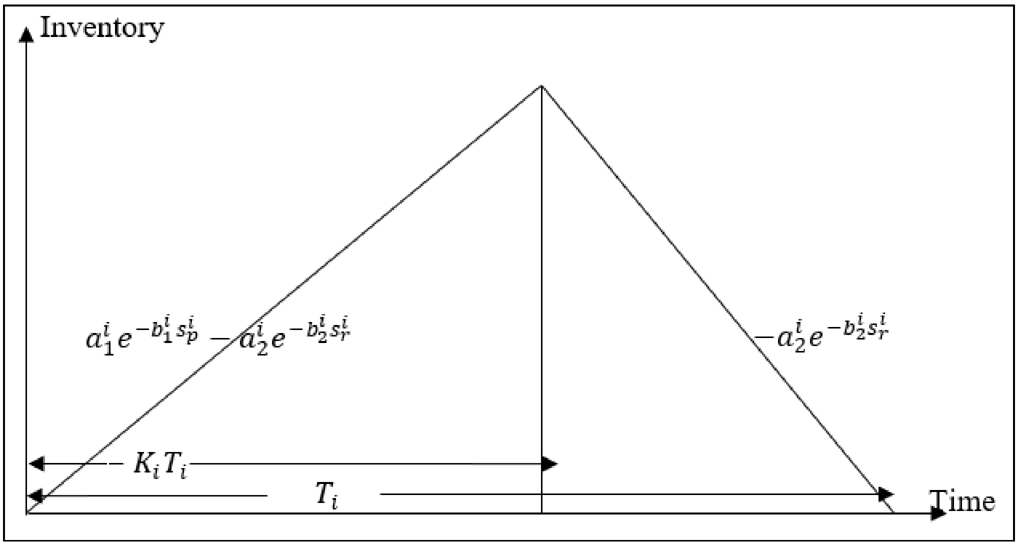

Figure 2: Retailer Inventory Level for i-Item

We have $I_{r2}^i(T_i) = 0 \Rightarrow K_i = \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}} < 1$ since the demand at manufacturer end is more than the demand at the retailer end.

Retailer's carrying cost for i-item$=h_r^i \left[\int_0^{K_iT_i} I_{r1}^i(t)dt + \int_{K_iT_i}^{T_i} I_{r2}^i(t)dt\right] = \frac{h_r^i\, a_2^i e^{-b_2^i s_r^i} T_i^2}{2}\left(1 - \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}}\right)$

3.3.1 Various Cases according to Trade Credit Periods

Manufacturer offered trade credit (M_i) to the retailer and in turn retailer offers trade credit (N_i) to his/her customer for i-item. Depending on the values of N_i, M_i, K_iT_i, and T_i, following four cases arises:

Case 1: $N_i < M_i < t_1^i$

In this situation, account is settled by the customer at N_i. So, interest is earned by the retailer at the rate I_e^i on the sale of i-item in the interval $[N_i, M_i]$. However, all items are not sold by the retailer at the end of the trade credit period M_i offered by manufacturer. Thus, retailer have to pay interest on the due payment for the duration $[M_i, T_i]$.

In the interval $[N_i, M_i]$, interest earn by the retailer per unit time for i-item is

$$IE_{r1}^i = \frac{s_r^i I_e^i}{T_i}\int_{N_i}^{M_i} a_2^i e^{-b_2^i s_r^i} t dt = \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i^2 - N_i^2)}{2T_i}$$

Interest paid by the retailer in the interval $[M_i, T_i]$ per unit time for i-item is

$$IP_{r1}^i = \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}}{T_i}\left[\int_{M_i}^{K_iT_i}\left(a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}\right) t dt + \int_{K_iT_i}^{T_i}\left(a_1^i e^{-b_1^i s_p^i} K_iT_i - a_2^i e^{-b_2^i s_r^i} t\right) dt\right]$$

$$= \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}}{2T_i}\left[\left(a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}\right)\left((K_iT_i)^2 - M_i^2\right) + a_2^i e^{-b_2^i s_r^i}(T_i - k_iT_i)^2\right]$$

In this case, profit of retailer is

$$APR^1 = \sum_{i=1}^m \frac{1}{T_i}\left[(s_r^i - s_p^i) a_2^i e^{-b_2^i s_r^i} T_i + \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i^2 - N_i^2)}{2} - \frac{h_r^i\, a_2^i e^{-b_2^i s_r^i} T_i^2}{2}\left(1 - \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}}\right) - A_r^i - \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}}{2}\left(\left(a_1^i e^{-b_1^i s_p^i} - a_2^i e^{-b_2^i s_r^i}\right)\left((K_iT_i)^2 - M_i^2\right) + a_2^i e^{-b_2^i s_r^i}(T_i - k_iT_i)^2\right)\right] \quad (12)$$

Objective Function: Here, objective of current study is to optimize the total profit of integrated system. Thus,

$$Max\ ISC$$
$$P_i > 0, t_1^i > 0$$

where $ISC = AP + APR^1$.

