Zagreb Polynomials, Co-Polynomials of Double and Strong Double Graphs

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Abstract: ***First Zagreb polynomial of a graph G with vertex set V(G) and edge set E(G) is defined as $M_1(G,x) = \sum_{uv\in E(G)} x^{d_u+d_v}$ and the first Zagreb index can be obtained from it as $M_1(G) = \frac{\partial M_1(G,x)}{\partial x}|_{x=1}$. In this paper Zagreb polynomials, co-polynomials and corresponding Zagreb indices, co-indices are obtained for path, cycle, double and strong double graphs.***

Keywords: Cycle graph, double graph, path graph, strong double graph, Zagreb co-polynomial, Zagreb index, Zagreb polynomial

1. Introduction

Let G be a simple, finite, connected graph with vertex set V(G) and edge set E(G). The degree of a vertex u ∈ V(G) is denoted by d_u and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv. A molecular graph is presentation of the structural formula of a chemical compound in terms of graph theory whose vertices correspond to the atoms of compound and edges correspond to chemical bonds [1]. The topological index is a numerical parameter mathematically derived from the graph structure, several such topological indices have been considered in theoretical chemistry and have found some applications in QSPR/QSAR study [2].

A complement $\bar{G}$ of a graph G consist of the same set of vertices, where two vertices v and w are adjacent by an edge vw if and only if they are not adjacent in G.Hence $vw\in E(\bar{G}) \Leftrightarrow vw\notin E(G)$.A complement graph consists of a number of edges and the degree of vertex v which are represented as $\bar{m} = \binom{n}{2} - m$ and $d_{\bar{G}}(v) = n-1-R_G(v)$ respectively. If the graph G has n vertices and m edges, then double graph D[G] has 2n vertices 4m edges. The first, second Zagreb indices and geometric-arithmetic index of double and strong double graph of path and cycle graphs were studied in [3]. The double graph of G is constructed by considering two copies of G in which a vertex v_i in first copy is adjacent to a vertex v_j in the second copy of v_i and v_j [4]. The strong double is a double in which a vertex v_i in the copy is adjacent to a vertex v_j in the second copy if i = j. It is denoted as SD[G] [5]. Zagreb indices and multiplicative Zagreb indices of double graphs of subdivision graphs were investigated by M. Togan et al. [6].

Degree based topological indices of strong double graphs have been studied in [7-8]. The relations between some Zagreb indices and Zagreb co-indices of graphs can be seen in papers [9-10]. General fifth Zagreb polynomial of benzene ring were studied in [11]. Li and W. Gao expressed the fourth Zagreb polynomial and sixth polynomial in terms eccentricity of a graph [12]. The Zagreb group indices and polynomials are studied in [13]. Zagreb indices and Zagreb polynomials have been studied in [14]. Distance based topological indices of double and strong double graphs were computed by Mirajkar [15]. The properties of strong double graphs of spanning trees of G were found in [16].

The Zagreb polynomials are defined as [17-18]

$$M_1(G,x) = \sum_{uv\in E(G)} x^{d_u+d_v}, \quad (1)$$
$$M_2(G,x) = \sum_{uv\in E(G)} x^{d_u\times d_v}, \quad (2)$$
$$M_3(G,x) = \sum_{uv\in E(G)} x^{|d_u-d_v|}, \quad (3)$$
$$M_4(G,x) = \sum_{uv\in E(G)} x^{d_u(d_u+d_v)}, \quad (4)$$
$$M_5(G,x) = \sum_{uv\in E(G)} x^{d_v(d_u+d_v)}, \quad (5)$$
$$\overline{M_1}(G,x) = \sum_{uv\notin E(G)} x^{(d_u+d_v)}, \quad (6)$$
$$\overline{M_2}(G,x) = \sum_{uv\notin E(G)} x^{(d_u\times d_v)}. \quad (7)$$
$$\overline{M_3}(G,x) = \sum_{uv\notin E(G)} x^{|d_u-d_v|}, \quad (8)$$
$$\overline{M_4}(G,x) = \sum_{uv\notin E(G)} x^{d_u(d_u+d_v)}, \quad (9)$$

And

$$\overline{M_5}(G,x) = \sum_{uv\notin E(G)} x^{d_v(d_u+d_v)} \quad (10)$$

The first Zagreb index can be calculated as the first derivative of first Zagreb polynomial, x = 1 by [19]

$$M_1(G) = \frac{\partial M_1(G,x)}{\partial x}|_{x=1}. \quad (11)$$

All the symbols and notations used in this paper are standard and taken mainly from books of graph theory [20-22]. In this paper we study Zagreb polynomials, co-polynomials of double, strong double graphs of path graph P_4 and cycle graph C_4.

