Temperature Distribution in Peripheral Region of Human Body with Uniformly Perfused Tumor: Finite Element Method based Study

Rajesh Singh

Department of Mathematics, JLN Govt. Degree College Haripur, (Manali) Distt. Kullu (H. P.) India
Email: *ranawat_rajesh[at]rediffmail.com*

Abstract: ***A mathematical study of temperature distribution in dermal part of human body with uniformly perfused tumor has been carried out. The region under study is divided into five layers in which epidermis and subcutaneous tissues parts contain one layer each and dermis region is divided into three layers where tumor exists. It is assumed that outer skin layer is exposed to the atmosphere and has heat loosed and gain accordingly. The study incorporates effect of metabolic heat generation and blood mass flow rate. The normal and benign tissues are assumed to have normal rates of blood mass flow and self controlled metabolic heat generation and the malignant portion incorporates increased rates of blood mass flow and uncontrolled metabolic heat generation.***

Keywords: Blood Mass Flow Rate, Metabolic Heat Generation, Finite Element Method, Normal and Benign Tissues

1. Introduction

The outer layers of human body (skin and subcutaneous tissues (SST)) play an important role to maintain body core temperature constant irrespective of the atmospheric temperature. The SST region mainly contains three layers; epidermis, dermis and subcutaneous tissues (Figure - 1). The epidermis contains no blood vessels whereas the density of blood vessels increases toward body core and it is uniformly distributed in subcutaneous tissues. The rate of blood mass flow, metabolic heat generation and sensible and insensible perspiration play an important role to maintain body core temperature which vary in response to change in atmospheric temperature. The body core temperature influences the thermoregulation in human dermal part. A broad integrated part influencing the thermoregulation in human dermal part has been shown in Figure 2 [7]. Chao and Yang [3] found solution for steady state and unsteady state case with all the parameters as constant.

Cooper and Trezek [2] obtained an analytic solution for brain tissues with negligible effect of blood mass flow rate and metabolic heat generation. Saxena [4, 6] obtained an analytic solution to one dimensional problem using position dependent values of blood mass flow rate and metabolic heat generation. Saxena and Arya [5] applied finite element method to solve the problem in three layers. Lateron Pardasani and Saxena [8] solve the problems involving malignant tumor. Pardasani and Shakya [9] studied thermal variation in infinite element domain of human peripheral region due to tumor. Khanday and Saxena [11, 12] have investigated mathematical estimation of cold effect in human dermal regions. They also studied one dimensional mathematical estimation of human physiological disturbance in human dermal parts at obtained extreme condition. Saxena and Gurung [13] obtained transient temperature distribution in human dermal parts with protective layer at low atmospheric temperature.