Case 2: $t_1^i < N_i < K_iT_i < M_i < T_i$

In this situation, account is settled by the customer at N_i. So, interest is earned by the retailer at the rate I_e^i on the sale of i-item in the interval $[N_i, M_i]$. However, all items are not sold by the retailer at the end of the trade credit period M_i offered by manufacturer. Thus, retailer have to pay interest on the due payment for the duration $[M_i, T_i]$.

In the interval $[N_i, M_i]$, interest earn by the retailer per unit time for i-item is

$$IE_{r2}^i = \frac{s_r^i I_e^i}{T_i}\int_{N_i}^{M_i} a_2^i e^{-b_2^i s_r^i} t dt = \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i^2 - N_i^2)}{2T_i}$$

Interest paid by the retailer in the interval $[M_i, T_i]$ per unit time for i-item is

$$IP_{r2}^i = \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}}{T_i}\int_{M_i}^{T_i}\left(a_2^i e^{-b_2^i s_r^i} T_i - a_2^i e^{-b_2^i s_r^i} t\right) dt = \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}(T_i - M_i)^2}{2T_i}$$

In this case, profit of the retailer is

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$$APR^2 = \sum_{i=1}^{m} \frac{1}{T_i}\left[(s_r^i - s_p^i)a_2^i e^{-b_2^i s_r^i} T_i + \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i^2 - N_i^2)}{2} - \frac{h_r^i a_2^i e^{-b_2^i s_r^i} T_i^2}{2}\left(1 - \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}}\right) - A_r^i - \frac{s_r^i I_p^i a_2^i e^{-b_2^i s_r^i}(T_i - M_i)^2}{2}\right] \quad (13)$$

Objective Function: Here, objective of current study is to optimize the total profit of integrated system. Thus,

$$Max\ ISC$$
$$P_i > 0, t_1^i > 0$$

where $ISC = AP + APR^2$.

Case-3: $K_iT_i<N_i<T_i<M_i$

In this situation, account is settled by the customer at N_i. So, interest is earned by the retailer at the rate I_e^i on the sale of i-item in the interval $[N_i, M_i]$. However, all items are sold by the retailer at the end of the trade credit period M_i offered by manufacturer. So, no interest is paid by the retailer to the manufacturer.

In the interval $[N_i, M_i]$, interest earn by the retailer per unit time for i-item is

$$IE_{r3}^i = \frac{s_r^i I_e^i}{T_i}\left[\int_{N_i}^{T_i} a_2^i e^{-b_2^i s_r^i} t dt + a_2^i e^{-b_2^i s_r^i} T_i (M_i - T_i)\right] = \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i T_i - N_i^2)}{2T_i}$$

The interest paid by the retailer per unit time is

$$IP_{r3}^i = 0$$

In this case, profit of the retailer is

$$APR^3 = \sum_{i=1}^{m} \frac{1}{T_i}\left[(s_r^i - s_p^i)a_2^i e^{-b_2^i s_r^i} T_i + \frac{s_r^i I_e^i a_2^i e^{-b_2^i s_r^i}(M_i T_i - N_i^2)}{2} - \frac{h_r^i a_2^i e^{-b_2^i s_r^i} T_i^2}{2}\left(1 - \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}}\right) - A_r^i\right] \quad (14)$$

Objective Function: Here, objective of current study is to optimize the total profit of integrated system. Thus,

$$Max\ ISC$$
$$P_i > 0, t_1^i > 0$$

where $ISC = AP + APR^3$.

Case 4: $T_i<N_i$

In this situation, account is settled by the customer at N_i. So, interest is earned by the retailer at the rate I_e^i on the sale of i-item in the interval $[N_i, M_i]$. However, all items are sold by the retailer at the end of the trade credit period M_i offered by manufacturer. So, no interest is paid by the retailer to the manufacturer.

