International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

Dynamic Modeling of Blood Flow and Pressure in the Cardiovascular System

Ashutosh Kumar Upadhyay

School of Computational and Integrative Sciences, Jawaharlal Nehru University, New Delhi 110067, India
Email: *ashuscis7[at]gmail.com*

Abstract: ***The cardiovascular system is a complex network of blood vessels responsible for transporting vital nutrients, oxygen, and hormones throughout the body. In this paper, we presented a mathematical model for simulating blood flow and pressure in the cardiovascular system, integrating principles from fluid dynamics and vessel mechanics. The model incorporates the continuum equation and Navier - Stokes equations to describe fluid behavior, while considering the viscoelastic properties of vessel walls. Through computational simulations, we investigated the effects of varying blood density, vessel wall thickness, and elasticity on blood flow distribution and pressure gradients. Our findings provided insights into the physiological implications of fluid - structure interactions within the cardiovascular system, highlighting the importance of dynamic modeling in cardiovascular research and clinical practice.***

Keywords: Cardiovascular system, Blood flow dynamics, Pressure distribution, Fluid - structure interactions, Dynamic modeling

1. Introduction

The cardiovascular system plays a fundamental role in maintaining homeostasis and ensuring proper organ function by facilitating the circulation of blood throughout the body. Central to this process is the intricate interplay between blood flow dynamics and vessel mechanics within the cardiovascular network [25, 34, 53]. Traditional models have offered valuable insights into cardiovascular physiology; however, they often oversimplify the complexities of blood flow behavior and vessel wall mechanics. Dynamic modeling approaches have emerged as promising tools for capturing the dynamic interplay between fluid dynamics and vessel mechanics, providing a more comprehensive understanding of cardiovascular function [13, 32, 36, 55, 74]. Many models have been shown for simulating blood flow and pressure in the cardiovascular system, leveraging principles from fluid dynamics and vessel mechanics. Blood pressure measurement assesses the force exerted on blood vessel walls and helps determine the rate of blood circulation. Blood vessels form a robust network of tubes that constantly transport blood throughout the body, playing a vital role in the circulatory system's function [4, 16, 46, 65, 73]. There are three main types of blood vessels: arteries, which carry blood away from the heart; capillaries, facilitating the exchange of substances between blood and tissues; and veins, returning blood to heart from capillaries. Aorta is largest artery, transports oxygen - rich blood from heart to the body. Its larger diameter enables a higher blood flow rate compared to other arteries [6, 23, 38, 43, 66]. Constriction of aorta requires heart to exert some force. Research predominantly focuses on pressure exerted on blood vessel walls. Studies have suggested that the narrowing of blood vessels is a contributing factor to high blood pressure development, leading to disruptions in blood flow. Many papers specifically examines high diastolic blood pressure, considering the varying density of blood flow between veins and arteries [14, 42, 50, 63, 70]. It investigates, impact of pressure on vessel walls' thickness and blood flow speed. The study on cardiovascular fluid dynamics and vessel mechanics encompasses a wide range of experimental and computational studies. Early models often relied on simplified assumptions, neglecting the complex interactions between blood flow and vessel walls [18, 44, 67, 72]. Recent advancements have led to the development of dynamic models that account for fluid - structure interactions within the cardiovascular system. These models integrate principles from continuum mechanics, fluid dynamics, and solid mechanics to simulate the behavior of blood flow and vessel deformation under physiological conditions. Key contributions include the incorporation of viscoelastic properties in vessel wall modeling, the investigation of hemodynamic factors influencing cardiovascular health, and the development of patient - specific models for clinical applications [1, 11, 24, 54, 62].

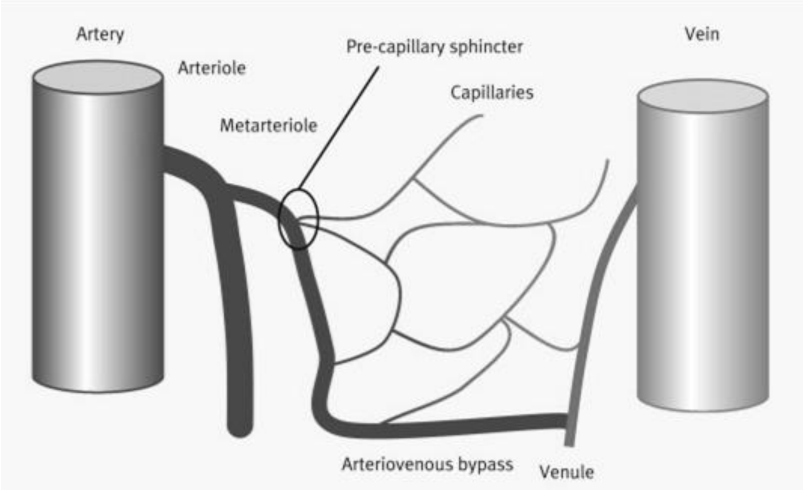

Figure 1: Blood circulation in artery and vein

Theoretical Framework and Formulation of the Problem: Our dynamic model is based on the continuum equation and Navier - Stokes equations, which describe the conservation of mass and momentum for fluid flow, respectively. The model accounts for incompressible Newtonian flow through a network of cylindrical vessels, considering the viscoelastic deformation of vessel walls [8, 9, 35, 56]. The governing equations are solved numerically using finite element methods, allowing for the simulation of blood flow and pressure distribution within the cardiovascular system [Figure (1)]. We introduced a mathematical model to characterize the pressure generated by blood flow within the layers of elastic, varying thickness in arteries, capillaries, and veins [33, 48, 68].

