Deciphering Pulsatile Behavior and Flow Characteristics in Stenosed Arteries: A Comprehensive Investigation

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Abstract: ***This study explores how Bingham plastic fluid behaves in narrowed arteries, using a two-layer model to analyze how the fluid pulsates within the artery. By employing perturbation techniques, the research investigates key factors like velocity, shear stress, and flow rate. The graphical representation of the results indicates that as the artery's radius increases, the velocity of fluid flow decreases, suggesting that narrower arteries have faster blood flow than wider ones. Furthermore, the graphs demonstrate that over time, there's a rise in resistance to flow within the narrowed arteries, implying that as arterial stenosis progresses, so does the resistance to blood flow. Understanding these dynamics is crucial for comprehending how blood moves through narrowed arteries, particularly in conditions like atherosclerosis, where plaque buildup leads to arterial blockages. By shedding light on how blood flow is affected by arterial narrowing, this study provides insights that could inform the diagnosis and treatment of cardiovascular diseases. Therapeutic approaches aimed at managing arterial stenosis may benefit from considering the fluid dynamics within narrowed arteries.***

Keywords: Bingham plastic fluid, Stenosed arteries, Two-layer model, Perturbation techniques, Velocity, Shear stress, Flow rate, Arterial radius, Arterial stenosis.

1. Introduction

Atherosclerosis tends to develop in specific regions of arteries, like bifurcations and flow divisions, influenced by the complex flow patterns in these areas. Studies indicate a link between atherosclerosis lesions and regions experiencing low or oscillating wall shear stress. Various cardiovascular diseases are linked to the flow conditions within blood vessels. Atherosclerosis, a form of arteriosclerosis, involves the accumulation of fatty substances, cholesterol, cellular waste products, calcium, and fibrin in the inner lining of arteries, resulting in arterial hardening and plaque formation [5,12,38,46]. A common manifestation of atherosclerotic cardiovascular disease is a heart attack or sudden cardiac death. Severe artery narrowing with sudden coronary changes can lead to sudden cardiac arrest or death, affecting a significant number of individuals globally each year, thus warranting extensive research. Within this research series, studies have investigated blood flow in narrowed arteries, focusing on pulsatile flow dynamics [24,30,39]. Some researchers have developed a mathematical model with two layers to study blood flow in asymmetrically stenosed arteries, considering slip velocity, and have also examined the impact of magnetic fields on pulsatile blood flow through inclined circular tubes subjected to periodic body acceleration. Additionally, steady blood flow through narrowed arteries using a non-Newtonian fluid model has been explored by various researchers. Researchers [8,22,35,54] explored a mathematical model to understand blood flow through asymmetrically stenosed arteries, considering velocity slip and the influence of a transverse force. Additionally, [2,6,19] conducted a study on blood flow through stenosed arteries in diseased conditions. Another group of researchers calculated flow resistance in small arteries with multiple stenoses and post-stenotic dilatation. Furthermore, studies investigated mathematical models of blood flow dynamics in blood vessels, focusing on how non-Newtonian fluids behave in arteries with varying cross-sections and representing flow through constricted vessels using fluid models [3,31,42,53]. These investigations contribute significantly to our comprehension of blood flow dynamics in stenosed arteries, potentially informing the development of diagnostic and treatment strategies for cardiovascular diseases. Treatments for atherosclerosis, such as endarterectomy and stenting, often rely on Doppler flow measurements in the common carotid artery [1,9,23,33]. Doppler velocity measurements compare the affected side with the contra-lateral side to gauge the degree of stenosis. In some cases, patients may exhibit equivalent or higher Doppler velocities post-stenting compared to before treatment, which some researchers attribute to decreased compliance of the artery wall in the stented region [13,21,25,52]. A mathematical model serves as a potent tool for describing real-world systems using mathematical language, a process known as mathematical modeling. Within the human body, the cardiovascular system acts as a crucial network responsible for distributing blood, comprising three primary components: blood, heart, and blood vessels [10,16,27,48]. This study reveals that as arterial stenosis progresses or persists, the resistance to blood flow also increases. Understanding these dynamics is crucial for gaining insights into how blood flows through narrowed arteries, which holds relevance to cardiovascular diseases like atherosclerosis. Atherosclerosis involves the accumulation of plaque in the arteries, leading to the narrowing or blockage of blood vessels. The findings from this research contribute to our understanding of how arterial narrowing affects blood flow, which can inform diagnostic methods and treatment strategies for cardiovascular diseases. Therapies aimed at reducing arterial stenosis or managing its effects may benefit from considering the dynamics of fluid flow within narrowed arteries.