In the interval $[N_i, M_i]$, interest earn by the retailer per unit time for i-item is

$$IE_{r4}^i = \frac{s_r^i I_e^i}{T_i}\left[a_2^i e^{-b_2^i s_r^i} T_i (M_i - N_i)\right]$$

The interest paid by the retailer per unit time for the i- item is

$$IP_{r4}^i = 0$$

In this case, profit of the retailer is

$$APR^4 = \sum_{i=1}^{m} \frac{1}{T_i}\left[(s_r^i - s_p^i)a_2^i e^{-b_2^i s_r^i} T_i + s_r^i I_e^i \left[a_2^i e^{-b_2^i s_r^i} T_i (M_i - N_i)\right] - \frac{h_r^i a_2^i e^{-b_2^i s_r^i} T_i^2}{2}\left(1 - \frac{a_2^i e^{-b_2^i s_r^i}}{a_1^i e^{-b_1^i s_p^i}}\right) - A_r^i\right] \quad (15)$$

Objective Function: Here, objective of current study is to optimize the total profit of integrated system. Thus,

$$Max\ ISC$$
$$P_i > 0, t_1^i > 0$$

where $ISC = AP + APR^4$.

4. Numerical Example

In this section, proposed model is illustrated with the help of numerical example. In this study, two items are considered. To carry out the numerical analysis, the values of the parameters in appropriate units are considered as follows:

Table 1: Data Related to Manufacturer

Item	s_p^i	A_p^i	I_p^i	δ_p^i	L^i	ϑ^i	a_1^i	b_1^i	h_p^i
1	97	487	196	1.78	3899	0.01	1.09×10^6	1.89	2.9
2	103	489	189	2.01	3979	0.01	1.19×10^6	1.99	3.9

Table 2: Data Related to Retailer

Item	s_r^i	h_r^i	I_p^i	I_e^i	A_r^i	M_i	N_i	a_2^i	b_2^i
1	111	4.2	0.08	0.06	379	0.68	0.57	1.25×10^6	1.79
2	129	5.5	0.08	0.06	399	0.68	0.57	1.49×10^6	1.81

Optimal values of decision variables are obtained with the help of mathematical software MATHEMATICA. On putting all these values in equation, we get the optimal values of (Q_{ni}^*, P_i^*) as follows:

Table 3: Optimal Values of Integrated System

Item	1	2
t_1^{i*}	0.31	0.37
P_i^*	615	632
$ISC(Q_{ni}, P_i)$	\$27890.89	

4.1 Sensitive analysis w.r.t carrying cost:

Table -4 shows the effect of manufacturer's carrying cost and retailer's carrying cost on profit of the integrated system.

Table 4: Effect of Carrying Cost on Profit of Integrated System

% Change in Parameter	% change in profit of integrated w.r.t manufacturer's carrying cost	% change in profit of integrated w.r.t retailer's carrying cost
-40	0.24	0.49
-20	0.14	0.29
20	-0.02	-0.19
40	-0.09	-0.51

From Table-4 reflects that profit of the integrated system decreases as the holding cost of manufacturer increases profit of the integrated system decreases. From the table, it is also observed that rise in holding cost of retailer also decreases the profit of the integrated system.

4.2 Sensitive analysis w.r.t to selling Price

Table-5 shows the effect of selling price of manufacturer and retailer on the profit of integrated system.

Table 5: Effect of selling price on profit of integrated system

% change in parameter	% change in profit of integrated system w.r.t manufacturer's selling price	% change in profit of integrated system w.r.t producer selling price
-40	7.80	9.30
-20	3.19	6.29
20	-4.19	-8.60
40	-9.09	-10.95

Table-5 shows the effect of selling price on the profit of the integrated system. Rise in selling at manufacturer or retailer end, decreases the demand of items. Hence, overall profit of the integrated system decreases. Results, also indicate that there is significant impact of the selling price on the profit of the integrated system.

5. Conclusion

In the current work, integrated inventory system is developed for the retailer and the manufacturer. Integrated system deals in multi-item. Manufacturer offers trade credit to the retailer and in turn retailer provides full trade credit to his/her customer. Demand rate is considered as the function of the selling price. The major consideration of the proposed model lies in the consideration of multi-item, supply chain management, volume flexibility, two-level trade credit policy, selling price dependent demand rate, cost of idle time at the end of manufacturer. Results indicate that profit of integrated system is highly affected by the change in selling price of retailer and the manufacturer. Carrying cost of retailer and manufacturer also have negative impact on the profit of the integrated system. In future research, current work can be extended by considering deterioration process at manufacturer end as well as retailer end. Effect of preservation technology can also be discussed in future to increase the practical utility of the model. Further, effect of inflation can also be considered in future. One immediate extension of the current work may be multi-manufacturer and multi-retailer.

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