Impact-Time Measurement Experiment

Rabindranath Chattopadhyay

(IAPT, IPS, MTA, ASI, ISEC) (India)
Email: *rchattopadhyay03[at]yahoo.co.in*

Abstract: *A simple experiment with spherical balls of different types and sizes and an ordinary digital weighing machine has been performed with a view to find out an approximate time of impact in each case of collision of each ball with horizontal rigid platform of the weighing machine and has been briefly described along with a basic theoretical discussion.*

Keywords: Impact, collision, elasticity, air-pressure, time

1. Introduction

Seven spherical balls, four of which are hollow hard plastic balls, one rigid moderately soft rubber-ball, two rigid metal balls were taken along with an ordinary digital weighing machine (SF-400D Model). Four hollow plastic balls are of different radii but of same kind of plastic-material while the two rigid metal balls are of different radius and different material, one aluminum and the other steel. Each ball has been released thrice from three different small height above the flat platform of weighing machine and the corresponding maximum of varying instantaneous weight and weight at rest, as displayed in the window of the machine are separately noted down. The difference between the static and maximum value of instantaneous weight as shown by the weighing-machine after impact is certainly related to time of impact.

Figure 1

In this experiment seven balls are assumed to be perfectly spherical in shape and therefore their centers are assumed to be the respective center of mass of each ball. Among these four balls are hollow and made of hard plastic material. One is rigid sphere made of soft rubber while the other two are metallic spheres, one is Aluminum-made and the other is Steel-made and are of different radii. A ruled scale is fixed aside the digital weighing machine that helps determine the initial height from which each ball is dropped. Three different initial heights as chosen are respectively 10cms., 20cms. and 30cms.

Figure 2

After release each ball moves vertically down and hit the platform of weighing machine. Before hitting of the ball the weighing-machine window shows reading zero. Immediately after hitting of ball it shows some reading that almost instantaneously reach a maximum. Their maximum value is noted down for each collision of each ball. Six to seven times repetition of this action has been made for each ball for each initial height and the average of those observed values are taken for each ball for each initial height. A few seconds after each such collision of each ball a steady unchanged reading in the weighing-machine window appears and that also be noted down corresponding to each ball. It is then observed that for each ball for each collision the static
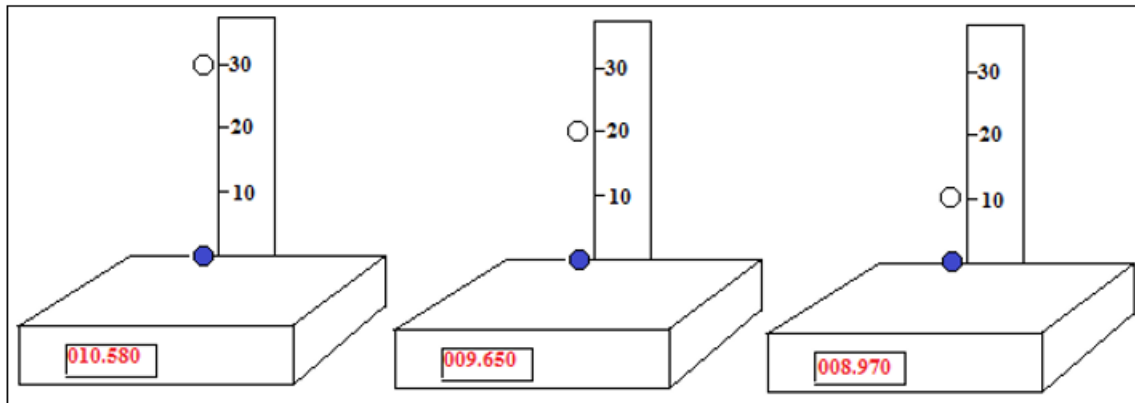

Figure 3

weight is always less than the corresponding maximum of varying weight as displayed in the window. The difference is found out and noted down for corresponding ball for corresponding each initial release-height.

Glimpse of a brief theoretical background: Free-falling ball falls from a certain height and gain a final velocity and momentum just before its impact with the platform of the weighing- machine. Immediately after collision with platform it stops within a very short span of time which cannot be measured with the help of such simple experimental setup at home. But as the rate of change of momentum gives the force it instantaneously adds up to its static weight and shows a maximum at some point of time. Additional weight of the ball increases from zero up to a certain maximum (as is recorded in the machine) and then again decreases down to zero.

So, the total time of impact is, of course on the presumption that the process of retardation on both side of the maximum is symmetrically uniform, is twice the time for reaching from zero to maximum of additional weight. Weighing machine basically functions on elasticity-based restoring force which very naturally is supposed to obey simple law of elasticity that stress generated within it is depression (i,e. strain)-dependent. For rigid balls again the bulk-elasticity is considered to come into play to retard the motion of it. For hollow sphere the air-pressure within the ball might have been considered to obey Boyle's law (as the heights of fall are relatively small and are assumed that the change in the total mechanical energy too is relatively small so that the temperature of the whole system does not change appreciably) and a bulk-elasticity type property holds for their collisions. Moreover as the balls undergo head-on collision straight from upward to descend down yielding depression the balls' elasticity-equivalent stress and strain relates to that of the load-cell elastic factor of the balance just like two springs connected in series they do function simultaneously.
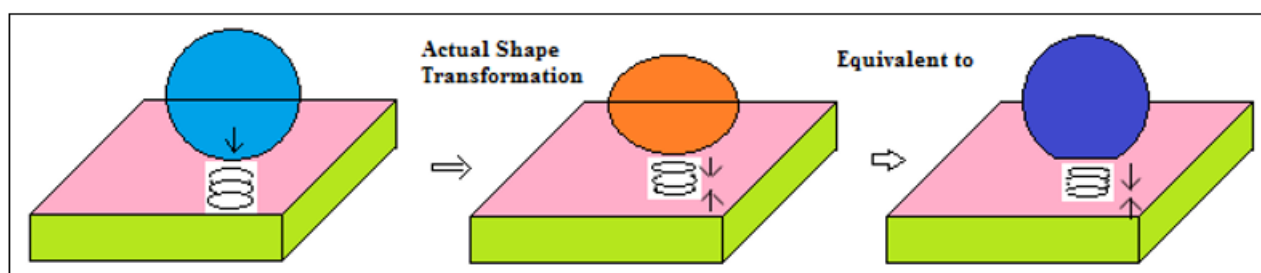