$$\mu\frac{\partial^2 u}{\partial r^2} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{1}$$

$$\bar{u}\frac{\partial^2 w}{\partial r^2} = \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{u}{a} \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial w}{\partial z} = 0 \tag{3}$$

We have introduced a novel parameter termed "elastic fiber. " This variable encapsulates the elastic properties within the structure under consideration [59, 78]. For instance, in the context of blood vessels, elastic fibers would denote the presence of proteins like elastin, which confer elasticity to the vessel walls [27, 51, 76].

$$a = \frac{\bar{u}}{R\,(L,t)} \tag{4}$$

Iinner layers of blood vessels refer to the innermost portions or surfaces of the blood vessel walls. These layers are crucial in determining the dynamics of blood flow within the vessels [2, 15, 26, 45, 47].

$$u = 0, \text{at } r = 0 \tag{5}$$

$$u = 0, \text{at } r = d. \tag{6}$$

When blood flows through blood vessels, it moves smoothly along the vessel walls, adjusting to changes in vessel diameter along its path. This sliding phenomenon ensures that blood flow remains continuous and unimpeded despite variations in vessel size [41, 69, 84].

$$u(r,z) = \frac{\mu}{2\rho}\frac{\partial p}{\partial z}\left(-r^2 + d^2\right) \tag{7}$$

When we replaced the concept of blood sliding smoothly through blood vessels while accounting for differences in vessel diameter, as described in equation (7), into the Navier - Stokes equations outlined in equation (2), we obtained a set of equations [22, 49, 75].

$$\frac{\partial^2 w}{\partial r^2} = \frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}\left(2 - \frac{r^2\mu}{a} + \frac{d^2\mu}{a}\right) \tag{8}$$

This condition implies that the blood adheres to the vessel walls and does not slip or move relative to them [5, 39, 61].

$$w = 0, \text{at } (r = 0) \tag{9}$$

$$w = 0, \text{at } (r = d) \tag{10}$$

Now process involves incorporating the constraints imposed by the condition into the mathematical framework, resulting in an expression that describes how the velocity of blood flow varies across radial dimension of vessel [28, 60, 82].

$$w(r,z) = \frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}\left(r^2 - \frac{\mu r^4}{12a} + \frac{d^2\mu r^2}{2a} - hr + \frac{\mu h^3 r}{12a} - \frac{d^2\mu r h}{2a}\right) \tag{11}$$

When we inserted equation (10) into equation (4), we derived what is known as the dynamic continuity equation. This process involves integrating the expression derived from equation (10) into the broader framework outlined in equation (4) [3, 12, 30, 79].

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial}{\partial z}\left(\frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}\left(r^2 - \frac{\mu r^4}{12a} + \frac{d^2\mu r^2}{2a} - hr + \frac{\mu h^3 r}{12a} - \frac{d^2\mu r h}{2a}\right)\right) \tag{12}$$

This condition describes how the velocity of blood flow, represented by v (r, z), is influenced or constrained at the boundaries or surfaces of the vessel layers. In simpler terms, it outlines how the flow of blood is affected by factors such as vessel geometry and the surrounding environment at these specific points within the blood vessel structure [7, 19, 31, 37].

$$\frac{\partial h}{\partial t} = -\frac{1}{2\rho\bar{u}}\frac{\partial^2 p}{\partial z^2}\left(\frac{L^3}{3} - \frac{\mu L^5}{60a} + \frac{d^2\mu L^3}{6a} - \frac{hL^2}{2} + \frac{\mu h^3 L^3}{24a} - \frac{d^2\mu L^2 h}{4a}\right) \tag{13}$$

By incorporating a non - dimensional variable and associated parameters, we are introducing a standardized way of representing certain aspects of the system under study [21, 54].

$$\bar{L} = \frac{L}{d}\ \bar{a} = \frac{a\beta}{d^2}\ \bar{h} = \frac{h}{d}\ \beta = \frac{R}{H}\ \bar{P} = -\frac{ph^2}{\mu h\frac{\partial h}{\partial t}}\ \bar{Z} = \frac{z}{L} \tag{14}$$

This non - dimensional allows for easier comparison and analysis across different scales [64, 83].

$$\frac{\rho\bar{u}H^2}{\mu} = \frac{\partial^2\bar{p}}{\partial\bar{z}^2}\delta\,(\mu, L) \tag{15}$$

$$\delta(\mu, L) = \frac{L^3}{2} - \frac{\mu\Re\bar{L}^5}{62\bar{a}} + \frac{\mu\Re\bar{L}^3}{64\bar{a}} - \frac{\bar{h}\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^5}{34\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{6\bar{a}} \tag{16}$$

This condition outlines how pressure varies within the vessel walls, considering factors such as vessel geometry and the surrounding environment.

$$\bar{p} = 0, \text{at } z = 1 \tag{17}$$

$$\frac{d\bar{p}}{d\bar{z}} = 0, \text{at } z = 0 \tag{18}$$

$$\bar{p} = \frac{2z_0\rho\bar{u}H^2}{\mu\left[\frac{L^3}{3} - \frac{\mu\Re\bar{L}^5}{60\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}} - \frac{\bar{h}\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^5}{24\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}\right]} \tag{19}$$

Hemodynamics refers to the study of how blood flows through the blood vessels in the body. The speed or rate at which blood moves through these vessels is affected by some factors, like blood pressure, vascular resistance and viscosity.

