Investigation of Pulsatile Behavior and Flow Characteristics in Stenosed Arteries: Analysis of Bingham Plastic Fluid Dynamics

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Abstract: ***This work delves into the effects of Bingham plastic fluid flow within stenosed arteries, employing two-layer model to examine the pulsatile behavior of fluid within artery. Utilizing perturbation techniques, the study addresses velocity, shear stress, and flow rate. Graphical representation of outcome indicates a decrease in velocity with increasing radius, alongside an increase in flow resistance over time. These findings enhance our understanding of fluid dynamics in stenosed arteries, with potential implications for the diagnosis and treatment of cardiovascular diseases, particularly atherosclerosis, characterized by plaque buildup and arterial blockages. The graphs illustrate that as the radius of the artery increases, the velocity of the fluid flow decreases. This suggests that narrower arteries experience faster blood flow compared to wider ones. Additionally, the graphs show that over time, there is an increase in resistance to flow within the stenosed arteries. This implies that as arterial stenosis progresses or persists, the resistance to blood flow also increases. Understanding these dynamics is crucial for gaining insights into how blood flows through stenosed arteries, which is relevant to cardiovascular diseases such as atherosclerosis. Atherosclerosis involves the buildup of plaque in the arteries, leading to narrowing or blockage of blood vessels. The findings from this study contribute to our understanding of how blood flow is affected by arterial narrowing, which can inform diagnostic methods and treatment strategies for cardiovascular diseases. Therapies aimed at reducing arterial stenosis or managing its effects may benefit from considering the dynamics of fluid flow within stenosed arteries.***

Keywords: Bingham plastic fluid, Stenosed arteries, Pulsatile flow, Velocity profile, Resistance to flow, Cardiovascular diseases, Atherosclerosis, Plaque buildup, Treatment strategies

1. Introduction

Atherosclerosis commonly develops in specific areas of arteries, such as bifurcations and flow divisions, due to the complex flow patterns occurring in these regions. Research suggests a correlation between the location of atherosclerosis lesions and low or oscillating wall shear stress. Additionally, various cardiovascular diseases are associated with flow conditions within blood vessels. Atherosclerosis is a type of arteriosclerosis characterized by the accumulation of fatty substances, cholesterol, cellular waste products, calcium, and fibrin within the inner lining of arteries. The buildup of cholesterol and lipid particles leads to arterial hardening and the formation of multiple plaques. A common symptom of atherosclerotic cardiovascular disease is a heart attack or sudden cardiac death [12,43,47]. Severe narrowing of the artery with sudden coronary alterations can result in sudden cardiac arrest or death, affecting hundreds of thousands of people worldwide annually, making it an important area of research. In this research series, various studies have explored blood flow in stenosed arteries, focusing on pulsatile flow dynamics. Some researchers have developed a two-layered mathematical model to analyze blood flow in asymmetric stenosed arteries with slip velocity and also investigated the effect of a magnetic field on pulsatile blood flow through inclined circular tubes with periodic body acceleration. Steady blood flow through stenosed arteries using a non-Newtonian fluid model was discussed by [4,16,24,34]. A mathematical model for blood flow through asymmetric stenosed arteries with velocity slip under the influence of a transverse was explored by [8,22,35,54]. A study of blood flow through stenosed arteries in diseased conditions was also done by [2,619]. Some researchers have calculated flow resistance for small arteries with multiple stenoses and post-stenotic dilatation. A study that explored mathematical models of blood flow dynamics in blood vessels. Specifically, it investigated how non-Newtonian fluids behave in arteries with varying cross-sections and examined the mathematical representation flow through constricted vessels using fluid models [3,31,42,53]. These studies contribute to our understanding of blood flow dynamics in stenosed arteries and may inform development of diagnostic treatment strategies for cardiovascular diseases. Treatment for atherosclerosis often involves procedures such as endarterectomy and stenting, which utilize Doppler flow measurements in the common carotid artery [1,9,23,33]. Doppler velocity measurements are compared between the affected side and the contra-lateral side to assess the degree of stenosis. In some cases, patients may exhibit equivalent or higher Doppler velocities after stenting compared to before treatment. Some researchers attribute the higher velocities post-stenting to decreased compliance of the artery wall in the stent region [13,21,25,52]. A mathematical model is a powerful tool used to describe real-world systems using mathematical language, a process known as mathematical modeling. Within the human body, the cardiovascular system serves as a vital network for distributing blood, consisting of three key components: blood, heart, and blood vessels. As blood courses through these vessels, it exerts pressure on their walls, which is termed blood pressure, influenced flow rate and pressure gradient [5,10,26,46]. Cardiovascular system has three major types of blood vessels. Arteries, responsible for carrying blood away from heart to various body regions. Arterioles, branch into smaller capillaries. Veins, forming pressure collecting system that returns oxygen-poor blood back to the heart. For analytical purposes, all vessels are generally assumed to be alike in nature, differing primarily in size, length, and cross-sectional area [18,54,55]. This conceptual framework provides a foundational understanding of both mathematical modeling principles and the intricate workings of the cardiovascular system. In this work it is obtained that as arterial stenosis progresses or persists, the resistance to blood flow also increases [15,32,44]. Understanding these dynamics is crucial for gaining insights into how blood flows through stenosed arteries, which is relevant to cardiovascular diseases such as atherosclerosis. Atherosclerosis involves the buildup of plaque in the arteries, leading to narrowing or blockage of blood vessels. The findings from this study contribute to our understanding of how blood flow is affected by arterial narrowing, which can inform diagnostic methods and treatment strategies for cardiovascular diseases. Therapies aimed at reducing arterial stenosis or managing its effects may benefit from considering the dynamics of fluid flow within stenosed arteries.