Figure 4

Elementary Theoretical Calculations: From the consideration of atomic structure any rigid body material collision summarily be assumed to be a type of scattering in Coulombic field of electron-complex within atomic bulks approaching each other. For single particle- scattering the angle by which the projectile particle gets scattered is given by

$$\varphi(b) = 2b \int_{\infty}^{r_b} \frac{\frac{dr}{r^2}}{(1 - \frac{b^2}{r^2} - \frac{V(r)}{E})^{\frac{1}{2}}} = 2b \int_{\infty}^{r_b} \frac{\frac{dr}{r^2}}{(1 - \frac{b^2}{r^2} - \frac{2a}{r})^{\frac{1}{2}}}$$

For $r_b = b$, the distance of closest approach and for $\varphi(b) = 0$ to $\varphi(b) = \pi$ i,e. head-on collision and reflection back approximate time of impact is given by

$$\tau = \int_0^\tau d\tau = \frac{mb}{(2mE)^{\frac{1}{2}}} \int_0^\pi d\varphi = \frac{\pi mb}{(2mE)^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots(1)$$

This calculation is for two-particles'-head-on collision. Now if we consider larger object such as spherical ball and a plane then there should be an arbitrary equivalent time of impact which could be certain number (a factor b_e say) multiplied by this time of impact. Then experimentally obtained value should be some number times this 'τ'.

Now the calculations of approximate time of impact as well as time of depression for different types of balls, taken in our experiment are given below; The simple equation of motion of a spherical ball after impact in general will be

$$\left(\frac{m}{2}\right)\frac{dv^2}{dx} = mg - F_{rest} \ldots\ldots\ldots\ldots..(2)$$

Here m' is the rest-mass of the ball (hollow or rigid whatever it may be; In case of hollow ball the mass is considered to be the totality of masses of ball-material and the air enclosed within it.) , v ' is the instantaneous after-impact speed of the ball, g' as usual the acceleration due to gravity and F_{rest}' is the material-elasticity-based restoring force that commences to play immediately with the impact. Following each impact a pair of vertically longitudinal strain and stress commence and remain acting until a maximum that stops motion is reached. One of the pair is with the ball-surface itself while the other is with the platform which again is instantaneously transmitted to internal load-cell sensor of the weighing-machine. These two strains and stresses thus produced may be regarded to act simultaneously hand-in-hand and thereby be represented like two springs connected in series undergoing successive compression and elongation.

For hollow spherical balls
$$F_{rest} = \frac{3\pi P_0 r^2}{2} \left(\frac{x}{r}\right)^3 \left\{1 - \frac{1}{2}\left(\frac{x}{r}\right)\right\} \ldots\ldots\ldots\ldots(3)$$

For rigid spherical balls
$$F_{rest} = \frac{K\pi r^2}{4} \left(\frac{x}{r}\right)^3 \left\{6 - 5\left(\frac{x}{r}\right) + \left(\frac{x}{r}\right)^2\right\} \ldots\ldots\ldots..(4)$$

Interpretation of symbols in the above equations:
P_0 -- the normal atmospheric pressure of air within each ball.
r -- radius of the ball , x -- instantaneous depression of colliding surface of the ball,
K -- bulk-modulus of the ball material

In this experiment the hollow spherical balls are all made of hard plastic and so merely a fall-height equals to 30 cms. or less is not enough to produce any appreciable depression so as to raise the air-pressure inside. In that case the hollow spherical balls too like the rigid ones are presumed to undergo instantaneous after-impact depression of their respective surface-materials only and therefore the elastic property of the material of the balls is to be considered only. From the combination of eqns. (2) and (4) and solving one gets,

$$v^2 = u^2 + 2gx - \frac{K\pi r^3}{2m}\left\{\frac{3}{2}\left(\frac{x}{r}\right)^4 - \left(\frac{x}{r}\right)^5 + \frac{1}{6}\left(\frac{x}{r}\right)^6\right\}(5)$$

Here, u' is the final vertical speed attained by a ball just at the time of impact ($v = u$ at $x = 0$).

As $x \ll r$ for simplicity then eqn. (5) may be rewritten as
$$v^2 = u^2 + 2gx - \left(\frac{3}{4}\right)\left(\frac{K\pi}{mr}\right)x^4 \ldots\ldots\ldots\ldots.(6)$$

Then putting the condition $v = 0$ at $x = \delta$ (the maximum depression) it is found that

$$\delta = \frac{\left(\frac{8mghr}{3K\pi}\right)^{\frac{1}{4}}}{1 - \frac{1}{4h}\left(\frac{8mghr}{3K\pi}\right)^{\frac{1}{4}}} \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

From eqn. (6) then one writes
$$v = \frac{dx}{dt} = \sqrt{2gh\left(1 + \frac{x}{h}\right) - \frac{3}{4}\left(\frac{K\pi h^4}{mr}\right)\left(\frac{x}{h}\right)^4}$$
$$\approx \sqrt{2gh\left(1 + \frac{x}{h}\right)} \text{ [as } x \ll h \text{ the higher power of the ratio is neglected]}$$

Then the time for reaching maximum depression 'δ' is found out to be
$$\tau_\delta = \sqrt{\left(\frac{2h}{g}\right)}\left[\sqrt{\left(1 + \frac{\delta}{h}\right)} - 1\right]\ldots\ldots\ldots\ldots(8)$$

Similarly for the depression of the weighing machine-platform and simultaneously transferring the stress to internal load sensor points there will be another time of internal depression

$$\tau_{is} = \left(\frac{m}{m'}\right)\left(\frac{N}{g}\right)\left(\frac{a}{A}\right)\ldots\ldots\ldots\ldots..(9)$$

Here m' is the maximum of mass displayed in the weighing machine immediately after the impact with the platform, N' is the final velocity-equivalent considering air-buoyancy, $\left(\frac{a}{A}\right)$ is the ratio of internal sensor-point area and external platform area on which the pressure due to momentum-imparting of the ball is exerted. N is given by

$$N = \sqrt{\frac{4rg}{3}\left(\frac{\rho_o}{\rho_{air}} - 1\right)\left\{1 - e^{-\left(\frac{3h}{2r}\right)\left(\frac{\rho_{air}}{\rho_o}\right)}\right\}} \ldots\ldots(10)$$

As both the depression, the depression of ball-surface and that of the weighing machine- platform occur simultaneously then obviously $\tau_\delta = \tau_{is}$.