2. Formulation of the problem

We've explored how blood flows in a circular artery with a stenosis in an axially symmetric, laminar, and pulsatile manner, as depicted in Figure (1). To analyze this, we utilized cylindrical polar coordinates (r*, θ*, z*) [29,41].

The equation governing momentum is as follows:

$$\rho\frac{\partial u^*}{\partial t^*} = (-\frac{\partial . p^*}{\partial z^*}) - (\frac{1}{r^*}\frac{\partial (r^*.\tau^*)}{\partial r^*}) \quad (1)$$

The mathematical equation representing the non-Newtonian properties of blood, referred to as the Herschel-Bulkley equation, can be expressed as follows:

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), \tau^* > \tau_y \quad (2)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, \tau^* \leq \tau_y \quad (3)$$

In this theoretical examination, we consider blood flow as a two-phase system. We assume that the outer layer, consisting of plasma, behaves according to Newtonian principles, while the inner core, containing the majority of erythrocytes within the artery, exhibits different behaviour [36,49]. The mathematical model developed in this study is characterized as follows:

$$\tau^* = (-\mu)\frac{\partial u^*}{\partial r^*}, if\ R_0^*(z^*,t^*) < r^* < R^*(z^*,t^*), \quad (4)$$

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), if\ R_p^*(z^*,t^*) < r^* < R_0^*(z^*,t^*), \quad (5)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, if\ 0 < r^* < R_p^*(z^*,t^*) \quad (6)$$

$$u^* = 0, \text{at } r^{.*} = R^{*.}(z^*,t^*), \quad (7)$$

$$\tau^{.*}, \text{is finite}, r^{*.} = 0. \quad (8)$$

$$r^* = R_0^*(z^*,t^*) \text{and } r^* = R_p^*(z^*,t^*).$$

The pressure gradient, which changes according to the non-dimensional spatial and temporal coordinates z* and t*, is represented as follows:

$$\frac{\partial}{\partial z^*}.p^{*.}(z^*,t^*) = -q^*(z^{*.}) * f(t^{*.})$$

with $q^*(z^*) = -\frac{\partial}{\partial z^*}p.^*(z^*,0),\ f(t^*) = [1 + A\sin(\omega t^*)]$.

In the subsequent analysis, we utilize the following dimensionless variables:

$$z = \frac{z^*}{a}, r = \frac{r^*}{a}, R(z,t) = \frac{R^*(z^*,t^*)}{a}, R_0(z,t) = \frac{R_0^*(z^*,t^*)}{a},\quad R_p(z,t) = \frac{R_p^*(z^*,t^*)}{a}, \tau = \frac{2\tau^*}{q_0 a},\quad \theta = \frac{2\tau_y}{q_0 a}, u = \frac{u^*}{\frac{q_0 a^2}{4\mu}}, t = t^*\omega, Q(z,t) = \frac{Q^*(z,t)}{\frac{\pi q_0 a^4}{8\mu}}, d. = \frac{d^{*.}}{a},\ \delta = \frac{\delta^{.*}}{a}, L_0 = \frac{L_{0.}^*}{a}, L = \frac{L^{*..}}{a}, \alpha^2 = \frac{\alpha^2\omega}{\frac{\mu}{\rho}}, q(z) = \frac{q^*(z^*)}{q_0} \quad (9)$$

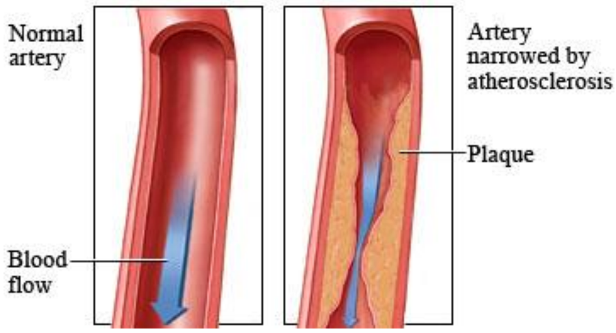

Figure 1: Stenosed Artery

Written in these dimensionless terms, equation (1) can be formulated as:

$$\alpha^2\frac{\partial u}{\partial t} = 4q(z)f(t) - 2(\frac{1}{r}\frac{\partial(r*\tau)}{\partial r}), 0 < [r] < R(z,t), \quad (10)$$