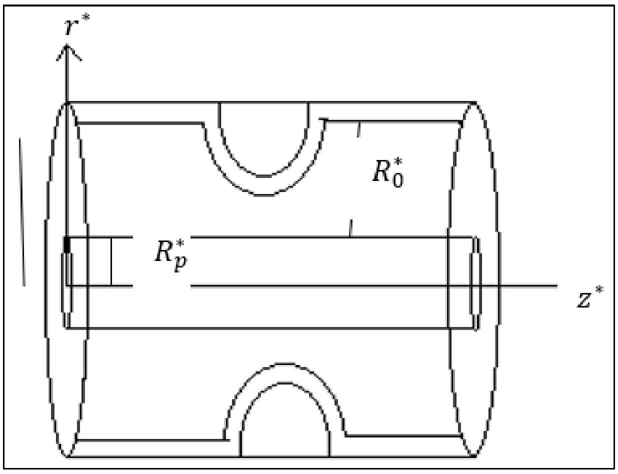

Figure 1: Stenosed Artery

2. Formulation of the problem

We have examined a scenario where blood flows through a circular artery with a stenosis in an axially symmetric, laminar, and pulsatile manner Figure. (1). To analyze this situation, we considered cylindrical polar coordinates (r*, Ø*, z*).

The equation governing momentum is as follows:

$$\rho \frac{\partial u^*}{\partial t^*} = (-\frac{\partial . p^*}{\partial z^*}) - (\frac{1}{r^*}\frac{\partial(r^* . \tau^*)}{\partial r^*}) \quad (1)$$

Mathematical expression describing non-Newtonian characteristics of blood, known as the Herschel-Bulkley equation, can be formulated as follows:

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), \qquad \tau^* > \tau_y \quad (2)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, \qquad \tau^* \leq \tau_y \quad (3)$$

In this theoretical analysis, we account for the blood flow as a two-phase system. The outer layer, composed of plasma, is assumed to exhibit Newtonian behavior, while the inner core, which contains the majority of erythrocytes within artery. Mathematical model derived in this study is characterized by as below;

$$\tau^* = (-\mu)\frac{\partial u^*}{\partial r^*}, \qquad if\ R_0^*(z^*,t^*) < r^* < R^*(z^*,t^*), \quad (4)$$

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), \qquad if\ R_p^*(z^*,t^*) < r^* < R_0^*(z^*,t^*), \quad (5)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, \qquad if\ 0 < r^* < R_p^*(z^*,t^*) \quad (6)$$

In addition to the momentum equation, boundary conditions are prescribed as below;

$$u^* = 0, \text{ at } r^{.*} = R^{*.}(z^*,t^*), \quad (7)$$

$$\tau^{.*}, \text{ is finite, } r^{*.} = 0. \quad (8)$$

$r^* = R_0^*(z^*,t^*)$ and $r^* = R_p^*(z^*,t^*)$.