$$Q_x = \frac{L^3}{t}\int_0^L V(r, z) * dz \tag{20}$$

$$V(r, z) = \frac{1}{2\bar{u}\mu L}\frac{dp}{dz}(Z^z - zL) + \frac{d\mu z}{Lh} \tag{21}$$

When we insert equation (20) into equation (21), we derive a new expression or equation. This process involves replacing the variables or terms in equation (21) with the corresponding terms from equation (20). By doing so, we obtain a modified equation that incorporates the information or relationships described in both equations (21) and (20).

$$Q_x = \frac{L^3}{t}\int_0^L \frac{1}{2\bar{u}\mu L}\frac{dp}{dz}(Z^z - zL) + \frac{d\mu z}{Lh} \tag{22}$$

By substituting equation (22) into equation (21), we arrive at a dimensionless flow rate,

$$\bar{Q}_x = \frac{tQ_x}{L^3} \tag{23}$$

$$\bar{Q}_x = \int_0^z \frac{\partial}{\partial z}\left(\frac{pL^3}{2\bar{u}\mu L^2}\right)\left(\bar{Z}^z - \bar{Z}L\right)d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}}d\bar{z} \tag{24}$$

$$\bar{Q}_x = \frac{1}{2}\int_0^z \frac{\partial\bar{p}}{\partial z}\left(\bar{Z}^z - \bar{Z}L\right)d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}}d\bar{z} \tag{25}$$

To derive the dimensionless pressure (p) from equation (26), we need to take the derivative of the equation with respect to the appropriate variables involved.

$$\frac{\partial\bar{p}}{\partial\bar{z}} = \frac{2\rho\bar{u}H^2}{\left[\left(\frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{60\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}}\right) - \left(\frac{\bar{h}^2\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^5}{24\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}\right)\right]} \tag{26}$$

When we substitute equation (26) into equation (25), we obtain a new expression that incorporates the information provided by both equations.

$$\bar{Q}_x = \frac{\rho\bar{u}H^2}{\left[\left(\frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{12\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}}\right) - \left(\frac{\bar{h}^2\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^5}{12\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}\right)\right]}\int_0^d\left(\bar{Z}^z - \bar{Z}L\right)d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}}d\bar{z} \quad (27)$$

$$\bar{Q}_x = \frac{\rho\bar{u}H^2}{\left[\left(\frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{12\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}}\right) - \left(\frac{\bar{h}^2\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^5}{12\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}\right)\right]}\left[\left(\frac{d^3}{3} - \frac{d^2}{2}L\right) + \frac{d^3\mu}{2\bar{h}}\right] \quad (28)$$

2. Results and Discussion

Computational simulations are conducted to investigate the effects of varying blood density, vessel wall thickness, and elasticity on blood flow dynamics and pressure gradients. The relationship between blood vessel pressure and the velocity of blood pumped by the heart is fundamental to understanding the dynamics of circulation within the human body. This relationship is intricately influenced by the cross - sectional area of different types of blood vessels, including the aorta, arteries, veins, and capillaries. The aorta is a largest artery responsible for carrying oxygen with blood from heart and exhibits high blood flow velocity, resulting in elevated pressure within artery. However, as blood traverses through arteries into veins, its velocity gradually decreases, leading to a reduction in pressure. This decrease in velocity and pressure is attributed to the increased total cross - sectional area of the vascular system as blood moves away from the heart. Notably, during the systole, or contraction phase of the heart, blood velocity in arteries is higher compared to diastole, the relaxation phase, reflecting the dynamic nature of blood flow throughout the cardiac cycle [54, 81]. Moreover, Figure 3 emphasizes significant pressure differences between veins and capillaries, particularly evident when comparing blood velocity across their cross - sectional areas.

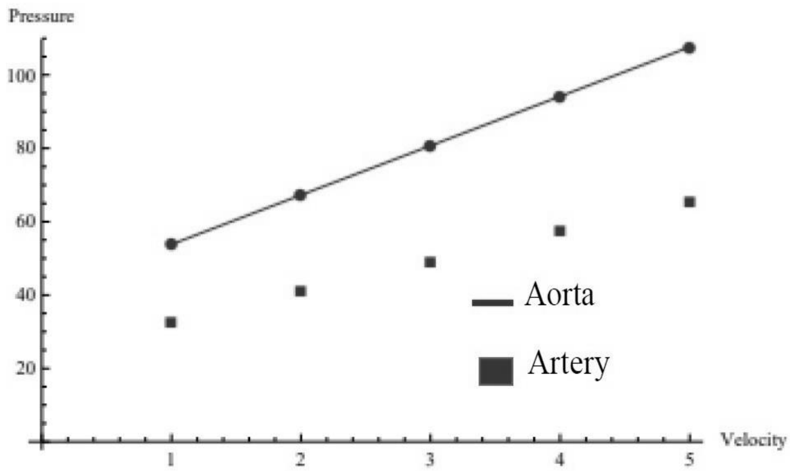

Figure 2: Relationship between blood pressure and the velocity of blood in aorta and artery