For each spherical ball three different fall-heights are taken and δ , τ_δ and N are calculated using eqns. (7), (8) and (10) respectively and are tabulated in Table-III. Then from those values and with the help of eqn.(8) the corresponding value of the ratio (a/A) is determined and shown against the corresponding value of the ratio (δ/h) in the same table.
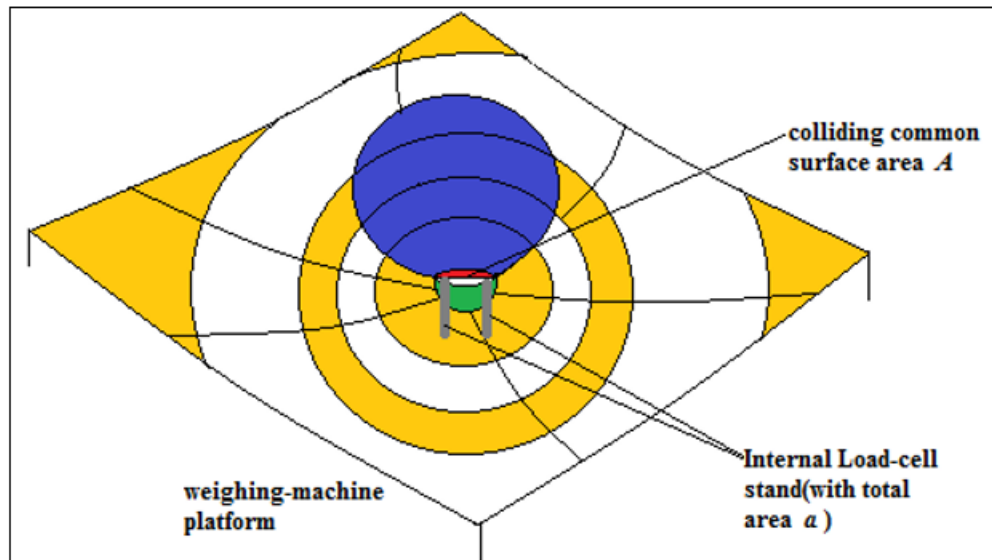
Figure 5

Table 1

For	Approximate Time-of-Impact
Single particle scattering	$T = \dfrac{\pi m b_e}{(2mE)^{\frac{1}{2}}} = \pi b_e / \sqrt{2gh}$
Weighing-machine load-cell points	$\tau_{is} = \left(\dfrac{m}{m\prime}\right)\left(\dfrac{N}{g}\right)\left(\dfrac{a}{A}\right)$
Rigid Sphere	$\tau_\delta = \sqrt{\left(\dfrac{2h}{g}\right)}[\sqrt{\left(1+\dfrac{\delta}{h}\right)} - 1]$
Hollow Air-filled Sphere	$T_{i(hs)} = 2\left(\dfrac{2mr}{3P_0 \pi gh}\right)^{\frac{1}{4}}$

Experimental Results in Tabular Form:

Table II

Nature, Diameter, Rest Wt. of ball \ Repetition No.	From Height	1 (gm)	2 (gm)	3 (gm)	4 (gm)	5 (gm)	6 (gm)	7 (gm)	Mean (gm)
Hollow Plastic (1) ball: D_1= 4.36cm W_{01}= 8.55gm	30cm	11.56	11.25	09.97	10.56	09.94	10.58	--	10.643
	20cm	09.62	10.51	10.18	09.65	09.62	10.15	10.17	09.986
	10cm	09.58	09.57	09.51	09.52	08.97	08.87	09.51	09.360
Hollow Plastic (2) ball: D_2= 4.37cm W_{02}= 9.62gm	30cm	11.64	11.54	12.31	11.76	12.63	11.84	11.65	11.910
	20cm	11.17	11.18	10.87	11.13	11.21	10.67	11.60	11.120
	10cm	10.65	09.17	10.61	10.18	10.15	10.61	10.16	10.220
Hollow Plastic (3) ball: D_3= 4.38cm W_{03}= 8.60gm	30cm	10.52	09.61	09.67	10.53	10.62	10.84	11.15	10.420
	20cm	10.15	10.12	09.89	09.63	10.15	09.87	10.15	09.990
	10cm	09.61	09.67	09.63	09.51	09.52	09.51	09.17	09.520
Hollow Plastic (4) ball: D_4= 3.63cm W_{04}= 2.87gm	30cm	03.48	03.84	03.25	03.84	03.49	03.63	03.35	03.550
	20cm	03.25	03.15	03.13	03.16	03.35	03.04	03.41	03.210
	10cm	03.17	03.12	02.96	03.11	03.06	03.14	02.97	03.080
Rigid soft rubber Ball (5), D_5= 3.37cm, W_{05}= 19.3gm	30cm	20.65	20.65	20.66	20.21	20.31	20.28	20.11	20.410
	20cm	20.21	20.24	20.13	20.15	20.16	20.21	20.31	20.200
	10cm	20.15	20.13	20.12	20.10	20.16	20.12	20.13	20.13
Rigid steel Ball (6), D_6= 0.64cm, W_{06}= 1.04gm	30cm	01.25	01.47	01.29	01.23	01.37	01.26	01.35	01.320
	20cm	01.17	01.28	01.18	01.25	01.18	01.32	01.27	01.240
	10cm	01.17	01.16	01.14	01.18	01.09	01.13	01.15	01.150
Rigid aluminum Ball (7), D_7= 1.915cm, W_{07}= 10.39gm	30cm	12.12	12.35	12.31	12.81	13.21	13.12	12.87	12.680
	20cm	12.12	12.15	11.45	11.35	11.45	11.63	12.41	11.790
	10cm	11.15	11.21	11.43	11.17	11.31	11.60	11.53	11.340