The pressure gradient, which varies with the non-dimensionalized spatial and temporal coordinates z∗ and t∗, is expressed below:

$$\frac{\partial}{\partial z^*} . p^{*.}(z^*,t^*) = -q^*(z^{*.}) * f(t^{*.})$$

with $q^*(z^*) = -\frac{\partial}{\partial z^*}p.^*(z^*,0),\ f(t^*) = [1 + A\sin(\omega t^*)]$.

Subsequent analysis, we employ following dimensionless variables:

$$z = \frac{z^*}{a}, r = \frac{r^*}{a}, R(z,t) = \frac{R^*(z^*,t^*)}{a}, R_0(z,t) = \frac{R_0^*(z^*,t^*)}{a}, \quad R_p(z,t) = \frac{R_p^*(z^*,t^*)}{a}, \tau = \frac{2\tau^*}{q_0 a}, \quad \theta = \frac{2\tau_y}{q_0 a}, u = \frac{u^*}{\frac{q_0 a^2}{4\mu}}, t = t^*\omega, Q(z,t) = \frac{Q^*(z,t)}{\frac{\pi q_0 a^4}{8\mu}}, d. = \frac{d^{*.}}{a},\ \delta = \frac{\delta^{.*}}{a}, L_0 = \frac{L_{0.}^*}{a}, L = \frac{L^{*..}}{a}, \alpha^2 = \frac{\alpha^2\omega}{\frac{\mu}{\rho}}, q(z) = \frac{q^*(z^*)}{q_0} \quad (9)$$

Expressed in these dimensionless terms, equation (1) can be stated as:

$$\alpha^2\frac{\partial u}{\partial t} = 4q(z)f(t) - 2(\frac{1}{r}\frac{\partial(r*\tau)}{\partial r}),\ 0 < [r] < R(z,t), \quad (10)$$

Equations (2) to (6) can be expressed as follows:

$$(-\frac{\partial u}{\partial r}) = 2\tau, \qquad [R_0(z,t) < r < R(z,t)], \quad (11)$$

$$(-\frac{\partial u}{\partial r}) = 2(\tau - \theta), \qquad [R_p(z,t) < r < R_0(z,t)], \quad (12)$$

$$(-\frac{\partial u}{\partial r}) = 0, \qquad [0 < r < R_p(z,t)], \quad (13)$$

Additionally, both u, τ must remain continuous. Geometric configuration of stenosis, represented as dimensionless terms, and defined as:

$$R = \begin{cases} 1 - A_1(t)[L_0^{(m.-1)}[(z-d)] - [(z-d)]^{m.}], & if\ [d \leq [z] \leq (d + L_{0.})], \\ 1, & \text{Otherwise.} \end{cases}$$

with $A_{1.}(t) = \frac{\delta[1-e^{(-t/T.)}]m^{\frac{m}{m-1}}}{aL_0^m(m-1)}, \quad m \neq 1$

where δ represents maximum height of stenosis within artery. It further explains that this maximum height is achieved at a specific position along the artery, namely $z = d + L_0/[m^{1/(m-1)}]$.

$$Q(z,t) = 4\int_0^{R(z,t)}[ru(z,r,t)]dr.$$

3. Analytical solution of the problem

This statement suggests that when the Womersley parameter is very small, certain variables such as velocity, shear stress, and the radii R_0 and R_p of the artery is approximated in a simplified term. This simplification is likely to make the mathematical expressions more manageable and easier to analyze;

$$u_{.}(z, r, t) = u_{0.}(z, r, t) + \alpha^2 u_1 + \dots \quad (14)$$
$$r_{.}(z, r, t) = r_{0.}(z, r, t) + \alpha^2 u_1 + \dots \quad (15)$$
$$R_{0.}(z, r, t) = R_{00.}(z, r, t) + \alpha^2 R_{10} + \dots \quad (16)$$
$$R_{p.}(z, r, t) = R_{0p.}(z, r, t) + \alpha^2 R_{1p} + \dots \quad (17)$$

By (14),(15) and (10), we obtained;

$$\frac{\partial}{\partial r}(r\tau_0) = 2rq(z).f(t) \quad (18)$$
$$\frac{\partial u_0}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \quad (19)$$

By integrating equation (18) applying conditions, it is obtained;

$$\tau_0 = q(z)f(t)R_p, 0 \le r \le R_p \quad (20)$$

For, $R_{.p} \le [r] \le R_{.0} : R_{.0} \le [r] \le R$,

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$$\tau_{0.} = [q(z)f(t)(r)] \quad (21)$$