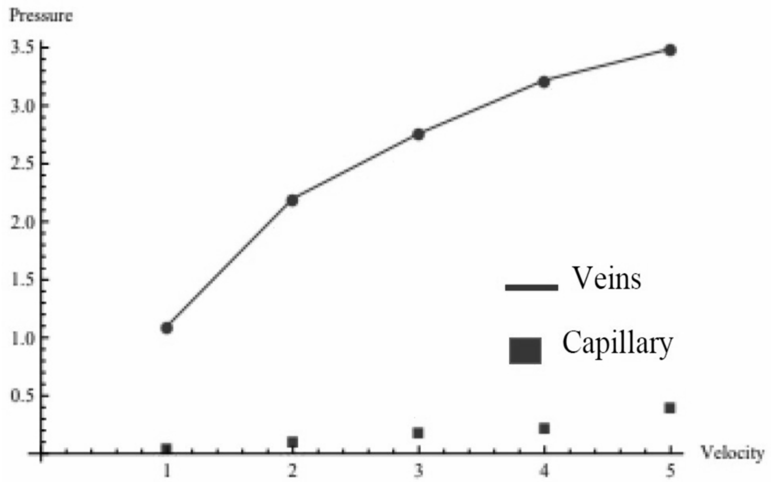

Figure 3: Relationship between blood pressure and the velocity of blood in veins and capillary

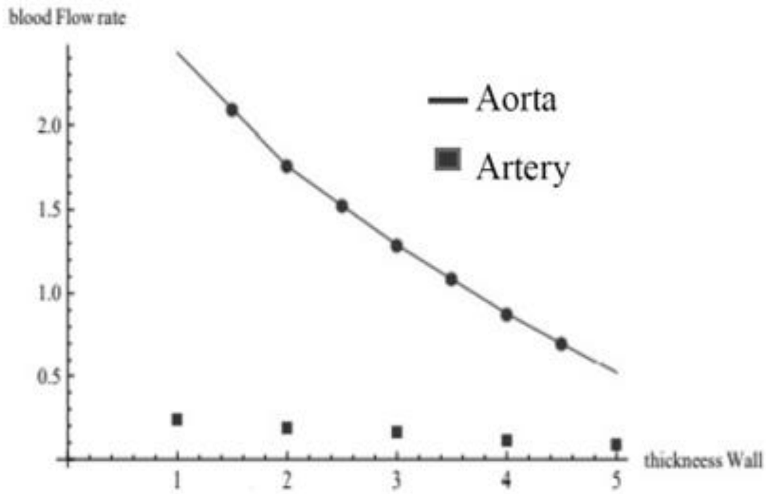

Figure 4: Relationship between blood flow rate and thickness of wall in aorta and artery

In capillaries, where crucial oxygen and nutrient exchange with tissues occur, pressure dramatically decreases due to their smaller diameter, facilitating efficient gas and nutrient exchange. Overall, these observations underscore the complex interplay between blood vessel pressure, flow velocity, and cross - sectional area, highlighting the intricate mechanisms that govern circulatory dynamics within the human body [80, 84]. Figure (2) and (3) illustrates the relationship between blood vessel pressure and the velocity of blood pumped by the heart. This relationship is influenced by the cross - sectional area of each type of blood vessel. Notably, during systole (contraction phase of the heart), blood velocity in arteries is higher compared to diastole (relaxation phase). Figure 3 highlights significant pressure differences between veins and capillaries, particularly evident when comparing blood velocity across their cross - sectional areas. Pressure decreases dramatically in capillaries due to their smaller diameter [17, 58, 77]. The dynamics of blood flow within the circulatory system is crucial for comprehending various physiological processes and their implications for health. One important observation in this regard is the relationship between the thickness of blood vessel walls and the rate of blood flow. It has been observed that blood vessels with thicker walls, such as the aorta, tend to have higher rates of blood flow [10, 29, 40]. This suggests that the structural integrity provided by thicker walls may contribute to more efficient propulsion of blood throughout the body. Conversely, blood vessels with thinner walls, like veins and capillaries, typically exhibit lower rates of blood flow. These observations highlight the intricate interplay between vessel morphology and hemodynamic parameters, underscoring the importance of vessel thickness in regulating blood flow dynamics. This understanding serves as a foundation for elucidating circulatory physiology and its role in maintaining homeostasis [20, 57, 71]. In Figure (4) and (5), we observed the relationship for rate of flow and the thickness of vessels. Volume of blood flowing vessels varies according to vessel thickness. With thicker walls, as seen in the aorta, blood flow rate is higher. Conversely, veins and capillaries, characterized by thinner walls, have lower flow rates

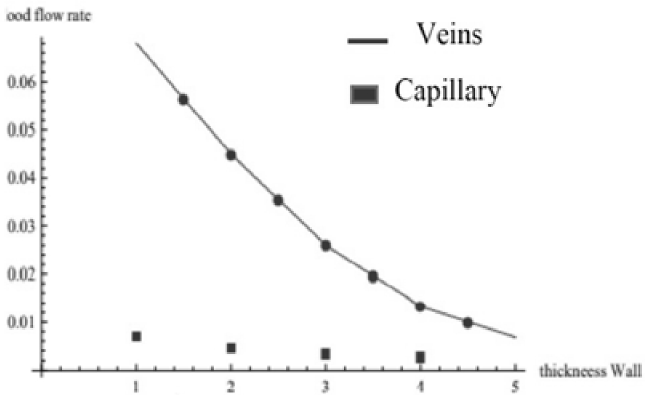

Figure 5: Relationship between blood flow rate and thickness of wall in vein and capillary

3. Conclusion

In this paper, we have explored the intricate relationship between blood vessel pressure, blood flow velocity, and vessel morphology within the context of circulatory dynamics. By developing a mathematical model, we have elucidated how the pressure exerted by the heart varies across different types of blood vessels, with the aorta exhibiting particularly high pressures due to its role in carrying oxygenated blood from the heart. Our analysis has underscored the crucial role of vessel diameter in regulating blood flow rates, with arterial blood flow generally being faster compared to venous and capillary flow. Additionally, we have highlighted the impact of vessel wall thickness on blood flow, observing higher flow rates in vessels with thicker walls, such as the aorta, and lower rates in vessels with thinner walls, like veins and capillaries. These findings deepen our understanding of circulatory physiology and provide insights into the mechanisms underlying cardiovascular health and disease. Through visual representations, we have visually depicted the relationship between blood flow rate and vessel thickness, further enhancing our comprehension of circulatory dynamics. This study contributed to the body of knowledge surrounding blood flow dynamics, paving the way for future research aimed at improving our understanding of circulatory function and its clinical implications.