Theoretically determined and semi-empirically found values of important parameters

Table III

m,r,K,ρ (CGS) (respectively), Material / Parameters	h (cm)	$m^{/}$(gm) (Mean)	$(m/m^{/})$	δ(cm)	τ_{δ} (millisec)	N(cm/sec)	(δ/h) X 10^{-3}	(a/A) X 10^{-3}	b_e (X 10^{-2})
8.55, 2.18, 2X10^{10} , 0.197 Hard Plastic	30	10.643	0.803	0.0694	0.286	234.36	2.310	1.44	2.2
	20	09.986	0.856	0.0628	0.317	193.34	3.140	1.83	1.99
	10	09.360	0.913	0.0528	0.376	138.13	5.280	2.88	1.7
9.62, 2.185, 2X10^{10} , 0.220 Hard Plastic	30	11.910	0.808	0.0716	0.295	235.15	2.390	1.48	2.28
	20	11.120	0.865	0.0647	0.326	193.77	3.230	1.86	2.0
	10	10.220	0.941	0.0544	0.388	138.29	5.440	2.88	1.7
8.6, 2.19, 2X10^{10} , 0.195 Hard Plastic	30	10.420	0.825	0.0696	0.287	234.12	2.320	1.41	2.2
	20	09.990	0.861	0.0629	0.317	193.14	3.150	1.82	1.99
	10	09.520	0.903	0.0530	0.378	138.00	5.300	2.93	1.68
2.87, 1.815, 2X10^{10} , 0.115 Hard Plastic	30	03.550	0.808	0.0505	0.208	226.27	1.680	1.04	1.6
	20	03.210	0.894	0.0456	0.230	188.64	2.280	1.27	1.4
	10	03.080	0.932	0.0384	0.274	136.24	3.840	2.03	1.2
19.30, 1.685, 5X10^{8} , 0.963 Rubber	30	20.410	0.946	0.2010	0.827	240.32	6.700	3.53	6.4
	20	20.200	0.955	0.1818	0.916	196.76	9.090	4.75	5.8
	10	20.130	0.959	0.1531	1.089	139.52	15.31	7.95	4.8
1.04, 0.32, 16X10^{11} , 7.577 Steel	30	01.320	0.788	0.0085	0.035	241.14	0.283	0.20	0.27
	20	01.240	0.839	0.0077	0.039	197.25	0.383	0.23	0.24
	10	01.150	0.904	0.0064	0.046	139.74	0.645	0.36	0.20
10.39, 0.9575, 7.5X10^{11} , 2.825 Aluminum	30	12.680	0.819	0.0240	0.099	241.18	0.800	0.49	0.74
	20	11.790	0.881	0.0217	0.109	197.25	1.084	0.61	0.68
	10	11.340	0.916	0.0182	0.130	139.71	1.820	0.99	0.58

Brief Description of the graphs drawn:
The following graphical plots have been made for an elaborate study and cross-checking the interrelations among variables and parameters involved;

1) h versus $m^{/}$, 2) (mh) versus $m^{/}$, 3) (δ/h) versus (a/A) , 4) $m^{/}$ versus δ, 5) ρ_o versus (δ/h), 6) N versus (δ/h), 7) N versus τ_{δ} ,8) δ versus τ_{δ} ,
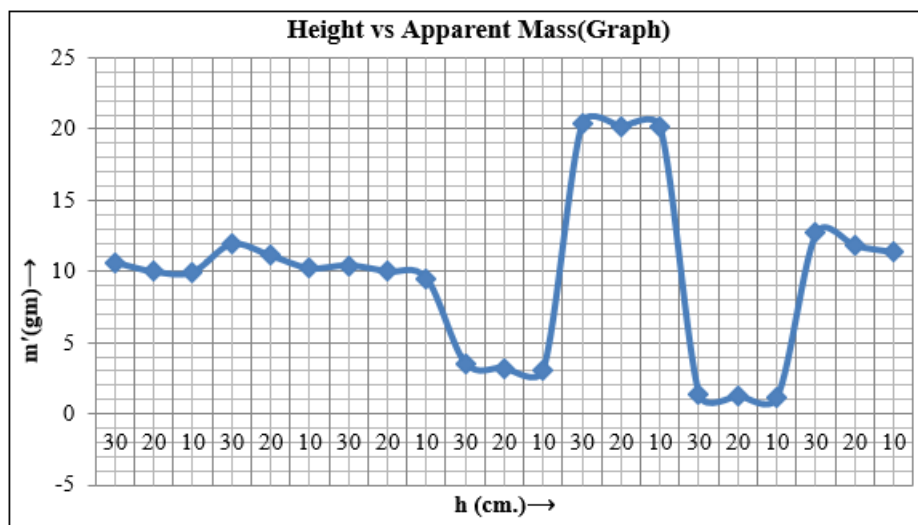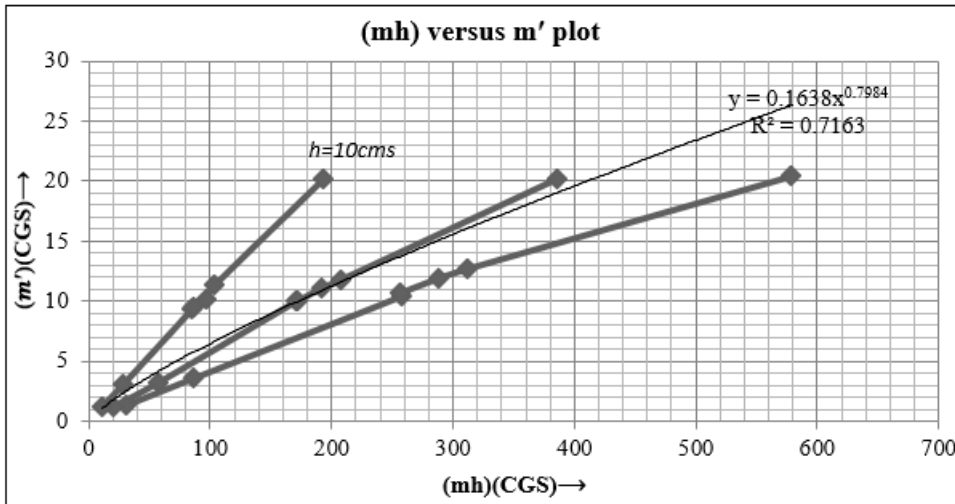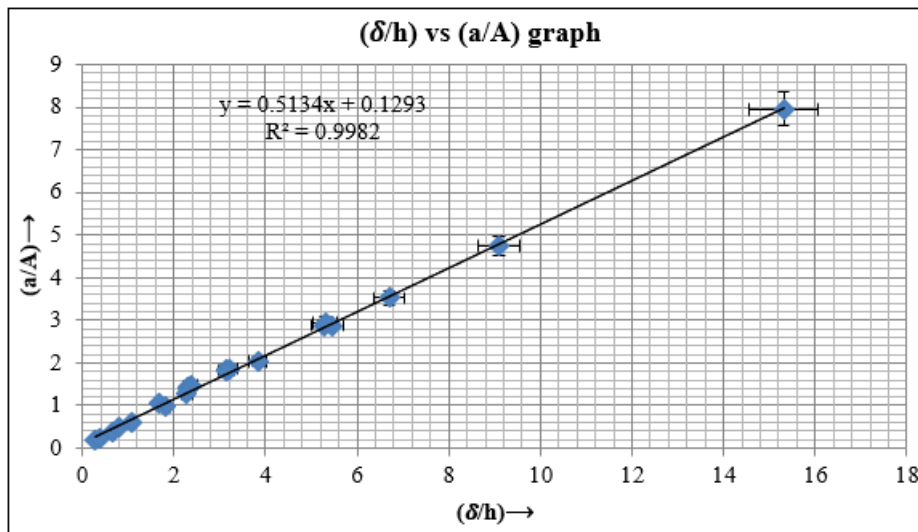
Figure 6