By substituting equations (14) and (15) into equations (11) to (13) and matching terms with the same power of α, we derive;

$$-\frac{\partial u_0}{\partial r} = 2\tau_0, -\frac{\partial u_1}{\partial r} = 2\tau_1, \text{if } R_0 \le r \le R. \quad (22)$$
$$-\frac{\partial u_0}{\partial r} = 2(\tau_0 - \theta), -\frac{\partial u_1}{\partial r} = 2\tau_1, \text{if } R_p \le r \le R_0. \quad (23)$$
$$\frac{\partial u_0}{\partial r} = 0, \frac{\partial u_1}{\partial r} = 0, \text{if } 0 \le r \le R_p. \quad (24)$$
$$u_{.0} = 0, u_{1.} = 0 \text{ at } r = R \quad (25)$$
$$u_{0.} = q(z).[f(t)](R^{2.} - r^{2.}) \quad (26)$$

By substituting equation (25) into equations (21) and (23), it was determined;

$$u_0 = [q(z)f(t)(R_{00}^2 - r^2) - 2\theta(R_{00} - r)] + q(z)f(t)(R^2 - R_{00}^2) \quad (27)$$

Now from (21), (24), (25) and (27)

$$u_0 = q(z).f(t)(R_{00}^2 - R_{0p}^2) + q(z).f(t)(R^2 - R_{00}^2) \quad (28)$$

$$r|_{\tau_0=\theta} = R_{0p} = \frac{\theta}{q(z)f(t)} \quad (29)$$

here τ_1 remains finite at r=0, (28) and (19), yields...

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}(R_{00}^2 - R_{0p}^2) - \theta(R_{00} - R_{0p})\right]R_{0p} - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2} \quad (30)$$

The continuity of τ_1 at $r=R_{0p}$ implies;

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}\left(R_{00}^2\frac{r}{2} - \frac{r^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{r}{4} + \frac{A_2}{r} \quad (31)$$

Similarly, because τ_1 is continuous at R_0, we have this expression;

$$\tau_1 = -\frac{1}{2}q(z)f'(t)\left(R^2\frac{r}{2} - \frac{r^3}{4}\right) + \frac{A_3}{r} \quad R_0 \le [r] \le R, \quad (32)$$

Using equation (25), equations (22) to (24) lead to;

$$u_1 = -q(z)f'(t)\left[\frac{R^2}{4}(R^2 - r^2) - \frac{(R^4 - r^4)}{16}\right] - A_3\log\left(\frac{r}{R}\right)$$

$$u_1 = X(r).$$
$$u_1 = X(R_{0p.})$$

$$A_2 = -\left[\frac{q(z)f'(t)}{8}R_{0p}\left(R_{00}^2\frac{R_{0p}}{2} - \frac{R_{0p}^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2}$$

$$A_3 = \left[q(z)f'(t)\frac{R_{00}}{8} - \frac{\theta}{3}\right]R_{00}^3 + A_2$$

$$X(r) = -2\left[q(z)f'(t)\left(\frac{R_{00}^2}{8}(r^2 - R_{00}^2) - \frac{1}{32}(r^4 - R_{00}^4)\right) - 2\theta(\frac{R_{00}}{4}(r^2 - R_{00}^2) - \frac{1}{9}(r^3 - R_{00}^3))\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{1}{2}(r^2 - R_{00}^2) - A_2\log(\frac{r}{R_{00}})$$

Fow rate has calculated and reformulating equation (18);

$$Q = 4\left(u(z, R_p, t)\frac{R_p^2}{2} + \int_{R_p}^{R_0} ru(z,r,t)dr + \int_{R_0}^{R} ru(z,r,t)dr\right). \quad (33)$$

$$\tau_\omega = (\tau_0 + \alpha^2\tau_1)|_{r=R} = q(z)f(t)R + \alpha^2(-\frac{1}{2}q(z)f'(t)\left(\frac{R^3}{4}\right) + \frac{1}{R}A_3) \quad (34)$$

The Newton-Raphson method is employed to determine value of R_{10}. This method starts with an initial guess for R_{10} and iteratively refines it until a satisfactory solution is reached. Velocity in peripheral layer at R_{00} is assumed to be its value in steady state, which is 0.03. $\tau^2(R_{00} + \alpha^2 R_{10}) = \tau_0^2(R_{00})$.

$$q(z) = \frac{Q_s}{R^4} + \frac{16}{7}\left(\frac{\theta Q_s}{R^5}\right)^{\frac{1}{2}} + \frac{64\theta}{49R},$$

when calculating q(z), $Q_s = 1.0$, indicating a reference value for the volumetric flow rate. Once q(z) is determined using this reference value, the function Q(z, t) can then be computed based on the obtained q(z) values.