References

[1] Akbar, S., Shah, S. R., "DURYSTA" the first biodegradable sustained release implant for the treatment of open - angle glaucoma, International Journal of Frontiers in Biology and Pharmacy Research, 01 (02), 1 - 7, (2021). https: //doi. org/10.53294/ijfbpr.2021.1.2.0042

[2] Akbar, S., Shah, S. R., "Mathematical Study for the Outflow of Aqueous Humor and Function in the Eye", International Journal of Scientific & Engineering Research, 11, 10, 743 - 750, (2020).

[3] Alshehri, Mo., Sharma, S. K., Gupta, P., Shah, S. R., "Detection and Diagnosis of Learning Disabilities in Children of Saudi Arabia with Artificial Intelligence", Research Square, 1 - 22, (2023). https: //doi. org/10.21203/rs.3. rs - 3301949/v1.

[4] Anamika, Shah, S. R., "Mathematical and Computational study of blood flow through diseased artery", International Journal of Computer Science, 5, (6), 1 - 6, (2017).

[5] Anamika, Singh A., Shah, S. R., "Mathematical Modelling Of Blood Flow through Three Layered Stenosed Artery", International Journal for Research in Applied Science and Engineering Technology, 5, (6), 1 - 6, (2017).

[6] Anamika, Singh A., Shah, S. R., "Mathematical Modelling of blood flow through tapered stenosed artery with the suspension of nanoparticles using Jeffrey fluid model", International journal of development research, 7 (6), 13494 - 13500, (2017).

[7] Anamika, Singh, A., Shah, S. R., "Bio - Computational analysis of blood flow through two phase artery", International Journal of Engineering Science and Computing, 7, (6), 13397 - 213401, (2017).

[8] Chaturvedi, P., Shah, S. R., "Assessing the Clinical Outcomes of Voxelotor Treatment in Patients with Sickle Cell Disease", International Journal of Applied Science and Biotechnology, 12 (1), 46 - 53, (2024). 10.3126/ijasbt. v12i1.64057.

[9] Chaturvedi, P., Shah, S. R., "Mathematical Analysis for the Flow of Sickle Red Blood Cells in Microvessels for Bio Medical Application, Yale Journal of Biology and Medicine, 96 (1), 13 - 21, (2023). 10.59249/ATVG1290.

[10] Cheturvedi, P., Kumar, R., Shah, S. R., "Bio - Mechanical and Bio - Rheological Aspects of Sickle Red Cells in Microcirculation: A Mathematical Modelling Approach, Fluids, 6, 322, 01 - 15, (2021). https: //doi. org/10.3390/fluids6090322.

[11] Geeta, Siddiqui S. U., Sapna, "Mathematical Modelling of blood flow through catheterized artery under the influence of body acceleration with slip velocity", Application and applied Mathematics An international journal, 8 (2), 481 - 494, (2013). digitalcommons. pvamu. edu: aam - 1333.

[12] Geeta, Siddiqui S. U., Shah, S. R. "A Biomechanical approach to the effect of body acceleration through stenotic artery", Applied Mathematics and

Computation, 109 (1), 27 - 41, (2015). https: //doi. org/10.1016/j. amc.2015.03.082.

[13] Geeta, Siddiqui S. U., Shah, S. R., "A Mathematical Model for two layered pulsatile blood flow through stenosed arteries", E - Journal of Science and Technology, 109 (11), 27 - 41, (2015).

[14] Geeta, Siddiqui S. U., Shah, S. R., "Effect of body acceleration and slip velocity on the pulsatile flow of casson fluid through stenosed artery", Advance in applied science research, 5 (3), 231 - 225, (2014).

[15] Geeta, Siddiqui S. U., Shah, S. R., "A Computational Analysis of a Two - Fluid non - Linear Mathematical model of pulsatile blood flow through Constricted Artery", E - Journal of science and Technology, 10 (4), 65 - 78, (2015).

[16] Gupta, P., Alshehri, Mo., Sharma, S. K., Shah, S. R., "Empowering the visually impaired: Translating Handwritten Digits into Spoken Language with HRNN - GOA and Haralick Features", J. of Disability Research, 3, 1 - 21, (2024).

[17] Kumar, P., Shah, S. R., "A Hydromechanical Perspective to Study the Effect of Body Acceleration through Stenosed Artery", International journal of mathematical engineering and management sciences, 6 (5), 1381 - 1390, (2021). 10.33889/IJMEMS.2021.6.5.083.

[18] Kumar, R., Chaturvedi, P., Akbar, S., Shah, S. R., "Prospective of Hydroxychloroquine and Zinc with Azithromycin for Nanoparticles Blood Flow in Covid - 19 Patients, International Journal of Nanotechnology in Medicine & Engineering, 6 (1), 01 - 07, (2021).