Figure 7

Figure 8
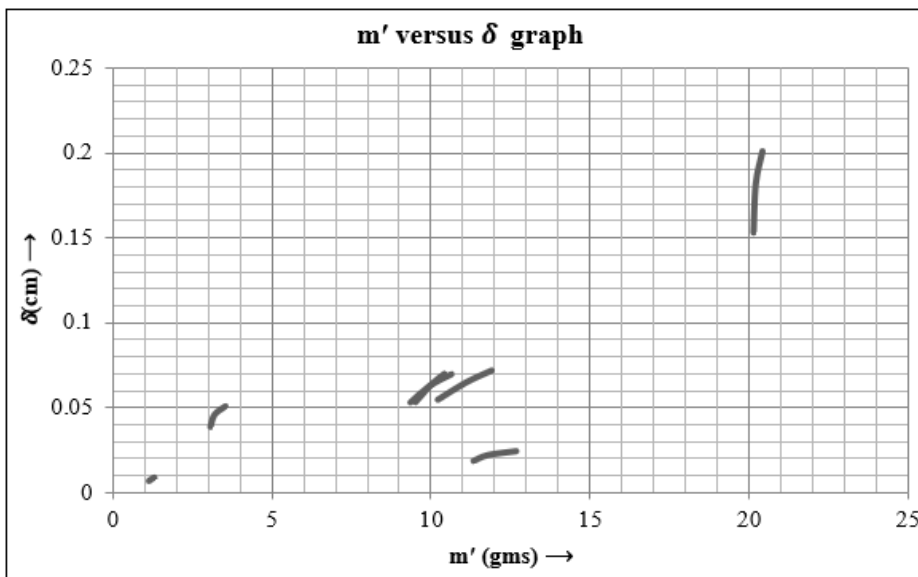
Figure 9
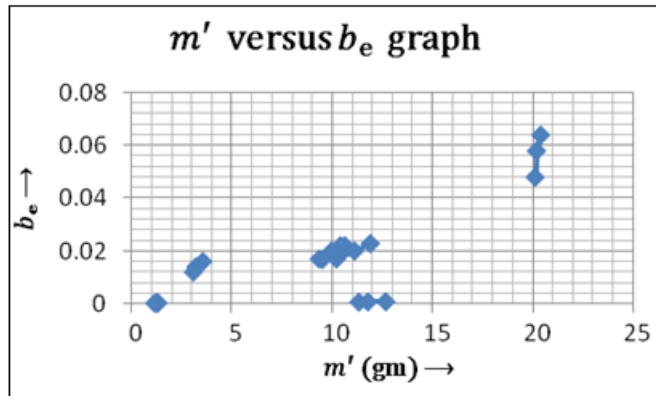
Figure 10
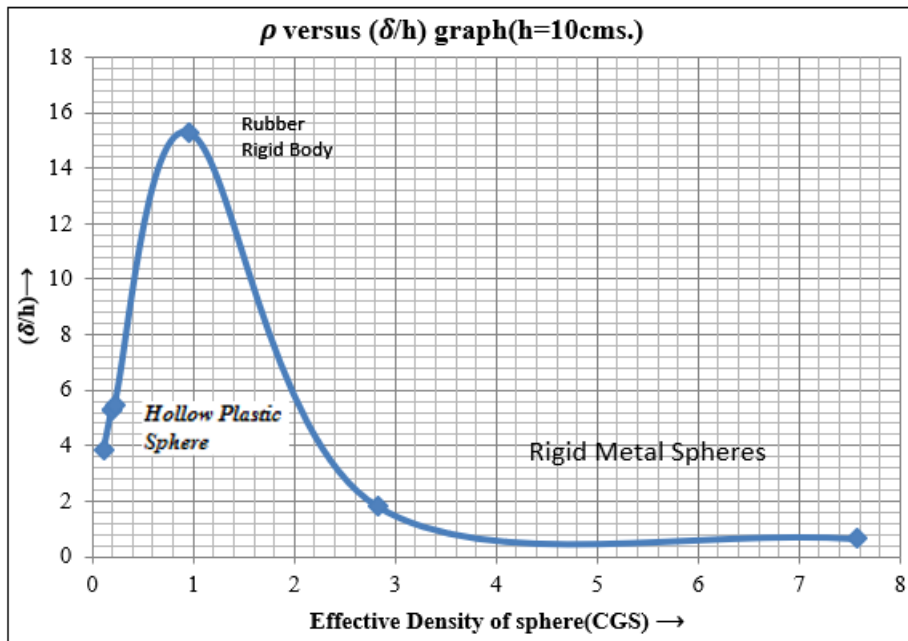
Figure 11(a)
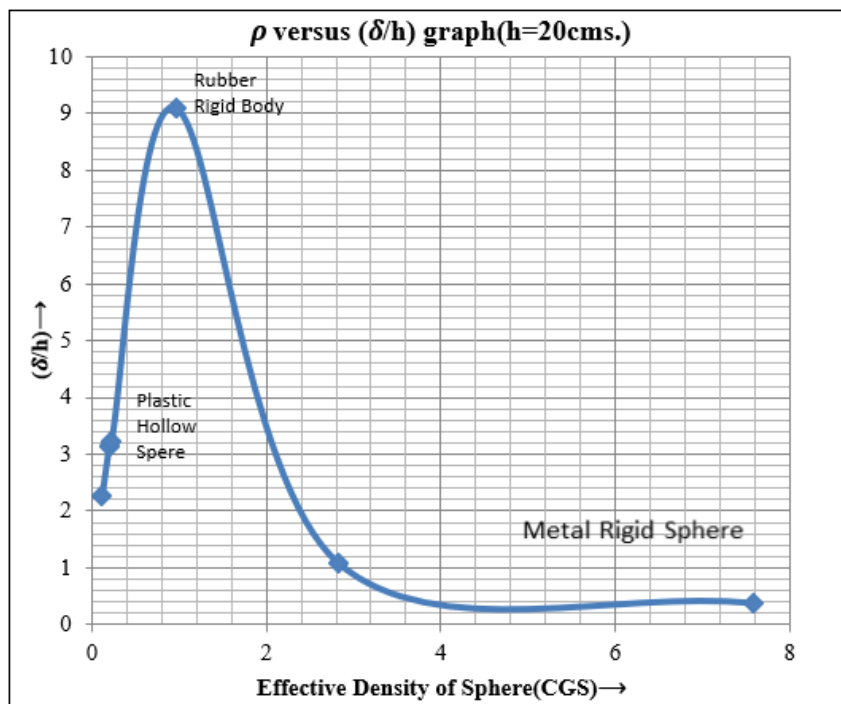
