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Economic Order Quantity Model for Deteriorating Items under Inflation with Time Dependent Demand and Partial Backlogging

Dr. Biswaranjan Mandal

Associate Professor of Mathematics, Acharya Jagadish Chandra Bose College, Kolkata, West Bengal, India
Email: *drbrmajcbc[at]gmail.com*

Abstract: ***One of the basic assumptions in the derivation of the classical Economic Order Quantity (EOQ) formula was that all the costs associated with the inventory system remain constant over time. Most of the inventory models developed so far do not include inflation. Today, inflation has become an unavoidable feature of the economy of almost all countries of the world and so several attempts have been made by researchers to reformulate the optimal inventory management policies taking into account inflation. The first attempt in this direction was by Buzacott [1]. Immediately, many other researchers have tried to extend Buzacott's approach to several other interesting situations taking into account the different inflation rates for the internal and external costs, infinite and finite replenishment rates, with or without shortages etc. The present paper deals with a finite time-horizon inventory replenishment model with time dependent demand for items deteriorating at a constant rate. The effects of inflation are incorporated along with shortages which are assumed to be partially backlogged. The results obtained are illustrated with numerical examples and a sensitivity analysis of the optimal solution with respect to some important parameters of the system have been presented.***

Keywords: EOQ, deteriorating, inflation, shortages and partial backlogging

Subject Classification: AMS Classification no. 90B05

1. Introduction

In inventory management system, the effect of deterioration is very important. Deterioration is defined as decay, change or spoilage that prevents the items from being used for its original purpose. Food items, drugs, pharmaceuticals and radioactive substances are few examples of items in which appreciable deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the model. Many authors have considered EOQ model for such decaying items. Ghare and Schrader [2] first analyzed the decaying inventory problem. Among the most recent investigations in this field, the works done by Mak [3], Raafat [4], Mandal and Pal [5], Chang et al [6], Biswaranjan [7] are noteworthy.

In classical inventory models, researchers have considered the demand rate to be constant over the entire time horizon which is generally assumed to be infinite. Dave and Patel [8] first considered the inventory model for deteriorating items with time-varying demand. Sachan [9] extended Dave and Patel's model allowing shortages. Later many research works in this field are developed by Goyal [10], Hariga [11], Biswajit et al [12], Biswaranjan [13], Yonit [14] and many more researchers developed EOQ models considering time-varying demand and deterioration.

Most of the classical inventory models assume that all relevant costs associated with the inventory systems remain constant over time. Perhaps low inflation in the economy of the Western Countries prior to the 1970's was the main cause of this approach. But during the last 30-40 years, the economic situation has changed rapidly and the annual inflation rate in most of the countries shot up to be in the range of 7% to 20% or even more. Today, inflation plays an important role in the economic field throughout the World. Buzacott [1] developed an approach of modelling such a situation by assuming a constant inflation rate. Biermann and Thomas [15] discussed an inventory model taking both inflationary trends and time-discounting. In this context, investigations of researchers like Chao-Ton-Su et al [16], Yang et al [17], Shaikh et al [18], Biswaranjan [19] are mentioned a few.

Existing literature on inventory problems reveal that investigations have been carried out by researchers either neglecting shortages altogether or assuming shortages which are completely backlogged. In this connection, mention may be made of the works done by Dave [20], Joaquín Sicilia et al [21] to name only a few. Later Chang et al [22], Wu KS et al [23], Biswaranjan [24, 25] have discussed an inventory model assuming partial backlogging. In practical situation, customers may like to wait for the backlogging period (shortages period), but there may be some who would not. Consequently, the opportunity cost due to lost sales must be taken into consideration. In the present paper, the backlogging rate has been assumed to be fixed fraction of the demand rate during the shortage period. However, in some inventory system, for items such as fashionable commodities, the length of the waiting time for the next replenishment become a prerequisite for customer whether backlogging would be accepted. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment.

In the present paper, we derive a finite time-horizon EOQ model for inventory of items that deteriorate at a constant rate assuming the time-varying demand and shortages which are partially backlogged. We also consider a constant rate of inflation for various costs associated with the inventory system. Finally, some numerical examples are given to illustrate the proposed model along with a sensitivity study.

2. Nomenclature

The present inventory model has been investigated under the following assumptions and notations:

Assumptions:
1) Replenishment rate is infinite.
2) Lead time is zero.
3) The demand rate f (t) is a function of time t.
4) The time horizon H is divided into 'm' equal parts, each of length T, so that T=H/m.
5) Shortages are allowed in all cycles.
6) The inflation rate is taken to be constant.
7) During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Hence the proportion of customers who would like to accept backlogging at time t decreases as the waiting time for the next replenishment (t_j – t) increases.