Equations (2) to (6) can be expressed as follows:

$$(-\frac{\partial u}{\partial r}) = 2\tau, [R_0(z,t) < r < R(z,t)], \quad (11)$$

$$(-\frac{\partial u}{\partial r}) = 2(\tau - \theta), [R_p(z,t) < r < R_0(z,t)], \quad (12)$$

$$(-\frac{\partial u}{\partial r}) = 0, [0 < r < R_p(z,t)], \quad (13)$$

Additionally, both u, τ must remain continuous. Geometric configuration of stenosis, represented as dimensionless terms, and defined as:

$$R = \begin{cases} 1 - A_1(t)[L_0^{(m.-1)}[(z-d)] - [(z-d)]^{m.}], & if\ [d \leq [z] \leq (d + L_{0.})], \\ 1, & \text{Otherwise.} \end{cases}$$

$$\text{with } A_{1.}(t) = \frac{\delta[1 - e^{(-t/T.)}]m^{\frac{m}{m-1}}}{aL_0^m(m-1)}, \quad m \neq 1$$

where δ represents maximum height of stenosis within artery. It further explains that this maximum height is achieved at a specific position along the artery, namely $z=d+L_0/[m^{1/(m-1)}]$.

$$Q(z,t) = 4\int_0^{R(z,t)}[ru(z,r,t)]dr.$$

3. Analytical Solution of the problem

This statement suggests that when the Womersley parameter is very small, certain variables such as velocity, shear stress, and the radii R_0 and R_p of the artery is approximated in a simplified term. This simplification is likely to make the mathematical expressions more manageable and easier to analyze;

$$u.(z, r, t) = u_{0.}(z, r, t) + \alpha^2 u_1 + \ldots \quad (14)$$

$$r.(z, r, t) = r_{0.}(z, r, t) + \alpha^2 u_1 + \ldots \quad (15)$$

$$R_{0.}(z, r, t) = R_{00.}(z, r, t) + \alpha^2 R_{10} + \ldots \quad (16)$$

$$R_{p.}(z, r, t) = R_{0p.}(z, r, t) + \alpha^2 R_{1p} + \ldots \quad (17)$$

By (14), (15) and (10), we obtained;

$$\frac{\partial}{\partial r}(r\tau_0) = 2rq(z).f(t) \quad (18)$$

$$\frac{\partial u_0}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \quad (19)$$

By integrating equation (18) applying conditions, it is obtained;

$$\tau_0 = q(z)f(t)R_p, 0 \leq r \leq R_p. \quad (20)$$

For, $R._p \leq [r] \leq R._0 : R._0 \leq [r] \leq R$,

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$\tau_{0.} = [q(z)f(t)(r)]$. (21)

By substituting equations (14) and (15) into equations (11) to (13) and matching terms with the same power of α, we derive;

$$-\frac{\partial u_0}{\partial r} = 2\tau_0, -\frac{\partial u_1}{\partial r} = 2\tau_1, \text{if } R_0 \le r \le R \quad (22)$$

$$-\frac{\partial u_0}{\partial r} = 2(\tau_0 - \theta), -\frac{\partial u_1}{\partial r} = 2\tau_1, \text{if } R_p \le r \le R_0 \quad (23)$$

$$\frac{\partial u_0}{\partial r} = 0, \frac{\partial u_1}{\partial r} = 0, \text{if } 0 \le r \le R_p \quad (24)$$

$$u_{.0} = 0,\ u_{1.} = 0 \text{ at } r = R \quad (25)$$

$$u_{0.} = q(z).[f(t)](R^{2.} - r^{2.}) \quad (26)$$

By substituting equation (25) into equations (21) and (23), it was determined;

$$u_0 = [q(z)f(t)(R_{00}^2 - r^2) - 2\theta(R_{00} - r)] + q(z)f(t)(R^2 - R_{00}^2) \quad (27)$$

Now from (21), (24), (25) and (27)

$$u_0 = q(z).f(t)\left(R_{00}^2 - R_{0p}^2\right) + q(z).f(t)(R^2 - R_{00}^2) \ (28)$$

$$r|_{\tau_0=\theta} = R_{0p} = \frac{\theta}{q(z)f(t)} \quad (29)$$

here τ_1 remains finite at r=0, (28) and (19), yields...