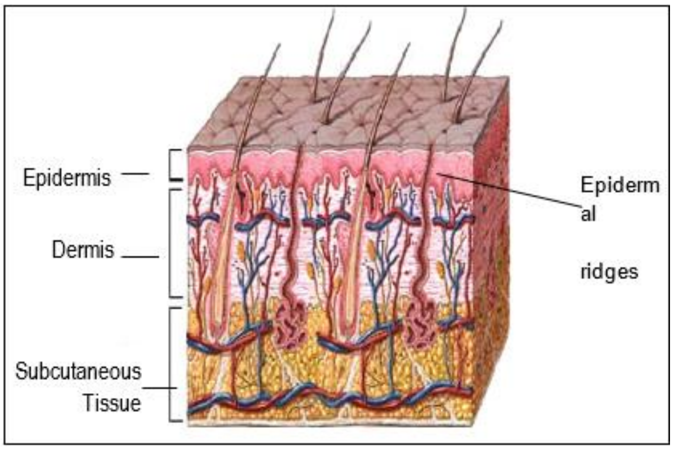

Figure 1: Skin and Subcutaneous tissue

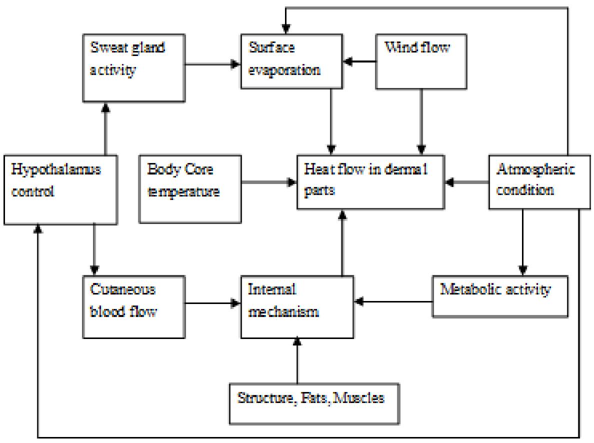

Figure 2: Effects of various factors on heat flow in dermal part.

In this paper we consider uniformly perfused tumor in dermis which is assumed to have higher rate of blood mass flow and metabolic heat generation. The tumor region is divided into three layers having necrotic region in tumor core which has less blood vessels and active regions on both side of tumor core having higher rate of blood mass flow and metabolic heat generation (Figure - 3).

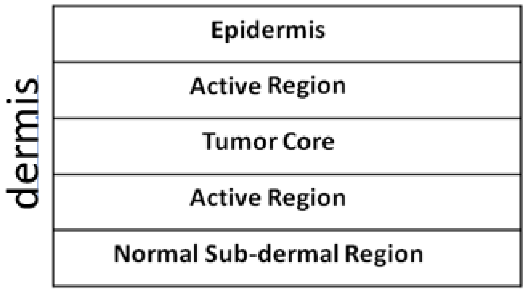

Figure 3: Skin and Subcutaneous tissue (SST) region with malignant tumor in dermis.

2. Mathematical Model

Perl [1] gave a mathematical model of heat and mass distribution in tissues. He combined the Fick's perfusion principle and Fick's second law of diffusion along with heat generation term to deduce the model. The partial differential equation derived by him is given by:

$$\rho c \,{\partial T}/{\partial t} = div(K grad T) + m_b c_b (T_A - T) + S \quad (1)$$

where

ρ, c, T, t, K, m_b, c_b, T_A and S are tissue density, specific heat of the tissue, unknown temperature, time, thermal conductivity, blood mass flow rate, specific heat of blood, arterial blood temperature (assumed to be equal to body core temperature) and rate of metabolic heat generation. The heat loss at skin surface is given by

$$-K \,{\partial T}/{\partial n} = h(T - T_a) + LE \quad (2)$$

The inner boundary is assumed to be at core temperature given by

$$T = T_b = 37^0 C \quad (3)$$

where

${\partial T}/{\partial n}$, h, T_a, L and E are derivative of T along normal to the boundary, heat transfer coefficient, atmospheric temperature, latent heat of evaporation and rate of evaporation.

We consider $M(M = m_b c_b)$ and S negligible in epidermis and constant throughout in subcutaneous tissues and linear function of the depths in various positions in dermis as blood mass flow rate and metabolic heat generation depend on density of blood vessels at different depth.

3. Solution to the Problem

In case of tumor equation (1) can be written for one dimensional steady state case as

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) + M(T_A - T) + \bar{S} = 0 \quad (4)$$

where

$\bar{S} = S + W$; S and W are self controlled and uncontrolled metabolic heat generation. Here we consider $M(M = m_b c_b)$ and $\bar{S}$ very small in necrotic region, three time in active region.