2. Materials and Methods

Let G be a finite, simple, connected graph with vertex set V(G) and edge set E(G). Molecular graph is a graph in which the vertices correspond to the atoms and the edges to the bonds of a molecule. Molecular graphs of path graph P_4, double graph $D[P_4]$, strong double graph $SD[P_4]$ and cycle graph C_4, double graph of C_4 and strong double graph of C_4 are shown figure (1-2). Degree d_u of a vertex u is the number of vertices adjacent to u. The order and size of strong double SD[G] are 2n and 4m+n respectively. The degree of a vertex v in SD[G] is $\deg_{SD[G]} = 2\deg_G[v] + 1$. If the graph has n vertices and the m edges, then the double graph D[G] has 2n vertices and 4m edges, in particular $\deg_{D[G]}(v,2) = 2\deg_G(v)$. The complement of G denoted by $\bar{G}$, is a simple graph on the same set of vertices V(G) in which two vertices u and v are adjacent, that is connected by an edge uv, iff they are not adjacent in G. Hence uv∈ $\bar{G}$⇔uv ∉E(G). Degree of a vertex u in G is denoted d_u; the degree of the same vertex is $\bar{G}$ is given by $deg_{\bar{G}}$ (u) = n-1-d_u. Degrees of vertices of path, cycle, double, strong double graphs are used in finding the Zagreb polynomials and co-polynomials.

3. Results and Discussion

It is observed from molecular graph of path graph P_4, there are $|E_{1,2}|$= n-2 and $|E_{1,2}|$= n-3 edges. For double graph of P_4 the edge partition is $|E_{2,4}|$= n and $|E_{4,4}|$= n-4 and in strong double graph of P_4, it is $|E_{3,4}|$= n, $|E_{4,4}|$= n-4 and $|E_{3,3}|$= n-6. The Zagreb polynomials, co-polynomials of P_4, D[P_4] and SD[P_4] are computed as follows.

Path graph P_4

Theorem 1.1. First Zagreb polynomial of P_4 is (n-2)x^3+(n-3)x^4.

Proof. First Zagreb polynomial of P_4

$M_1(G,x)= \sum_{u\in V(G)} x^{d_u+d_v}$

$= |E_{1,2}|x^{1+2}+|E_{2,2}|x^{2+2}$

$= (n-2)x^3+(n-3)x^4.$

Theorem 1.2. Second Zagreb polynomial of P_4 is (n-2)x^2+(n-3)x^4.

Proof. Second Zagreb polynomial of P_4

$M_2(G,x)= \sum_{u\in V(G)} x^{d_u\times d_v}$

$= |E_{1,2}|x^{1\times 2}+|E_{2,2}|x^{2\times 2}$

$= (n-2)x^2+(n-3)x^4.$

Theorem 1.3. Third Zagreb polynomial of P_4 is (n-2)x

Theorem 1.4. Fourth Zagreb polynomial of P_4 is (n-2)x^3+(n-3)x^8.

Theorem 1.5. Fifth Zagreb polynomial of P_4 is (n-2)x^6+(n-3)x^8.

Theorem 1.6. First Zagreb co-polynomial of P_4 is (n-2)x^3+(n-3)x^2.

Proof. First Zagreb co-polynomial of P_4

$\overline{M_1}(G,x)= \sum_{u\in\notin E(G)} x^{d_u+d_v}$

$= |E_{1,2}|x^{2+1}+|E_{2,2}|x^{1+1}$

$= (n-2)x^3+(n-3)x^2.$

Theorem 1.7. Second Zagreb co-polynomial of P_4 is (n-2)x^2+(n-3)x.