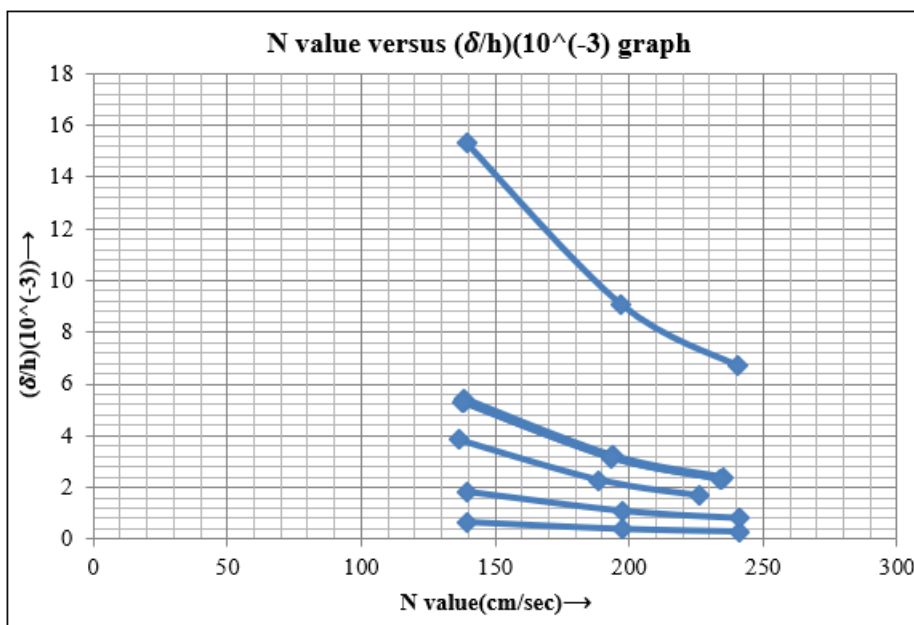**Figure 11(b)**
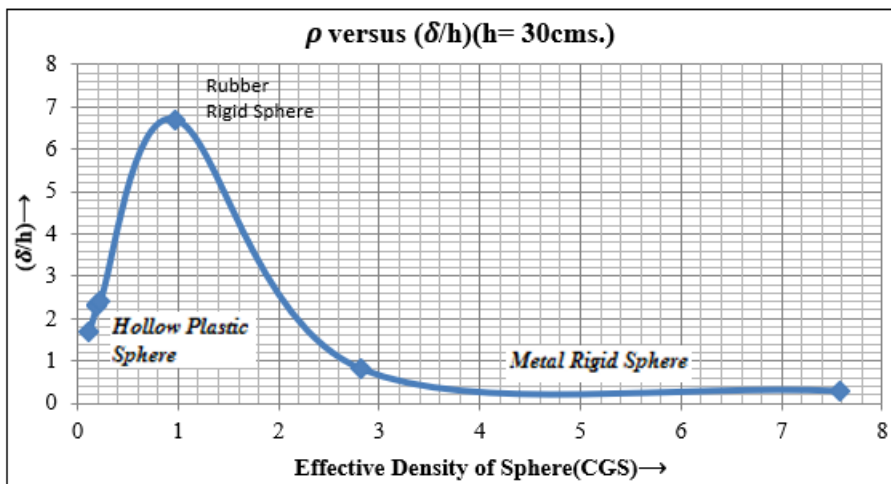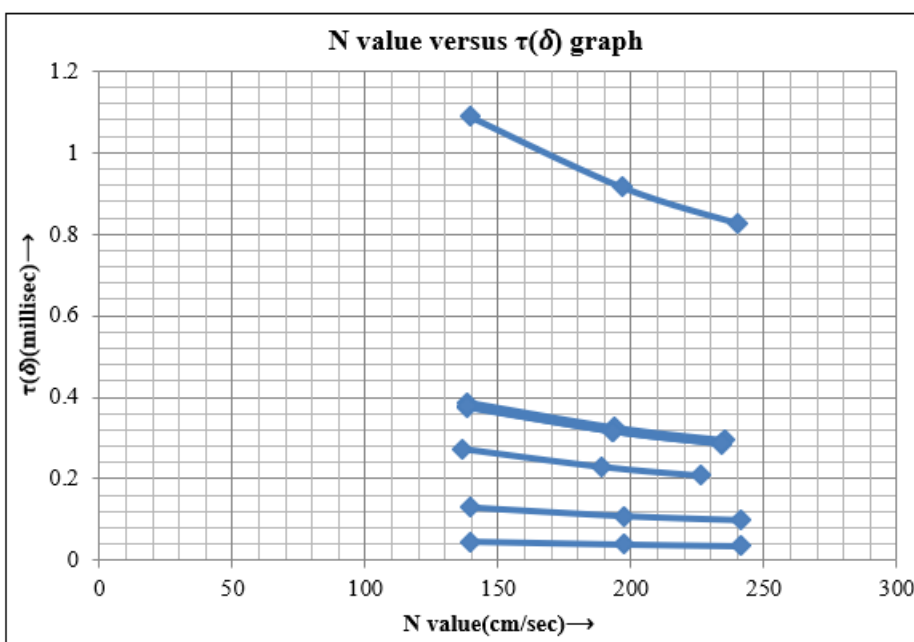
Figure 11(c)
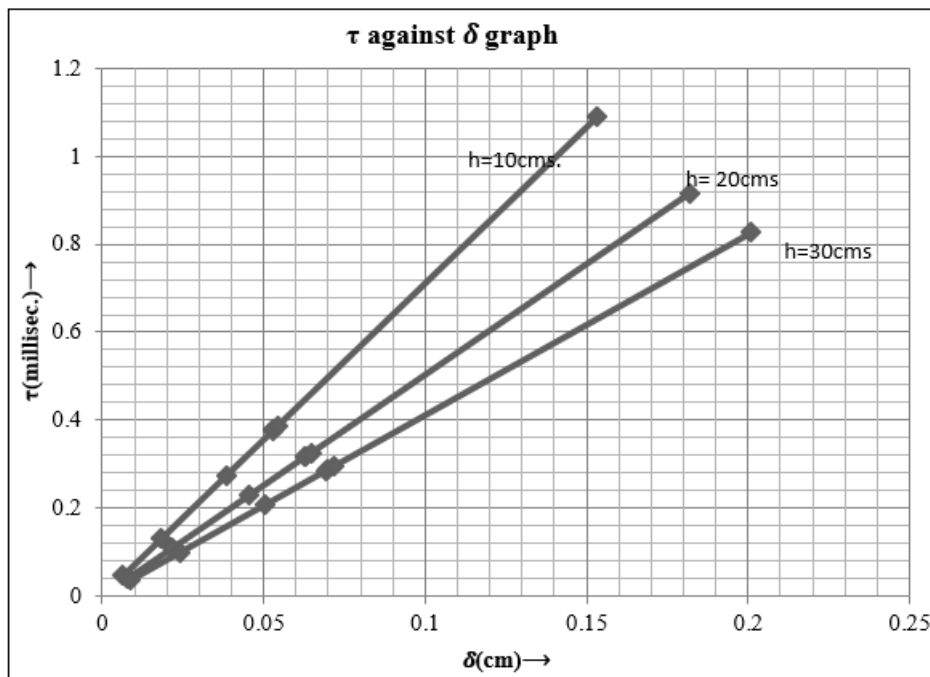