The partial differential equation (4) along with boundary conditions (2) is transformed into following equivalent Euler - Lagrange variational form is given by

$$I = \frac{1}{2}\int_0^{x_5}\left[K\left(\frac{dT}{dx}\right)^2 + M(T_A - T)^2 - 2\bar{S}T\right]dx + \lambda\left[\frac{1}{2}h(T - T_a)^2 + LET\right] \quad (5)$$

where

$$\lambda = \begin{cases} 1 & for \quad x = x_0 = 0 \\ 0 & otherwise \end{cases}$$

We can write I for five layers as

$$I = \sum_{i=1}^{5} I_i$$

and

$$I_i = \frac{1}{2}\int_{x_{i-1}}^{x_i}\left[K_i\left(\frac{dT^i}{dx}\right)^2 + M_i\left(T_A^{(i)} - T^{(i)}\right)^2 - 2\bar{S}_i T^{(i)}\right]dx + \lambda\left[\frac{1}{2}h(T_0 - T_a)^2 + LET_0\right]_{x=0} \quad (6)$$

Here $x_0 = 0$. The expression not included in the integral is for $i = 1$ at $x = 0$. The different values of I for epidermis, dermis and subcutaneous tissues can be written as follow.

$$I_1 = \frac{1}{2}\int_{x_0}^{x_1}\left[K_1\left(\frac{dT^{(1)}}{dx}\right)^2 + M_1\left(T_A^{(1)} - T^{(1)}\right)^2 - 2\bar{S}_1 T^{(1)}\right]dx + \lambda\left[\frac{1}{2}h(T_0 - T_a)^2 + LET_0\right]_{x=0} \quad (7)$$

$$I_{i(i=2,3,4)} = \frac{1}{2}\int_{x_{i-1}}^{x_i}\left[K_i\left(\frac{dT^i}{dx}\right)^2 + M_i\left(T_A^{(i)} - T^{(i)}\right)^2 - 2\bar{S}_i T^{(i)}\right]dx \quad (8)$$

$$I_5 = \frac{1}{2}\int_{x_4}^{x_5}\left[K_5\left(\frac{dT^5}{dx}\right)^2 + M_5\left(T_A^{(5)} - T^{(5)}\right)^2 - 2\bar{S}_5 T^{(5)}\right]dx \quad (9)$$

We describe the complete layer wise formulation as given below.

i. For epidermis $(x_0 < x < x_1)$

$$T^{(1)} = T_0 + \frac{(T_1 - T_0)}{x_1} x, \quad K = K_1 \text{ (constant)},$$

$$M = M_1 = 0, \quad \overline{S} = \overline{S_1} = 0$$

ii. At interface - I $(x = x_1)$

$$T^{(1)} = T^{(2)} = T_1, \quad K_1 = K_2, \quad M_1 = M_2, \quad \overline{S_1} = \overline{S_2},$$

$$K_1 \frac{\partial T^{(1)}}{\partial x} = K_2 \frac{\partial T^{(2)}}{\partial x}$$

i. For dermis $(x_{i-1} < x < x_i)$ $i = 2, 3, 4,$

$$T^{(i)} = \frac{T_{i-1}x_i - T_i x_{i-1}}{x_i - x_{i-1}} + \frac{(T_i - T_{i-1})}{x_i - x_{i-1}} x;$$

$K = K_i$ (Constant with different values)

$$M = M_i = \left(\frac{x - x_{i-1}}{x_i - x_{i-1}} \right) m, \quad \overline{S} = \overline{S_i} = \left(\frac{x - x_{i-1}}{x_i - x_{i-1}} \right) s,$$

$$T_A = T_A^{(i)} = \left(\frac{x - x_{i-1}}{x_i - x_{i-1}} \right) T_b$$

ii. At interface - II $(x = x_4)$

$$T^{(4)} = T^{(5)} = T_4, \quad K_4 = K_5, \quad M_4 = M_5, \quad \overline{S_4} = \overline{S_5},$$

$$T_A = T_b, \quad K_4 \frac{\partial T^{(4)}}{\partial x} = K_5 \frac{\partial T^{(5)}}{\partial x}$$

iii. For subcutaneous tissues $(x_4 < x < x_5)$

iv. $$T^{(5)} = \frac{T_4 x_5 - T_5 x_4}{x_5 - x_4} + \frac{(T_5 - T_4)}{x_5 - x_4} x;$$