Proof. Second Zagreb co-polynomial of P_4

$\overline{M_2}(G,x)= \sum_{u\in\notin E(G)} x^{d_u\times d_v}$

$= |E_{1,2}|x^{2\times 1}+|E_{2,2}|x^{1\times 1}$

$= (n-2)x^2+(n\text{-}3)\ x.$

Theorem 1.8. Third Zagreb co-polynomial of P_4 is (n-2)x+(n-3)

Theorem 1.9. Fourth Zagreb co-polynomial of P_4 is (n-2)x^6+(n-3)x^2.

Theorem 1.10. Fifth Zagreb co-polynomial of P_4 is (n-2)x^3+(n-3)x^2.

Theorem 2.1. First Zagreb polynomial of double graph of P_4 is 4nx^{12}+4(n-4)x^{16}.

Theorem 2.2. Second Zagreb polynomial of double graph of P_4 is 4nx^{32}+4(n-4)x^{64}.

Theorem 2.3. Third Zagreb polynomial of double graph of P_4 is 4n x^4.

Theorem 2.4. Fourth Zagreb polynomial of double graph of P_4 is 4nx^{48}+4(n-4)x^{128}.

Proof. Fourth Zagreb polynomial double graph of P_4

$M_4(D[G],x)= \sum_{uv\in E(G)} x^{d_u(d_u+d_v)}$

$= 4\sum_{uv\in E(D[G])} x^{2d_u(2d_u+2d_v)}$

$=4|E_{2,4}|x^{2d_u(2d_u+2d_v)}+4|E_{4,4}|x^{2d_u(2d_u+2d_v)}.$

$= 4nx^{48}+4(n-4)x^{128}.$

Theorem 2.5. Fifth Zagreb polynomial of double graph of P_4 is 4nx^{48}+4(n-4)x^{128}.

Proof. Fifth Zagreb polynomial of double graph of P_4.

$M_5(D[G],x)= \sum_{uv\in E(G)} x^{d_v(d_u+d_v)}$

$= 4\sum_{uv\in E(D[G])} x^{2d_u(2d_u+2d_v)}$

$=4|E_{2,4}|x^{2d_v(2d_u+2d_v)}+4|E_{4,4}|x^{2d_v(2d_u+2d_v)}.$

$= 4nx^{48}+4(n-4)x^{128}.$

Theorem 2.6. First Zagreb co-polynomial of double graph of P_4 is 4nx^6+4(n-4)x^{-2}.

Theorem 2.7. Second Zagreb co-polynomial of double graph of P_4 is 4nx^{-3}+4(n-4)x.

Theorem 2.8. Third Zagreb co-polynomial of double graph of P_4 is 4nx^4-(n-4).

Theorem 2.9. Fourth Zagreb co-polynomial of double graph of P_4 is $(4n)x^{40}+4(n-4)x^{18}$.

Proof. Fourth Zagreb co-polynomial of double graph of P_4

$\overline{M_4}(G,x)= \sum_{u\in\notin E(G)} x^{d_u(d_u+d_v)}$

$= 4\sum_{u\in\notin E(G)} x^{2d_u(2d_u+2d_v)}$

$=4|E_{2,4}|x^{5(5+3)} +4|E_{4,4}|x^{3(3+3)}$

$= (4n)x^{40}+4(n-4)x^{18}.$

Theorem 2.10. Fifth Zagreb co-polynomial of double graph of P_4 is 4nx^{24}+4(n-4)x^{18}.

Proof. Fifth Zagreb co-polynomial of double graph of P_4

$\overline{M_5}(G,x)= \sum_{u\notin E(G)} x^{d_v(d_u+d_v)}$

$= 4\sum_{u\in\notin E(G)} x^{2d_v(2d_u+2d_v)}$

$=4|E_{2,4}|x^{3(5+3)} +4|E_{4,4}|x^{3(3+3)}$

$= 4nx^{24}+4(n-4)x^{18}.$

Theorem 3.1. First Zagreb polynomial of strong double graph of P_4 is nx^{16}+(n-4)x^{18}+(n-6)x^{14}.

Proof. First Zagreb polynomial of strong double graph of P_4

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$M_1(SD[G],x)= \sum_{uv\in E(G)} x^{d_u+d_v}$

$= |E_{3,4}|x^{[2d_u+1]+[2d_v+1]}+|E_{4,4}|x^{[2d_u+1]+[2d_v+1]}+|E_{3,3}| x^{[2d_u+1]+[2d_v+1]}$

$= nx^{16}+(n-4)x^{18}+(n-6)x^{14}$.