4. Results and Discussion

This study is about significance of two key parameters, flow rate and shear stress, in analysis of fluid flow through stenosed artery. Study derives analytical solution in velocity, flow rate, and stress, considering appropriate boundary conditions. These expressions, denoted by equations (33) and (34), are evaluated numerically using MATLAB software across a range of relevant parameter values. To ensure accuracy in the numerical computations, a detailed quantitative analysis is performed, involving specific parameter values relevant to the study. This comprehensive approach allows for a thorough understanding of the fluid dynamics within stenosed arteries. Figure (2) depicted of the relationship between blood velocity and the radius considering different m. This relationship is crucial for understanding blood flow dynamics within stenosed arteries, where narrowing of the vessel affects flow characteristics. The figure demonstrates that as the radius of the blood vessel increases, the velocity of blood decreases. This finding aligns with the principles of fluid dynamics, where larger cross-sectional areas lead to reduced flow velocities. The figure illustrates that the velocity of blood tends to increase as the parameter "m" increases. This observation sheds light on how variations in the shape parameter influence flow behavior within stenosed arteries. The insights provided by Figure (2) contribute to a deeper understanding of how blood flow characteristics are impacted by vessel geometry and rheological properties, which is essential for diagnosing and treating cardiovascular conditions.

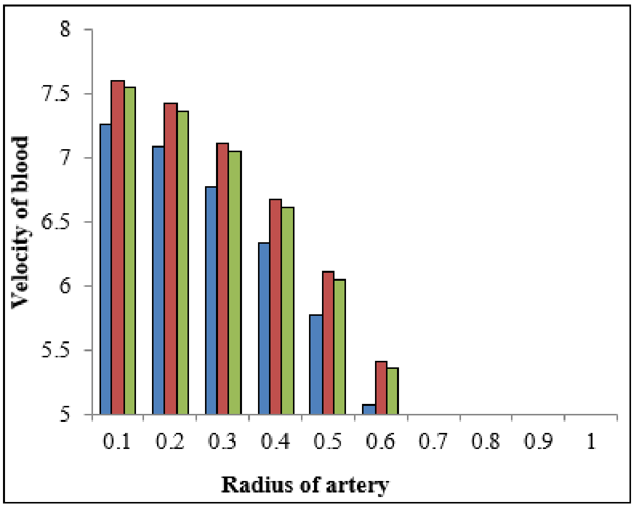

Figure 2: Velocity of blood with m

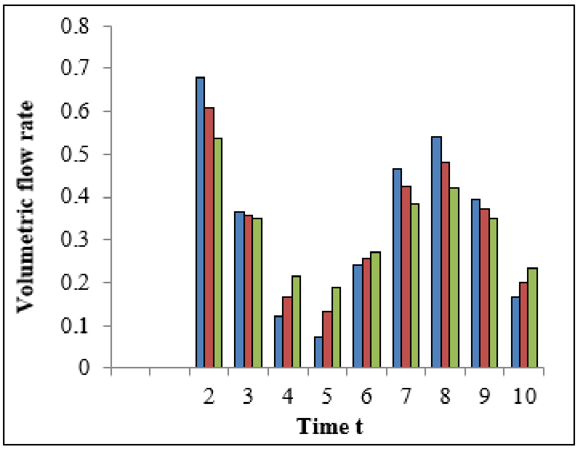

Figure 3: Volumetric flow rate with A

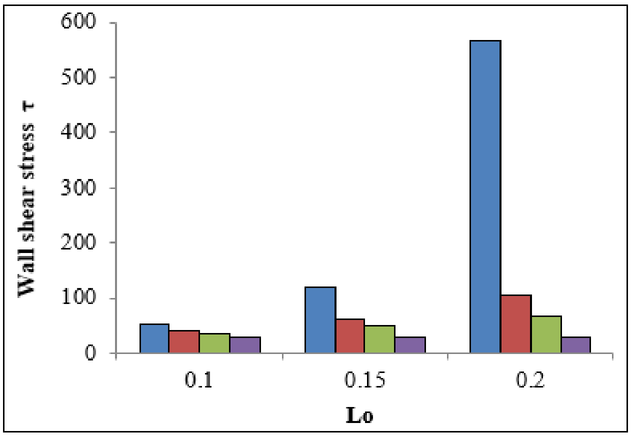

Figure 4: Wall shear stress with time

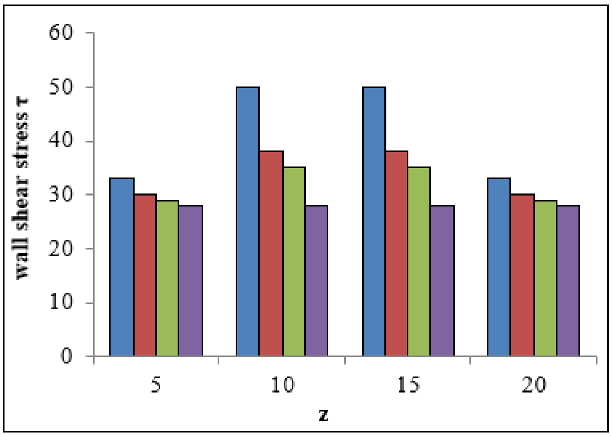

Figure 5: Wall shear stress with time

Figure (3) shows its portrayal of relationship between flow rate and time for parameter 'A'. This relationship is crucial for understanding how changes in flow rate occur over time, which is essential for assessing dynamic flow behavior in physiological systems such as blood circulation. The figure illustrates that the volumetric flow rate initially decreases, followed by an increase, and then another decrease over a certain time interval. This pattern suggests a complex temporal variation in flow dynamics, possibly influenced by factors such as changes in pressure gradients or flow resistance within the system. Additionally, the figure demonstrates that flow rate tends to increase with higher values of the A. This observation implies that alterations in the driving force, represented by the A, can lead to significant changes in flow characteristics. The insights provided by Figure (3) contribute to a better understanding of how flow rates evolve over time under varying conditions, which is pertinent for