$K = K_5$ (constant), $\overline{S} = \overline{S_5} = s$, $M = M_5 = m$ (constant),

$$T_A = T_A^{(5)} = T_b = 37^0 C$$

v. Inner boundary $(x = x_5)$

$$T_5 = T_b = 37^0 C, \quad K = K_5, \quad M = m, \quad \overline{S} = s, \quad T_A = T_b$$

Now using above mentioned physical and physiological parameters equations (7), (8) and (9) becomes

$$I_1 = A_1 + B_1 T_0 + D_1 T_0^2 + E_1 T_1^2 + F_1 T_0 T_1 \quad (10)$$

$$I_{i(i=2,3,4)} = A_i + B_i T_{i-1} + C_i T_i + D_i T_{i-1}^2 + E_i T_i^2 + F_i T_{i-1} T_i \quad (11)$$

$$I_5 = A_5 + B_5 T_4 + C_5 T_5 + D_5 T_4^2 + E_5 T_5^2 + F_5 T_4 T_5 \quad (12)$$

where

A_1, B_1, D_1, E_1, F_1, A_i, B_i, C_i, D_i, E_i, F_i and A_5, B_5, C_5, D_5, E_5, F_5 are defined in Appendix - I

Now optimizing I, we differentiate it with respect to each nodal temperature and equate them to zero so that

$$\frac{\partial I}{\partial T_i} = 0 \quad (i = 0, 1, 2, 3, 4,)$$

and

$$T_5 = T_b = 37^0 C$$

We get following system of equations.

$$2P_2 T_0 + F_1 T_1 = -P_1 \quad (13)$$

$$F_1 T_0 + 2P_4 T_1 + F_2 T_2 = -P_3 \quad (14)$$

$$F_2 T_1 + 2P_6 T_2 + F_3 T_3 = -P_5 \quad (15)$$

$$F_3 T_2 + 2P_8 T_3 + F_4 T_4 = -P_7 \quad (16)$$

$$F_4 T_3 + 2P_{10} T_4 = -F_5 T_5 - P_9 \quad (17)$$

and P_i ($i =$ 1, 2, 3, …….10) are given in Appendix - I

The above system of equations from equation (13) to equation (17) can be written as

$$AT = B \quad (18)$$

where

$$A = \begin{bmatrix} 2P_2 & F_1 & 0 & 0 & 0 \\ F_1 & 2P_4 & F_2 & 0 & 0 \\ 0 & F_2 & 2P_6 & F_3 & 0 \\ 0 & 0 & F_3 & 2P_8 & F_4 \\ 0 & 0 & 0 & F_4 & 2P_{10} \end{bmatrix}, \quad T = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} -P_1 \\ -P_3 \\ -P_5 \\ -P_7 \\ -F_5 T_5 - P_9 \end{bmatrix}$$

The system of equation (18) is solved by using MATLAB to give nodal temperatures.

4. Numerical Results

The solution of equation (18) is obtained by using the following values of physical constants [2, 6, 8].

K_1 = 0.030 cal/cm - min^0; K_i =0.0845 cal/cm - min^{0}C ($i =$ 2, 4,), K_3 =0.030 cal/cm - min^{0}C, K_5 =0.060 cal/cm - min^{0}C, h = 0.009 cal/cm^2 - min^{0}C, L = 579 cal/g.

At different temperature M and $\overline{S}$ are zero in epidermis whereas for tumor in dermis M and $\overline{S}(=W)$ are three times in active region and half in necrotic region and for subcutaneous tissues respective values are to be taken as given below.

Paper ID: SR24611172613 DOI: https://dx.doi.org/10.21275/SR24611172613

The numerical computations have been made for following three cases of atmospheric temperature with the respective values of m, s and E [2, 6, 8].