For this situation, the backlogging rate is defined as $\dfrac{1}{1+\alpha(t_j-t)}$, where the backlogging parameter α is positive, $S_j \le t \le t_j$, j = 1, 2, ……….m.

Notations:
A: Ordering cost of inventory per order,
C_1 : Holding cost per unit per unit time,
C_2: Shortage cost per unit per unit time,
C_3: Opportunity cost due to lost sales per unit per unit time,
P: Purchase cost per order,
θ : A constant fraction of the on-hand inventory deteriorates per unit time.
There is no repair or replacement of the deteriorated inventory during H,
k: Constant rate of inflation,
t_j: Time at which the jth replenishment is made, j = 0, 1, 2, …….m
S_j: Time at which shortages start during the jth cycle, j = 1, 2, ….m
I (t): The inventory level at time t (>0).

3. Mathematical Formulation

According to the assumptions and notations mentioned above, it is cleared that, the amount of inventory is depleted by the combined effect of the demand and deterioration during the interval $[t_{j-1}, S_j]$ of the jth replenishment cycle. In addition, the depletion of inventory occurs due to the demand backlogged during the interval [S_j, t_j].

Therefore, the instantaneous state of inventory can be described by the following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t), t_{j-1} \le t \le S_j \tag{1}$$

and

$$\frac{dI(t)}{dt} = -\frac{f(t)}{1+\alpha(t_j-t)}, S_j \le t \le t_j \text{, j = 1, 2, …..m} \tag{2}$$

The total replenishment cost during the entire time horizon is given by the following

$$C_R = A\sum_{j=0}^{m}\exp(kt_j) \tag{3}$$

The total purchasing cost in (0, H) is

$$C_P = \sum_{j=1}^{m} P_{j-1} \tag{4}$$

where P_{j-1} = the present worth of the purchasing cost at time t_{j-1} for the period [t_{j-1}, S_j] and at time t_j for the period [S_j, t_j], (j =1, 2, …….m)

$$= p\exp(kt_{j-1})\int_{t_{j-1}}^{S_j}\exp(\theta t)f(t)dt + p\exp(kt_j)\int_{S_j}^{t_j}\frac{f(t)}{1+\alpha(t_j-t)}dt \tag{5}$$

The inventory holding cost during the entire time horizon H is

$$C_H = \sum_{j=1}^{m} H_{j-1} \tag{6}$$

Where H_{j-1} the present worth of the holding cost over the period [t_{j-1}, S_j], j=1, 2, …..…m

$$= C_1\int_{t_{j-1}}^{S_j}\frac{\exp(-\theta t_{j-1})-\exp(-\theta t)}{\theta}\exp(kt)\exp(\theta t)f(t)dt \tag{7}$$

The total shortage cost during the period (0, H) is given by

$$C_S = \sum_{j=1}^{m} G_{j-1} \tag{8}$$

Where G_{j-1} = the present worth of the shortage cost during the period [S_j, t_j]

$$= C_2\int_{S_j}^{t_j}(t_j-t)\exp(kt)\frac{f(t)}{1+\alpha(t_j-t)}dt \text{, j = 1, 2, ….m} \tag{9}$$

The total opportunity cost due to lost sale over the entire time horizon H is given by

$$C_O = \sum_{j=1}^{m} B_{j-1} \tag{10}$$

where B_{j-1} = the present worth of the opportunity cost due to lost sales during [S_j , t_j], (j = 1, 2, ….m)

$$= C_3\int_{S_j}^{t_j}\{1-\frac{1}{1+\alpha(t_j-t)}\}\exp(kt)f(t)dt \text{, j = 1, 2, ...m}$$

$$= C_3\alpha\int_{S_j}^{t_j}(t_j-t)\exp(kt)\frac{f(t)}{1+\alpha(t_j-t)}dt \text{, j = 1, 2, …m} \tag{11}$$

Therefore, the total cost of the inventory system over the entire time horizon H is the following

$$TC = C_R + C_P + C_H + C_S + C_O \quad (12)$$

Now substituting the values for C_R, C_P, C_H, C_S and C_O from relations (3), (4), (6), (8) and (10); using (5), (7), (9) and (11), the following expression for TC is obtained

$$TC = A\sum_{j=0}^{m}\exp(kt_j) + p\sum_{j=1}^{m}\exp(kt_{j-1})\int_{t_{j-1}}^{S_j}\exp(\theta t)f(t)dt + p\sum_{j=1}^{m}\exp(kt_j)\int_{S_j}^{t_j}\frac{f(t)}{1+\alpha(t_j-t)}dt +$$
$$C_1\sum_{j=1}^{m}\int_{t_{j-1}}^{S_j}\frac{\exp(-\theta t_{j-1})-\exp(-\theta t)}{\theta}\exp(kt)\exp(\theta t)f(t)dt + (C_2+C_3\alpha)\sum_{j=1}^{m}\int_{S_j}^{t_j}(t_j-t)\exp(kt)\frac{f(t)}{1+\alpha(t_j-t)}dt \quad (13)$$

We have to determine the values of m and S_j (j = 1, 2, …..m) which minimize the total cost of the inventory system TC.