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}\left(R_{00}^2 - R_{0p}^2\right) - \theta\left(R_{00} - R_{0p}\right)\right]R_{0p} - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2} \ (30)$$

The continuity of τ_1 at r=R_{0p} implies;

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}\left(R_{00}^2\frac{r}{2} - \frac{r^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{r}{4} + \frac{A_2}{r} \ (31)$$

Similarly, because τ_1 is continuous at R_0, we have this expression;

$$\tau_1 = -\frac{1}{2}q(z)f'(t)\left(R^2\frac{r}{2} - \frac{r^3}{4}\right) + \frac{A_3}{r} \quad R_0 \le [r] \le R \quad (32)$$

Using equation (25), equations (22) to (24) lead to;

$$u_1 = -q(z)f'(t)\left[\frac{R^2}{4}(R^2 - r^2) - \frac{(R^4 - r^4)}{16}\right] - A_3 \log\left(\frac{r}{R}\right)$$

$$u_1 = X(r).$$

$$u_1 = X\left(R_{0p.}\right)$$

$$A_2 = -\left[\frac{q(z)f'(t)}{8}R_{0p}\left(R_{00}^2\frac{R_{0p}}{2} - \frac{R_{0p}^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2}$$

$$A_3 = \left[q(z)f'(t)\frac{R_{00}}{8} - \frac{\theta}{3}\right]R_{00}^3 + A_2$$

$$X(r) = -2\left[q(z)f'(t)\left(\frac{R_{00}^2}{8}(r^2 - R_{00}^2) - \frac{1}{32}(r^4 - R_{00}^4)\right) - 2\theta(\frac{R_{00}}{4}(r^2 - R_{00}^2) - \frac{1}{9}(r^3 - R_{00}^3))\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{1}{2}(r^2 - R_{00}^2) - A_2\log(\frac{r}{R_{00}})$$

Fow rate has calculated and reformulating equation (18);

$$Q = 4\left(u(z, R_p, t)\frac{R_p^2}{2} + \int_{R_p}^{R_0} ru(z,r,t)dr + \int_{R_0}^{R} ru(z,r,t)dr\right). \ (33)$$

$$\tau_\omega = (\tau_0 + \alpha^2\tau_1)|_{r=R} = q(z)f(t)R + \alpha^2(-\frac{1}{2}q(z)f'(t)\left(\frac{R^3}{4}\right) + \frac{1}{R}A_3) \ (34)$$

The Newton-Raphson method is employed to determine value of R_{10}. This method starts with an initial guess for R_{10} and iteratively refines it until a satisfactory solution is reached. Velocity in peripheral layer at R_{00} is assumed to be its value in steady state, which is 0.03. $\tau^2(R_{00} + \alpha^2 R_{10}) = \tau_0^2(R_{00})$.[28,40]

$$q(z) = \frac{Q_s}{R^4} + \frac{16}{7}\left(\frac{\theta Q_s}{R^5}\right)^{\frac{1}{2}} + \frac{64\theta}{49R},$$

when calculating q(z), Q_s = 1.0, indicating a reference value for the volumetric flow rate. Once q(z) is determined using this reference value, the function Q(z, t) can then be computed based on the obtained q(z) values.

4. Results and Discussion

This study focuses on the significance of two crucial parameters, flow rate and shear stress, in the analysis of fluid flow through stenosed arteries. It derives analytical solutions for velocity, flow rate, and stress, considering appropriate boundary conditions. These solutions, represented by equations (33) and (34), are then numerically evaluated using MATLAB software across a range of relevant parameter values. To ensure accuracy in the numerical computations, a detailed quantitative analysis is conducted, involving specific parameter values pertinent to the study [14,43,50].