1) T_a = 15^{0}C; s = 0.0357 cal/cm^3 - min; m = 0.003 cal/cm^3 - min^{0}C; E = 0 gm/cm^2 - min.
2) T_a = 23^{0}C; s = 0.018 cal/cm^3 - min; m = 0.018 cal/cm^3 - min^{0}C; E = 0, 0.24 ×10^{-3}, 0.48×10^{-3} gm/cm^2 - min.
3) T_a = 33^{0}C; s = 0.018 cal/cm^3 - min; m = 0.0315 cal/cm^3 - min^{0}C; E = 0.24 ×10^{-3}, 0.48×10^{-3}, 0.72×10^{-3} gm/cm^2 - min.

The values of element thickness used here are given below
x_0 = 0 cm, x_1 = 0.1 cm, x_2 = 0.3 cm, x_3 = 0.5 cm, x_4 = 0.7 cm, x_5 = 1.1 cm.

5. Conclusion

Graphs (Fig.4, 5 and 6) are drawn to show the effect of uniformly perfused tumor on temperature profiles for different cases. Figure - 4 depicts the results for low atmospheric temperature (15^{0}C) and having no significant evaporation from the surface. Figure - 5 shows the temperature variation at modest atmospheric temperature (23^{0}C) with zero and small sweat evaporation. The third and last case considers higher atmospheric temperature (33^{0}C) and accordingly higher sweat evaporation. The impact of atmospheric temperature can be visualized in the graphs and it is clear that at same evaporation rate the temperature variation in the tumor region is higher at low atmospheric temperature as compare to high atmospheric temperature.

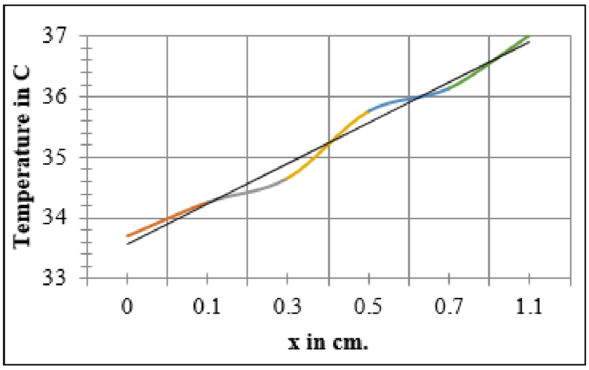

Figure 4: Graph between temperature and position for T_a = 15^{0}C and E=0 gm/cm^2 - min.

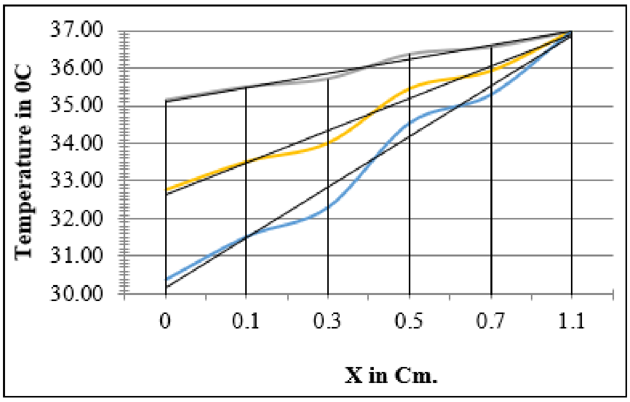

Figure 5: Graph between temperature and position for T_a = 23 ^{0}C and E=0, 0.24×10^{-3}, 0.48×10^{-3} gm/cm^2