[19] Kumar, R., Shah, S. R., "A mathematical approach to study the blood flow through tapered stenosed artery with the suspension of nanoparticles" Destech Transactions on Engineering and Technology Research, 01, 1 - 6, (2017). 10.12783/dtetr/amsm2017/14809

[20] Kumar, R., Shah, S. R., "Mathematical Modeling of Blood Flow with the Suspension of Nanoparticles Through a Tapered Artery With a Blood Clot", Frontiers in Nanotechnology, 2, 596475, 1 - 5, (2020). https: //doi. org/10.3389/fnano.2020.596475.

[21] Kumar, R., Shah, S. R., "Performance of blood flow with suspension of nanoparticles though tapered stenosed artery for jeffrey fluid model" International Journal of Nanoscience, 17, 6, 1850004 (1 - 7), (2018). 10.1142/S0219581X18500047.

[22] Kumar, V., Shah, S. R., "A Mathematical study for heat transfer phenomenological processes in human skin", International Journal of Mechanical Engineering, 7 (6), 683 - 692, (2022).

[23] Kumar, V., Shah, S. R., "Mathematical modelling to study the heat transfer between core and skin", SRMS, Journal of Mathematical Sciences, 7, 7 - 12, (2024). https: //doi. org/10.29218/srmsmaths. v7i2.02 .

[24] Kumar, V., Shah, S. R., "Thermobiological Mathematical Model for the study of temperature response after cooling effects", SSRG International Journal of Applied physics, 9 (2), 2022, 7 - 11. doi. org/10.14445/23500301/IJAP - V9I2P102.

[25] Kumar., V., Shah, S. R., "A mathematical approach to investigate the temperature distribution on skin surface with sinusoidal heat flux condition, Int. J. of Multidisciplinary Research and Development, 9 (5), 2022, 141 - 146.

[26] Lenin, J. S., Shah, S. R., "Mathematical Analysis of Stem Cell Dynamics in Acute Myeloid Leukemia: Towards Precision Medicine Strategies", International Journal of Science and Research, 13 (05), 528 - 535, (2024).

[27] Malik, M. Z., Kumar, R., Shah, S. R., "Effects of (Un) lockdown on COVID - 19 transmission: A mathematical study of different phases in India, medRxiv The preprint server for health science, 1 - 13, (2020), **doi:** https: //doi. org/10.1101/2020.08.19.20177840.

[28] Sadique, Mo., Shah, S. R., "Mathematical model to study the effect of PRG4, hyaluronic acid and lubricin on squeeze film characteristics of diseased synovial joint", International Journal of Mechanical Engineering, 7 (6), 832 - 848, (2022). https: //www.kalaharijournals. com/resources/JUNE - 94. pdf.

[29] Sadique, Mo., Shah, S. R., "Mathematical model to study the study the squeeze film characteristics of synovial joints in diseased human knee joint", World Scientific Annual Review of Biomechanics, 1 (2330004) 1 - 21, (2023). https: //doi. org/10.1142/S2810958923300044.

[30] Sadique, Mo., Shah, S. R., "Mathematical study for the synovial fluid flow in Osteoarthritic knee joint", Journal of Engineering and Applied Sciences, 17 (2), 15 - 21, (2022). https: //medwelljournals. com/abstract/?doi=jeasci.2022.15.21.

[31] Sardar, M. N. Islam, Shabab, A., Shah, S. R., "Aqueous Humor Flow in the Schlemm's Canal via Rectangular Channel: Mathematical Modeling for Analytical Results, Numerical Experiments, and Implications for Treatments (Chapter) " Advances on Mathematical Modeling and Optimization with Its Applications, Edition 1st Edition, First Published, Imprint CRC Press, Pages 10, eBook ISBN 9781003387459, (2024). DOI: 10.1201/9781003387459 - 5.

[32] Sardar, S. K., Sharma, M. N. Islam, Sadique, Mo., Shah, S. R., "Effect of Significant Parameters on Squeeze Film Characteristics in Pathological Synovial Joints", Mathematics (MDPI), 11 (1468) 1 - 23, (2023). https: //doi. org/10.3390/math11061468.

[33] Shabab A., Shah, S. R., "Mathematical Analysis of Stem Cell Dynamics in Acute Myeloid Leukemia: Towards Precision Medicine Strategies", Journal of Current Medical Research and Opinion, 07 (04), 2216 - 2225, (2024).

[34] Shabab, A., Shah, S. R., "Mathematical Modeling of Blood Flow Dynamics in the Cardiovascular System: Assumptions, Considerations, and Simulation Results", Journal of Current Medical Research and Opinion, 7 (4), 2216 - 2225, (2024).

[35] Shabab, A., Shah, S. R., "The Effects of Prostaglandin Analogs on Intraocular Pressure (IOP) in Human Eye for Open Angle Glaucoma. Int. J. of Innovative

Technology and Exploring Engineering, 10 (2), 176 - 180, (2020).

[36] Shabab, A., Shah, S. R., Mohammed Alshehri, Sharma, S. K., Gupta, P., “A Mathematical Study for Promoting Disability Inclusion in Glaucoma: A Comprehensive Approach”, Journal of Disability Research, 3, 1 - 12, (2024).

[37] Shah, S. R., “A biomechanical approach for the study of deformation of red cells in narrow capillaries”, IJE: Transaction A: Basics, 25 (4), 303 - 313, (2012).

[38] Shah, S. R., “A case study of non - Newtonian viscosity of blood through artherosclerotic artery”, Asian Journal of Engineering and Applied Technology, 1 (1), 47 - 52, (2012).