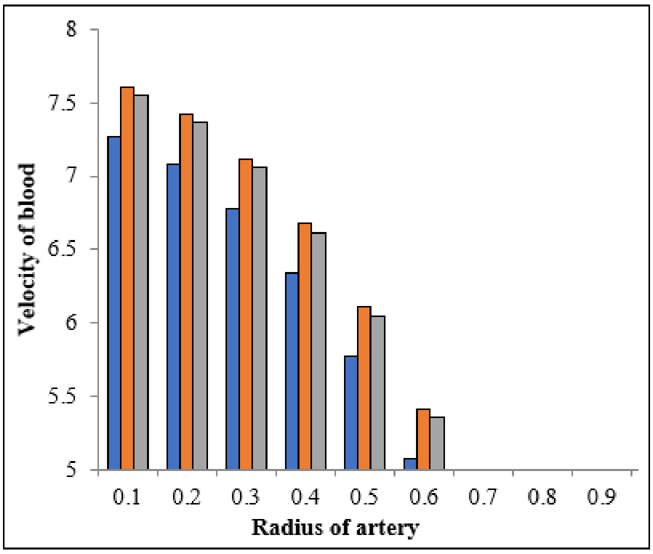

Figure (2): Velocity of blood with m

Paper ID: SR24508021708 DOI: https://dx.doi.org/10.21275/SR24508021708

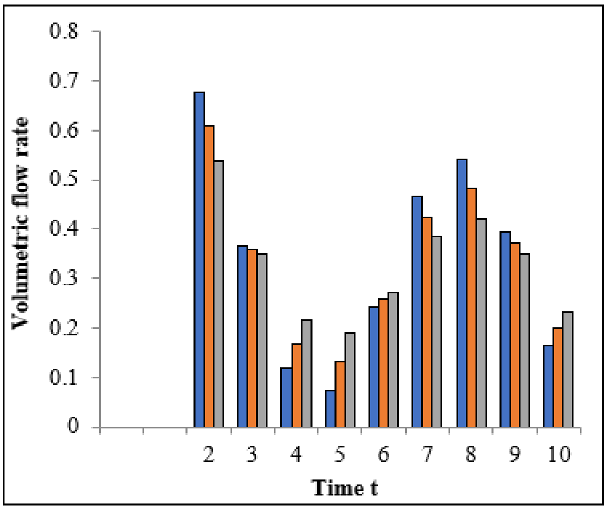

Figure (3): Volumetric flow rate with A

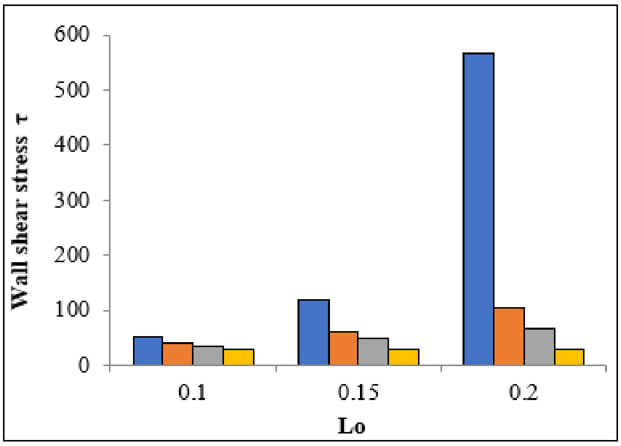

Figure 4: Wall shear stress with time

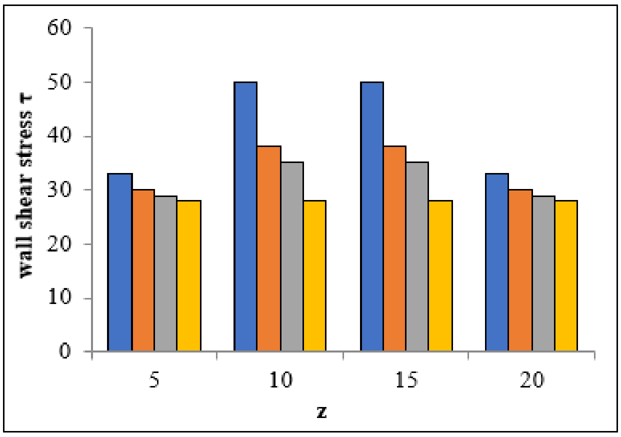

Figure 5: Wall shear stress with time

This comprehensive approach allows for a thorough understanding of the fluid dynamics within stenosed arteries. Figure (2) illustrates the relationship between blood velocity and the radius for different values of "m". This relationship is pivotal for comprehending blood flow dynamics within stenosed arteries, where vessel narrowing impacts flow characteristics. The figure demonstrates that as the radius of the blood vessel increases, the velocity of blood decreases. This finding aligns with fluid dynamics principles, where larger cross-sectional areas result in reduced flow velocities [18,20,45]. Additionally, the figure shows that blood velocity tends to increase as the parameter "m" increases. This observation provides insights into how variations in the shape parameter influence flow behavior within stenosed arteries. The insights provided by Figure (2) contribute to a deeper understanding of how blood flow characteristics are influenced by vessel geometry and rheological properties, which is essential for the diagnosis and treatment of cardiovascular conditions [11,26,37].

Figure (3) depicts the relationship between flow rate and time for parameter 'A'. This relationship is essential for comprehending how changes in flow rate unfold over time, which is critical for assessing dynamic flow behavior in physiological systems like blood circulation. The figure illustrates that the volumetric flow rate initially decreases, then increases, and subsequently decreases again over a specific time interval [7,34,51]. This pattern indicates a complex temporal variation in flow dynamics, potentially influenced by factors such as alterations in pressure gradients or flow resistance within the system. Furthermore, the figure shows that flow rate tends to rise with higher values of parameter 'A'. This observation suggests that modifications in the driving force, as represented by parameter 'A', can induce significant alterations in flow characteristics. The insights offered by Figure (3) enhance our understanding of how flow rates evolve over time under different conditions, which is crucial for analyzing and optimizing fluid transport processes in biological and engineering systems [4,15,47,54]. Figure (4) and Figure (5) presents the relationship between wall shear stress, stenosis height, and time, offering insights into the mechanical