Theorem 3.2. Second Zagreb polynomial of strong double graph of P_4 is $nx^{63}+(n-4)x^{81}+(n-6)x^{49}$.

Proof. Second Zagreb polynomial of strong double graph of P_4

$M_2(SD[G],x)= \sum_{uv\in E(G)} x^{d_u\times d_v}$

$= |E_{3,4}|x^{[2d_u+1]\times[2d_v+1]}+|E_{4,4}|x^{[2d_u+1]\times[2d_v+1]}+|E_{3,3}| x^{[2d_u+1]\times[2d_v+1]}$

$= nx^{63}+(n-4)x^{81}+(n-6)x^{49}$.

Theorem 3.3. Third Zagreb polynomial of strong double graph of P_4 is $nx^2+(2n-10)$.

Theorem 3.4. Fourth Zagreb polynomial of strong double graph of P_4 is $nx^{112}+(n-4)x^{162}+(n-6)x^{98}$.

Theorem 3.5. Fifth Zagreb polynomial of strong double graph of P_4 is $nx^{144}+(n-4)x^{162}+(n-6)x^{98}$.

Theorem 3.6. First Zagreb co-polynomial of strong double graph of P_4 is $(n-6)x^{18}+n\,x^{16}+(n-4)x^{14}$.

Theorem 3.7. Second Zagreb co-polynomial of strong double graph of P_4 is $(n-6)x^{81}+n\,x^{63}+(n-4)x^{49}$.

Theorem 3.8. Third Zagreb co-polynomial of strong double graph of P_4 is $nx^2+2n-10$.

Theorem 3.9. Fourth Zagreb co-polynomial of strong double graph of P_4 is $(n-6)x^{162}+nx^{144}+(n-4)x^{98}$.

Proof. Fourth Zagreb co-polynomial of double graph of P_4

$\overline{M_4}(SD[G],x)= \sum_{u\in\notin E(G)} x^{d_u(d_u+d_v)}$

$= \sum_{u\in\notin E(G)} x^{(2d_u+1)[(2d_u+1)+(2d_v+1)]}$

$=|E_{3,3}|x^{(2d_u+1)[(2d_u+1)+(2d_v+1)]}+|E_{3,4}|x^{(2d_u+1)[(2d_u+1)+(2d_v+1)]}+|E_{4,4}| x^{(2d_u+1)[(2d_u+1)+(2d_v+1)]}$

$= (n-6)x^{162}+nx^{144}+(n-4)x^{98}$.

Theorem 3.10. Fifth Zagreb co-polynomial of strong double graph of P_4 is $(n-6)x^{162}+nx^{112}+(n-4)x^{98}$.

Proof. Fifth Zagreb co-polynomial of double graph of P_4

$\overline{M_5}(SD[G],x)= \sum_{u\in\notin E(G)} x^{d_u(d_u+d_v)}$

$=|E_{3,3}|x^{(2d_v+1)[(2d_u+1)+(2d_v+1)]}+|E_{3,4}|x^{(2d_v+1)[(2d_u+1)+(2d_v+1)]}+|E_{4,4}| x^{(2d_v+1)[(2d_u+1)+(2d_v+1)]}$

$= (n-6)x^{162}+nx^{112}+(n-4)x^{98}$.

Cycle graph C_4

It is observed from molecular graph of cycle graph C_4 there are $|E_{2,2}|=$ n edges. For double graph of $D[C_4]$, the edge partition is $|E_{4,4}|=2n$ and in strong double graph $SD[C_4]$ it is $|E_{55}|=3n$. The Zagreb polynomials, co-polynomials of cycle graph C_4, $D[C_4]$ and $SD[C_4]$ are computed as follows.