4. Solution Procedure

For a given value of m, the t_j are given by

$$t_j = j\,H/m, \; j = 0, 1, 2, \ldots\ldots m \quad (14)$$

because of the assumption of the equal replenishment cycles.

Moreover, for a fixed value of m, the necessary condition for TC to be minimum are

$$\frac{\partial TC}{\partial S_j} = 0, \; j = 0, 1, 2, \ldots m$$

Which gives

$$C_1\frac{\exp(-\theta t_{j-1})-\exp(-\theta S_j)}{\theta}\{1+\alpha(t_j-S_j)\}\exp\{(\theta+k)S_j\} - p\exp(kt_j)$$
$$+ p\{1+\alpha(t_j-S_j)\}\exp(kt_{j-1}+\theta S_j) - (C_2+C_3\alpha)(t_j-S_j)\exp(kS_j) = 0 \quad j = 1, 2, \ldots m \quad (15)$$

Particular Cases

If shortages are fully backlogged, i.e. $\alpha = 0$, the equations (15) are transformed to

$$C_1\frac{\exp(-\theta t_{j-1})-\exp(-\theta S_j)}{\theta}\exp\{(\theta+k)S_j\} - p\exp(kt_j) + p\exp(kt_{j-1}+\theta S_j) - C_2(t_j-S_j)\exp(kS_j) = 0, \; j = 1, 2, \ldots m \quad (16)$$

Which are the necessary conditions for the optimality of S_j in the situation of complete backlogging.

For absence of deterioration i.e. $\theta = 0$, the equations (15) are transformed to

$$C_1(S_j - t_{j-1})\{1+\alpha(t_j-S_j)\}\exp(kS_j) - p\exp(kt_j) +$$
$$p\{1+\alpha(t_j-S_j)\}\exp(kt_j) - (C_2+C_3\alpha)(t_j-S_j)\exp(kS_j) = 0$$
$$j = 1, 2, \ldots m \quad (17)$$

Which are the necessary conditions for the optimality of S_j in the situation of absence of deterioration.

If r_j represents the fraction of the jth replenishment interval for which there is no shortage. Then S_j can be expressed as

$$S_j = t_{j-1} + r_j\,(t_j - t_{j-1}), \quad j = 1, 2, \ldots m \quad (18)$$

Therefore, after substituting the expressions of t_j and Sj, given from (14) and (18), in the equations (15), we get the following

$$\frac{C_1}{\theta}\{1+\alpha(1-r_j)H/m\}\exp\{k(r_j+j-1)H/m\}\{\exp(\theta H/m)r_j - 1\}$$
$$+ p\{1+\alpha(1-r_j)H/m\}\exp(-kr_jH/m) - \exp\{(\theta+k)(r_j+j-1H/m\} - p\exp(kjH/m)$$
$$-(C_2+C_3\alpha)(1-r_j)H/m\exp\{k(r_j+j-1)H/m\} = 0, \; j = 1, 2, \ldots m \quad (19)$$

For a given positive integer m, the above equations (19) can be solved by any iterative method when the values for the other parameters are prescribed. Hence to solve the proposed model, the usual practice of substituting m = 1, 2, 3, …etc in (19) and finding the corresponding values of $r_j (0 \le r_j \le 1)$ for which TC is minimum has to be followed and a list of corresponding values of TC from (13) has to be prepared. The values of m and T in the minimum TC as obtained above are the optimal values for m and T respectively.

When r_j (j = 1, 2, …..m) =1, shortages are not allowed in all cycles of the inventory model.

Again r_j (j = 1, 2, …..m) = 0 are the conditions for an EOQ model where inventories are not carried at all.

5. Solution Algorithm

The solution procedure outlined in the previous section is summarized below in an algorithm form:

Step 0: Prescribe time-varying demand function f (t), Step 1: Set m = 1

Step 2: m = m + 1

Step 3: Let r_j (m), (j = 1, 2, …..m) be the solution of (19),

Step 4: Set t_j = j H/m, j = 0, 1, 2, …..m

Step 5: Set $S_j = (r_j + j - 1)$ H/m, j = 1, 2, …m Step 6: Compute TC (m), (using (13)),
Step 7: If TC (m) < TC (m –1), go to step 2,
Step 8: If TC (m) ≥ TC (m – 1), set m* = m – 1, TC* = TC (m –1) and T* = T (m – 1),
Step 9: Stop.