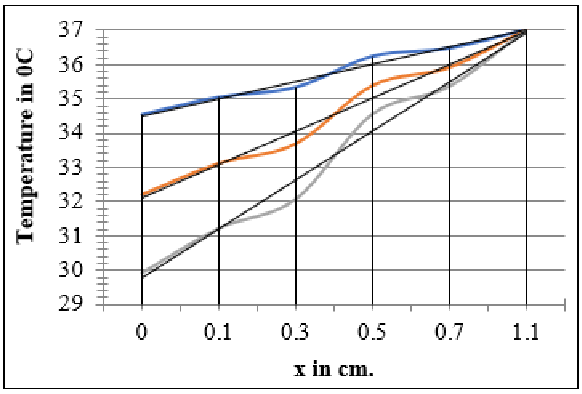

Figure 6: Graph between temperature and position for T_a= 33 ^{0}C and E= 0.24×10^{-3}, 0.48×10^{-3}, 0.72×10^{-3} gm/c

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Appendix - I

$$A_1 = \frac{1}{2}\left(hT_a^2\right),\quad B_1 = LE - hT_a\quad D_1 = \frac{1}{2}\left(\frac{K_1}{x_1} + h\right),\quad E_1 = \frac{K_1}{2x_1},\quad F_1 = -\frac{K_1}{x_1}$$

$$B_i = \frac{mT_b}{Y^2}\left[L_1 - x_i L_2\right] + \frac{s}{Y}\left[L_3 - x_i L_4\right]\quad C_i = \frac{mT_b}{Y^2}\left[x_{i-1}L_2 - L_1\right] + \frac{s}{Y}\left[x_{i-1}L_4 - L_3\right]$$

$$D_i = \frac{K_2}{2X} + \frac{m}{2Y}\left[M_1 + x_i^2 M_2 - 2x_i M_3\right]\quad E_i = \frac{K_2}{2X} + \frac{m}{2Y}\left[M_1 + x_{i-1}^2 M_2 - 2x_{i-1} M_3\right]$$

$$F_i = -\frac{K_2}{X} - \frac{m}{Y}\left[M_1 + x_{i-1}x_i M_2 - (x_i + x_{i-1})M_3\right]$$

For i = 2, 3, 4.

where

$$L_1 = \frac{X^3}{4} + \frac{x_1^2 X}{2} - \frac{2}{3}x_1 X^2,\quad L_2 = \frac{X^2}{3} + x_1^2 - x_1 X,\quad L_3 = \frac{X^2}{3} - \frac{x_1 X}{2},\quad L_4 = \frac{X}{2} - x_1,\quad M_1 = \frac{X^2}{4} - \frac{2x_1 X}{3},$$

$$M_2 = \frac{1}{2} - \frac{x_1}{X},\quad M_3 = \frac{X}{3} - \frac{x_1}{2}\quad X = (x_i - x_{i-1})$$

and $Y = (x_8 - x_1)$, $A_5 = \dfrac{mT_b^2 X_1}{2}$ $B_5 = -\left[\dfrac{mT_b Y_1}{2} + \dfrac{sY_1}{2}\right]$, $C_5 = \left[\dfrac{mT_b(3x_4 - x_5)}{2} + \dfrac{s(3x_4 - x_5)}{2}\right]$

$$D_5 = \frac{K_5}{2X_1} + m\left[\frac{X_1}{6} + \frac{x_5^2}{2X_1} - \frac{x_5}{2}\right],\quad E_5 = \frac{K_5}{2X_1} + m\left[\frac{X_1}{6} + \frac{x_5^2}{2X_1} - \frac{x_5}{2}\right],\quad D_5 = -\frac{K_5}{X_1} - m\left[\frac{X_1}{3} + \frac{x_4 x_5}{X_1} - \frac{Y_1}{2}\right],$$

$$X_1 = (x_5 - x_4),\ Y_1 = (x_4 + x_5),\ P_1 = B_1,\ P_2 = D_1,\ P_3 = B_2,\ P_4 = (E_1 + D_2)\ P_5 = (C_2 + B_3),\ P_6 = (E_2 + D_3),$$

$$P_7 = (C_3 + B_4),\ P_8 = (E_3 + D_4),\ P_9 = (C_4 + B_5),\ P_{10} = (E_4 + D_5).$$