Figure 12

Figure 13

Figure 14

Fig.6, being a purely experimental plot depicts that instantaneous maximum mass $(m^{/})$ as displayed in the weighing-machine digital window decreases with decrease in fall-height and shows also that '$m^{/}$' has also rest-mass-dependence. Fig.7 shows the pattern of variation of $m^{/}$ against (mh) [the product of rest-mass and fall-height] and reveals that $m^{/}$ increases with the increase in (mh) but not straight-linearly. The observed nonlinearity is perhaps due to the nature of the material of the balls and their respective elasticity which are different for different balls. Slopes of the curves in general are found to depend on the physical characteristic and fall-height of the balls. Fig.8, which is a semi-empirical plot between two ratios (δ/h) and (a/A) is seen to be a straight line having a slope approximately equal to '0.513' and with R^2 – value being almost 100%. This certainly indicates that whole experimental procedure and its corresponding theoretical calculations are justifiable to a great extent and can be marked as validated so as to match extensively with the internal structural details and subsequent calibration-parameters. This graphical plot (Fig.9) is a semi-empirical one and expresses the variation of theoretically (model-based) calculated value of maximum depression (δ) against (15) experimentally obtained value of maximum of displayed apparent mass $(m^{/})$. Key-feature of this graph is that it contains section-wise seven graphs corresponding to seven spherical balls while both the axis inherently depends on (mh) and slopes are different owing to different nature of materials the balls are made of with. Variation-pattern of the slopes clearly indicates the differences among metal, hard plastic and rubber showing a maximum for rubber while minimum for metals. In Fig.11(a),(b) and (c) plots of (δ/h) against effective density(ρ) for three different fall-heights have been made for a curious checking how does it manifest where it is surprisingly interesting enough to note that the three curves resemble Planck's curves for black-body radiation without yet no connection with that. The peak of the ratio (δ/h) is for rubber at a middle value towards lower end of effective density while plastic balls have an intermediate value at lowest end of density and metal balls have lowest value of the said ratio at extreme end of density. The nature of the curve may be called semi-Gaussian.