[39] Shah, S. R., “A Mathematical Model for the analysis of blood flow through diseased blood vessels under the influence of porous parameter”, Journal of Biosciences and Technology, 4 (6), 534 - 541, (2013).

[40] Shah, S. R., “A mathematical study of blood flow through radially non - symmetric multiple stenosed arteries under the influence of magnetic field”, Int. J. of Advanced Research in Biological Sciences, 2 (12), 379 - 386, (2015)

[41] Shah, S. R., “A mathematical study of blood flow through stenosed artery”, International Journal of Universal Science and Engineering, 1 (1), 26 - 37, (2015).

[42] Shah, S. R., “A study of blood flow through multiple atherosclerotic arteries”, International Journal for Mathematics, 1, (12), 1 - 6, (2015).

[43] Shah, S. R., “A study of effects of magnetic field on modified Power - law fluid in modeled stenosed artery” Journal of Bioscience and Technology, 1 (4), 187 - 196, (2010).

[44] Shah, S. R., “An innovative solution for the problem of blood flow through stenosed artery using generalized bingham plastic fluid model”, Int. J. of research in applied and natural social sciences, 1 (3), 97 - 140, (2013).

[45] Shah, S. R., “An innovative study for non - Newtonian behavior of blood flow in stenosed artery using Herschel - Bulkely flud”, Int. J. of biosiences and biotechnology, 5 (5), 233 - 240, (2013). 10.14257/ijbsbt.2013.5.5.24.

[46] Shah, S. R., “Capillary - tissue diffusion phenomena for blood flow through a stenosed artery using herschel - bulkley fluid” International journal of research in Biochemistry and Biophysics, 1 (1), 1 - 8 (2011).

[47] Shah, S. R., “Effect of clopidogrel on blood flow through stenosed artery under diseased condition”, International Journal of Experimental Pharmacology, 4 (1), 887 - 893, (2014).

[48] Shah, S. R., “Effects of Acetylsalicylic Acid on blood flow through an artery under Atherosclerotic condition”, International Journal of Molecular medicine and advances sciences, 7 (6), 19 - 24, (2011). 10.3923/ijmmas.2011.19.24.

[49] Shah, S. R., “Effects of antiplatelet drugs on blood flow through stenosed blood vessels”, Journal of Biomimetics, Biomaterials and Tissue Engineering, 18, 21 - 27, (2013). https: //doi. org/10.4028/www.scientific. net/JBBTE.18.21.

[50] Shah, S. R., “Impact of radially non - symmetric multiple stenoses on blood flow through an artery”, International Journal of Physical and Social Sciences, 1 (3), 1 - 16, (2011).

[51] Shah, S. R., “Mathematical analysis of blood flow through atherosclerotic arterial segment having non - symmetric mild stenosis”. International Journal of Research in Pure and Applied Physics, 1, 1 - 5, (2011).

[52] Shah, S. R., “Mathematical Study of Blood Flow through Atherosclerotic Artery in the Presence of Porous Effect”, International Journal of Modern Sciences and Engineering Technology, 2, (12), 12 - 20, (2015).

[53] Shah, S. R., “Non - Newtonian flow of blood through an atherosclerotic artery”, Research journal of applied sciences.6 (1), 76 - 80, (2011). 10.3923/rjasci.2011.76.80.

[54] Shah, S. R., “Performance modeling and analysis of magnetic field on nutritional transport capillary tissue system using modified Herschel - Bulkely fluid”, Int. J. of Advanced research in physical sciences, 1 (1), 33 - 41, (2014).

[55] Shah, S. R., “Performance Study on Capillary - Tissue Diffusion Phenomena for Blood Flow through Stenosed Blood Vessels”, American journal of pharmatech research, 2 (2), 695 - 705, (2012).

[56] Shah, S. R., “Response of *blood flow through* an atherosclerotic *artery* in the presence of *magnetic field* using Bingham plastic fluid” International Journal of Pharmaceutical and Biomedical Research, 2 (3), 96 - 106, (2011).

[57] Shah, S. R., “Role of Non - Newtonian behavior in blood flow through normal and stenosed artery”, Research journal of Biological sciences, 6 (9), 453 - 458, (2011).

[58] Shah, S. R., “Significance of Aspirin on Blood Flow to Prevent Blood Clotting through Inclined Multi - Stenosed Artery”, Letters In Health and Biological Sciences, 2 (2), 97 - 100, (2017). DOI: 10.15436/2475 - 6245.17.018.

[59] Shah, S. R., “Study of dispersion of drug in blood flow with the impact of chemical reaction through stenosed artery”, International journal of Biosciences, 21 (3), 21 - 29, (2022). http: //dx. doi. org/10.12692/ijb/21.3.199 - 208.

[60] Shah, S. R., “Study of modified Casson’s fluid model in modeled normal and stenotic capillary - tissue diffusion phenomena” International journal of computational engineering & management, 11, 51 - 57, (2011).

[61] Kumar, R., Shah, S. R., “Study of blood flow with suspension of nanoparticles through tapered stenosed artery”, Global Journal of Pure and Applied Mathematics, 13 (10), 7387 - 7399, (2017).

[62] Siddiqui S. U., Shah, S. R., "A Physiologic Model for the problem of blood flow through Diseases blood vessels", International journal of advances in Applied Sciences, 5 (2), 58 - 64, (2016). DOI: 10.11591/ijaas. v5. i2. pp58 - 64.

[63] Shah, S. R., “A biomechanical approach for the study of Two - phase blood flow through stenosed artery”, International Journal of research studies in biosciences, 1 (2), 24 - 32, (2013).