stresses experienced by the vessel wall in stenosed arteries. Wall shear stress holds critical importance in vascular physiology, influencing endothelial function and potentially contributing to pathological processes like atherosclerosis. The figure demonstrates that as the stenosis height increases, wall shear stress also increases. This observation underscores how geometric irregularities within the artery, such as stenosis, can impact the distribution of shear stress along the vessel wall, potentially leading to localized areas of heightened stress. Moreover, the figure indicates that shear stress tends to rise with higher values of time, suggesting a temporal evolution of shear stress dynamics. This finding implies that changes in flow conditions over time can influence the mechanical forces exerted on the vessel wall, which may have implications for vascular health and disease progression [17,32,44]. Overall, the insights provided by Figure (4) enhance our understanding of how stenosis geometry and temporal flow variations affect wall shear stress, thereby aiding in the assessment and management of cardiovascular conditions.

5. Conclusion

This research delved into the effects of Bingham plastic fluid flow within stenosed arteries, employing a two-layer model to analyze the pulsatile characteristics of fluid within the artery. By utilizing perturbation techniques, the study explored the velocity profile, wall shear stress, and resistance to flow. The graphical representations illustrated a correlation between decreased velocity and increased radius, alongside an escalation in flow resistance over time. These findings contribute to a deeper understanding of fluid dynamics in stenosed arteries, with potential implications for diagnosing and treating cardiovascular diseases, particularly

Paper ID: SR24508021708 DOI: https://dx.doi.org/10.21275/SR24508021708

atherosclerosis. The results indicated that narrower arteries tend to experience swifter blood flow compared to wider ones, and as arterial stenosis progresses, the resistance to blood flow intensifies. This understanding of dynamics is crucial for managing cardiovascular diseases like atherosclerosis. Additionally, the study adopted blood as a Bingham plastic fluid, deriving numerical expressions for velocity, wall shear stress, and volumetric flow rate. Comparing these results highlights the significance of the Bingham plastic fluid model, especially in capturing parameters like shear stress and flow rate. Furthermore, the study revealed that the Bingham plastic fluid model outperforms Newtonian fluid models, emphasizing its relevance in understanding blood flow dynamics within stenosed arteries.

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