studying and optimizing fluid transport processes in biological and engineering systems. Figure (4) depicted in its representation of the relationship between wall shear stress, height of the stenosis and time, with implications for understanding the mechanical stresses experienced by the vessel wall in stenosed arteries. Wall shear stress is a critical parameter in vascular physiology, influencing endothelial function and potentially contributing to pathological processes such as atherosclerosis. The figure illustrates that as the height of the stenosis increases, wall shear stress also increases. This finding is significant as it highlights how geometric irregularities within the artery, such as stenosis, can impact the distribution of shear stress along the vessel wall, potentially leading to localized areas of high stress. Additionally, it is demonstrates that shear stress tends to increase with higher values of time, indicating a temporal evolution of shear stress dynamics. This observation suggests that changes in flow conditions over time can influence the mechanical forces exerted on the vessel wall, which may have implications for vascular health and disease progression. Overall, the insights provided by Figure (3) contribute to our understanding of how stenosis geometry and temporal flow variations affect wall shear stress, aiding in the assessment and management of cardiovascular conditions.

5. Conclusion

This study investigated the impact of Bingham plastic fluid flow within stenosed arteries, utilizing a model for two-layer to analyze the pulsatile characteristics of fluid within artery.

Through perturbation techniques, the study examines velocity profile, wall shear stress, and resistance to flow. Graphical representations demonstrated a correlation between decreased velocity and increased radius, as well as an escalation in flow resistance over time. These findings deepen our comprehension of fluid dynamics in stenosed arteries, with potential implications for diagnosing and treating cardiovascular diseases, particularly atherosclerosis. The results suggested that narrower arteries experience swifter blood flow compared to wider ones, and as arterial stenosis progresses, resistance to blood flow intensifies. Understanding these dynamics is pivotal for understanding how blood flows through stenosed arteries, crucial for managing cardiovascular diseases like atherosclerosis. Moreover, the study adopted blood as a Bingham plastic fluid, deriving numerical expressions for velocity, wall shear stress, and volumetric flow rate. Comparing these results underscores the significance of the Bingham plastic fluid model, particularly in capturing parameters like shear stress and flow rate. Furthermore, study reveals that the Bingham plastic fluid model outperforms Newtonian fluid models, emphasizing its relevance in understanding blood flow dynamics within stenosed arteries.

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