6. Numerical Examples

To illustrate the preceding theory, the following examples are considered.

Example 1:
Let f (t) = 10exp (0.98t), A = 100, C_1= 4, C_2 = 8, C_3 = 5, p = 10, θ = .01, k = 0.1, α = 0.2, H = 4 in appropriate units.

Equations (19) are solved for r_j (j = 1, 2, ….m) for different values of m and then substituting these values of m and r_j (j = 1, 2, ….m) in (13), the corresponding values of the total inventory cost TC are obtained. The output results are shown in Table 4.2.1

Table 4.2.1: Optimal solution of example 1

M	T	TC
1	4	9983.16
2	2	9026.24
3	1.33	8645.69
4	1	8507.29
5*	0.80*	8472.46*
6	0.67	8490.26
7	0.57	8538.20
8	0.50	8604.95

The optimal values are m* = 5, T* = 0.80 and TC* = 8472.46

Example 2:
Let f (t) = 500exp (– 0.98t), A = 100, C_1= 4, C_2 = 8, C_3 = 5, p = 10, θ = .01, k = 0.1,
α = 0.2, H = 4 in appropriate units.
Computed results are given in the following Table 4.2.2.

Table 4.2.2: Optimal solution of example 2

m	T	TC
1	4	7372.89
2	2	6929.45
3	1.33	6766.91
4*	1*	6727.72*
5	0.80	6747.34
6	0.67	6798.95
7	0.57	6869.77

The optimal values are m* = 4, T* = 1 and TC* = 6727.72

7. Sensitivity Analysis

In this section, we have examined the effects of changing the backlogging parameter 'α', inflation parameter 'k' and deterioration parameter 'θ' on the optimal cost and the optimal number of replenishments. A sensitivity analysis is performed by considering the above numerical examples. Computed results are shown in the following Tables:

Table 4.2.3: Variation of the optimal solution with 'α'

		α				
		0	0.1	0.3	0.5	0.7
Example 1	TC*	8513.68	8493.99	8448.88	8394.50	8378.46
	m*	5	5	5	5	5
Example 2	TC*	6748.80	6738.85	6715.21	6685.06	6656.82
	m*	4	4	4	4	4

Analysing the results in the above table 4.2.3, the following observations may be made.
1) Increase the value of α will result in a decrease in the optimal total cost (TC*) for both the increasing and decreasing demand rate functions.
2) It is also noted that the total cost is quite sensitive to changes in the value of α, whereas the optimal replenishment cycle time (m*) is practically insensitive to changes in the value of backlogging parameter α .

Table 4.2.4: Variation of the optimal solution with ' k'

		k				
		0	.05	0.2	0.3	0.4
Example 1	TC*	6239.32	7436.64	11360.52	15263.80	20468.90
	m*	5	5	5	5	4
Example 2	TC*	6111.90	6478.63	7427.08	8211.04	9143.02
	m*	5	4	3	3	2

The following inferences may be made from the Table 4.2.4:

The optimal total cost TC* increases while the optimal replenishment time m* decreases with increase in the value of the inflation parameter k. The results obtained show that both TC* and m* are quite sensitive to changes in the value of k.

Table 4.2.5: Variation of the optimal solution with 'θ'

		θ				
		0	.03	.05	.08	.1
Example 1	TC*	8386.59	8714.31	8925.07	9139.79	9397.51
	m*	5	5	5	5	5
Example 2	TC*	6533.82	6813.14	6898.95	7013.49	7075.01
	m*	4	4	4	4	4

From the above table 4.2.5, the following observation can be made:
1) When the deterioration parameter θ increases, the optimal total cost (TC*) of the inventory system increases for both the increasing and decreasing demand functions.
2) It can also be seen that the effect on TC* due to changes in the value of the parameter θ is quite appreciable. On the other hand, its influence on the optimal replenishment cycle time (m*) is negligible.

8. Concluding Remarks

In this paper, an inflationary trended inventory model for decaying items with time-varying demand is developed for a fixed and finite planning horizon considering partial backlogging. In particular, the backlogging rate is considered to be a decreasing function of the waiting time for the next replenishment. This assumption is more realistic in the market. The effect of backlogging parameter (α), inflation rate (k) and deterioration parameter (θ) are discussed in the present model. Numerical results indicate that when k and θ increase, the optimal total cost increases, where as it decreases with the increase in value of α. Moreover, the optimal inventory cost is quite sensitive for the changes of α and θ, while it is highly influenced by the changing value of the parameter k.

In a future study, the proposed model can further incorporate more realistic assumptions such as variable inflation rate and time-discounting, infinite rate of replenishment and probabilistic demand.

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