The same ratio is again plotted against N-value in Fig.12 and a family of curves with different rate of variation of slopes are obtained. The curves do fit for power series expansion with negative of power around '2' of N-value. Compatibly with previous graph the rate of variation of slope is maximum for rubber and minimum for metals while intermediate for plastics. Immediately following is the plot of depression-time $(\tau(\delta))$ versus N-value where one gets exactly similar family of curves [Fig.13] as that of the previous one is obtained. Finally, Fig.14 is drawn as a plot of δ versus τ where it is observed that the three graphs for three different chosen fall-heights are perfect straight lines having different slopes for different fall-height. The approximate values of the slopes are 7millisecs/cm, 5.13millisecs/cm and 4.125 millisecs/cm respectively for fall-height $h=$ 10cms., 20 cms. and 30 cms. Fig.13 may be regarded to be a consequence of intermingling among Figs.11 and 12 and partly 7.

2. Brief Discussion

This is basically a paper of elementary standard reporting the manually observed results of a simple experiment concerning the impact of spherical rigid balls and subsequent changes in their apparent mass in term of weight in digital weighing machine. Balls made of different materials, nature and radius and also obviously mass have been chosen to investigate the possibility of finding distinction among the balls on the basis mainly of their nature, material-content as regard to the impact parameters which are essentially the impact-depression and impact-time. As of-course, because it is a very simple experiment and yet to undergo relatively deeper analysis pointing towards moderately serious type of investigation, like other similar experimental observations a preliminary model-

based theoretical frame-work is needed here too which has been briefly discussed in the earlier section of this paper. The theoretical formulations considered here are, though not too rigorous, may be regarded as to interconnect results to sought for observables and parameters approximately satisfactorily as far as science is concerned.

Seven graphical plots and a triplet of similar plots have been made of variables and parameters among which two are purely experimental and the rests are semi-empirical. Presumably complete analysis could have been done with four or five graphical plots and here, instead eight different plots have been considered mainly for two reasons;

1) Firstly it is a simple way of cross-verification of the pattern of co-variation among different observed variables and calculated parameters.
2) Secondly besides being of pedagogic value graph is the most effective means of complete manifestation of essential revelations to be envisaged.

The most prominent direct findings, which are expectation-compatible from theoretical background are as follows:

1) The apparent mass ($m^{/}$) of a spherical ball, as displayed in the weighing machine depends explicitly on both rest-mass and fall-height and implicitly on the material-property including the elasticity of it. Fig.7 gives more information than Fig.6. Fig.6 shows, of-course tallying with data-table [Table-II, col. 1,2 and 10] the normal trend of $m^{/}$ is to be directly proportional to both h and m. The nonlinearity of variation of $m^{/}$against (mh) in Fig.7, on one hand justifies results from Fig.6 while on other hand indicates indirectly its relation to any implicit parameter be it material-nature or other.

2) Both δ (the maximum of after-impact depression) divided by fall-height (h) and the ratio (a/A) [a calibration-index of weighing machine] depends exactly similarly on the bulk-modulus of the material concerned [Fig.8].

3) $m^{/}$, besides its proportionality with δ also depends on something else and that's why the discretely scattered graphs (Fig.9) are not straight lines ones and form small groups with similar slope-variation. There are vividly three groups, one for metals within a range of 0.005cm< δ <0 .025cm and other for plastic within a range of 0.04cm < δ < 0.07cm approximately while the last one for rubber within a range of 0.15cm < δ < 0.20cm.

4) Fig.11, as has been mentioned in the previous section, is drawn to inquire whether the radius of the ball has any relation with the parameter δ. Comparing the graph with data [Table-II] it is observed that the radius has little role to play in the pattern of variation.

5) For getting more accurate value the air-buoyancy is considered and on that basis N [Eqn.10] instead of merely ($\sqrt{2h/g}$) has been calculated and used. If one compares Fig.12 with Fig.13 he or she will get the impression that they are very much similar to each other in nature and the reason behind that similarity is none other than the result one obtains from graphical plot of τ (δ) against δ [Fig.14].

τ is defined to be the depression-time during which the depression after impact reaches its maximum and naturally the time of impact or the impact-time is simply twice τ i,e. $\tau_i = 2\tau_\delta$. τ is found to be linearly proportional to δ the proportionality-constant, the slope of which straight-linear graph does depend on the fall-height being higher for smaller fall-height and the vice-versa. This sort of observation has some deeper meaning and that is the time for reaching the maximum depression is relatively longer for smaller h. This fact is perhaps caused by the strain-dependent stress produced due to commencement of elastic force with the onset of depression or strain and the corresponding potential function is known generally to be much higher order variant of intermolecular or inter-atomic distance.

Thus, the elastic force of restoration increases cumulatively with N resulting in longer time of depression for lower impact-velocity or rather smaller fall-height. In addition to all these it seems to be relevant to mention about the factor $'b_e'$ the corresponding values of which have been found out equating 'T' with 'τ_δ' [Table-I].The values thus found out and enlisted in Table-III could be thought to represent some statistical weight in terms of some atomic packing fraction-equivalent entity.

3. Conclusion

The experiment has been performed with a view to show how interesting can such a simple experiment be as far as the results and the corresponding analysis is concerned and therefore has a pedagogic value for undergraduate students. The impact- time that has been semi-empirically investigated is of the order of milliseconds for rubber ball to microseconds for metal balls and the impact-depression is found to have values of the order of fraction of millimeter to several hundred micrometers. The elementary theoretical framework that has been taken into account in this paper seems to work satisfactorily enough because the results thus obtained are all quite reasonable and likely to be pursued in other similar experimental observations. Elastomechanical function of digital weighing machine in general can be explored and improved further with the help of the findings of this paper. With its help perhaps it can also be ventured to modify for remodeling the manufacturing process so as to produce a versatile force-measuring device in dynamical state of an object with more accuracy.