[64] Shah, S. R., "Clinical influence of hydroxychloroquine with azithromycin on blood flow through blood vessels for the prevention and Treatment of covid - 19", Int. J. of Bio., pharmacy and allied science, 10 (7), 2195 - 2204, (2021).

[65] Siddique, S. U., Shah, S. R., "Achievement of Pentoxifylline for Blood Flow through Stenosed Artery", Journal of Biomimetics, Biomaterials and Tissue Engineering, 13 81 - 89, (2012). https: //doi. org/10.4028/www.scientific. net/JBBTE.13.81.

[66] Siddiqui S. U., Sapna, Km., "Herschel - Bulkely fluid model for stenosis shape aspects of blood flow through an artery", Ultra Science, International journal of physical sciences, 18 (3), 407 - 416, (2006).

[67] Siddiqui, S. U., Shah, S. R., "A Comparative Study for the Non - Newtonian Behaviour of Blood Flow through Atherosclerotic Arterial Segment", Int. J. of Pharmaceutical Sci. Review and Research, 9 (2), 120 - 125, (2011).

[68] Siddiqui, S. U., Shah, S. R., "Two - phase model for the study of blood flow through stenosed artery", International Journal of Pharmacy and Biological Sciences, 1 (3), 246 - 254, (2011).

[69] Singh, A., Shah, S. R., Siddiqui S. U., "Effects of inclined multi - stenoses arteries on blood flow characteristics using bingham plastic fluid", International Journal for Mathematics, 1, (12), 7 - 14, (2015).

[70] Singh, A., Shah, S. R., Siddiqui S. U., "Mathematical Modeling and Numerical Simulation of Blood Flow through Tapered Artery", International Journal of Innovative Science, Engineering & Technology, 3, (2), 710 - 717, (2016). https: //ijiset. com/vol3/v3s2/IJISET_V3_I2_97. pdf.

[71] Singh, A., Shah, S. R., Siddiqui S. U., "Mathematical Modelling and Analysis of Blood Flow through Diseased Blood Vessels", International Journal of Engineering and Management Research, 5, (6), 366 - 372, (2015). https: //doi. org/10.3389/fnano.2020.596475.

[72] Singh, A., Shah, S. R., Siddiqui S. U., "Performance of blood flow through two phase stenosed artery using Herschel - Bulkley model", Int. J. of Applied And Pure Science and Agriculture, 2, (2), 228 - 240, (2016).

[73] Singh, A., Siddiqui S. U., Shah, S. R., "A Mathematical Model to study the similarities of blood fluid models through inclined multi - stenosed artery", International Journal of Engineering Research and Modern Eduacation, 2, (1), 108 - 115, (2017). https: //core. ac. uk/download/pdf/144783771. pdf.

[74] Singh, A., Siddiqui S. U., Shah, S. R., "Mathematical Modeling of peristaltic blood flow through a vertical blood vessel using prandtl fluid model", Int. J. of Mathematics and Computer Research, 4, (9), 710 - 717, (2016).

[75] Singh, S., "A two - layered model for the analysis of arterial rheology" International Journal of Computer Science and Information Technology, 4, 37 - 42. (2011).

[76] Singh, S., "Analysis of non - newtonian fluid flow in a stenosed artery", International journal of physical sciences, 4 (11), 663 - 671, (2009). https: //academicjournals. org/article/article1380630965_Sapna. pdf

[77] Singh, S., "Clinical significance of aspirin on blood flow through stenotic blood vessels" Journal of Biomimetics, Biomaterials and Tissue Engineering, 10 (17) 24 - 35, (2011). https: //doi. org/10.4028/www.scientific. net/JBBTE.10.17.

[78] Singh, S., "Effects of shape of stenosis on arterial rheology under the influence of applied magnetic field" Int. J. of Biomedical Engineering and Technology, 6 (3), 286 - 294, (2011). DOI: 10.1504/IJBET.2011.041466.

[79] Singh, S., "Influence of magnetic field on blood flow through stenosed artery using casson's fluid model", International Journal of BioEngineering, CardioPulmonary Sciences and Technology, 1, 1 - 7, (2010).

[80] Singh, S., "Numerical modeling of two - layered micropolar fluid through a normal and stenosed artery", International journal Engineering, 24 (2), 177 - 187, (2011). DOI: 10.5829/idosi. ije.2012.25.04a.02.

[81] Singh, S., "Numerical modelling for the modified Power - law fluid in stenotic capillary - tissue diffusion phenomena", Archives of Applied Science Resaerch, *An* Int. *peer reviewed* J. *of* Appl. Sci., 2 (1), 104 - 112, (2010).

[82] Singh, S., "The effect of Saline Water on viscosity of blood through stenosed blood vessels using Casson's fluid model", Journal of Biomimetics, Biomaterials and Tissue Engineering, 9, 37 - 45, (2011). https: //doi. org/10.4028/www.scientific. net/JBBTE.9.37.

[83] Singh, S., and Shah, R. R., "A numerical model for the effect of stenosis shape on blood flow through an artery using power - law fluid", Advance in applied science research, *An* Int. *peer reviewed* J. *of* Sci., 1, 66 - 73, (2010).

[84] Thomas, S., Kumar, R., Shah, S. R., "Understanding the impact of feedback regulations on blood cell production and leukemia dynamics using model analysis and simulation of clinically relevant scenarios", Applied Mathematical Modelling, (2024). https: //doi. org/10.1016/j. apm.2024.01.048.