Theorem 4.1. First Zagreb polynomial of C_4 is nx^4.
Theorem 4.2. Second Zagreb polynomial of C_4 is nx^4.
Theorem 4.3. Third Zagreb polynomial of C_4 is n.
Theorem 4.4. Fourth Zagreb polynomial of C_4 is nx^8.
Theorem 4.5. Fifth Zagreb polynomial of C_4 is nx^8.
Theorem 4.6. First Zagreb co-polynomial of C_4 is nx^2.
Theorem 4.7. Second Zagreb co-polynomial of C_4 is nx.
Theorem 4.8. Third Zagreb co-polynomial of C_4 is n.
Theorem 4.9. Fourth Zagreb co-polynomial of C_4 is nx^2.
Theorem 4.10. Fifth Zagreb co-polynomial of C_4 is nx^2.
Theorem 5.1. First Zagreb polynomial of double graph of C_4 is $8x^{16}$.

Proof. First Zagreb polynomial of double graph of C_4

$M_1(D[G],x)= \sum_{uv\in E(G)} x^{d_u+d_v}$

$= 4|E_{4,4}|x^{2d_u+2d_v}$

$= 8nx^{16}$.

Theorem 5.2. Second Zagreb polynomial of double graph of C_4 is $8nx^{64}$.

Proof. Second Zagreb polynomial of double graph of C_4

$M_2(D[G],x)= \sum_{uv\in E(G)} x^{d_u\times d_v}$

$= 4|E_{4,4}|x^{2d_u\times 2d_v}$

$= 8nx^{64}$.

Theorem 5.3. Third Zagreb polynomial of double graph of C_4 is 8n.
Theorem 5.4. Fourth Zagreb polynomial of double graph of C_4 is $8nx^{128}$.
Theorem 5.5. Fifth Zagreb polynomial of double graph of C_4 is $8nx^{128}$.
Theorem 5.6. First Zagreb co-polynomial of double graph of C_4 is $8nx^{12}$.
Theorem 5.7. Second Zagreb co-polynomial of double graph of C_4 is $8nx^{36}$.
Theorem 5.8. Third Zagreb co-polynomial of double graph of C_4 is 8n.
Theorem 5.9. Fourth Zagreb co-polynomial of double graph of C_4 is $8nx^{72}$.
Proof. Fourth Zagreb co-polynomial of double graph of C_4

$M_4(D[G],x)= \sum_{uv\notin E(G)} x^{d_u(d_u+d_v)}$

$= 4|E_{4,4}|x^{2d_u(2d_u+2d_v)}$

$= 8nx^{72}$.

Theorem 5.10. Fifth Zagreb co-polynomial of double graph of C_4 is $8nx^{72}$.

Proof. Fifth Zagreb co-polynomial of double graph of C_4

$M_5(D[G],x)= \sum_{uv\notin E(G)} x^{d_u(d_u+d_v)}$

$= 4|E_{4,4}|x^{2d_v(2d_u+2d_v)}$

$= 8nx^{72}$.

Theorem 6.1. First Zagreb polynomial of strong double graph of C_4 is $(2n+4)x^{22}$.

Proof. First Zagreb polynomial of strong double graph of C_4

$M_1(SD[G],x)= \sum_{uv\in E(G)} x^{d_u+d_v}$

$=|E_{5,5}|x^{(2d_u+1)+(2d_v+1)}$

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$= (2n+4)x^{22}$.

Theorem 6.2. Second Zagreb polynomial of strong double graph of C_4 is $(2n+4)x^{121}$.

Proof. Second Zagreb polynomial of strong double graph of C_4

$M_2(SD[G],x) = \sum_{uv \in E(G)} x^{d_u \times d_v}$

$= |E_{5,5}| x^{(2d_u+1)\times(2d_v+1)}$

$= (2n+4)x^{121}$.

Theorem 6.3. Third Zagreb polynomial of strong double graph of C_4 is 2n+4.

Theorem 6.4. Fourth Zagreb polynomial of strong double graph of C_4 is $(2n+4)x^{242}$.

Theorem 6.5. Fifth Zagreb polynomial of strong double graph of C_4 is $(2n+4)x^{242}$.

Theorem 6.6. First Zagreb co-polynomial of strong double graph of C_4 is $(2n+4)x^{-8}$.

Theorem 6.7. Second Zagreb co-polynomial of strong double graph of C_4 is $(2n+4)x^{16}$.

Theorem 6.8. Third Zagreb co-polynomial of strong double graph of C_4 is (2n+4).

Theorem 6.9. Fourth Zagreb co-polynomial of strong double graph of C_4 is $(2n+4)\,x^{32}$.

Proof. Fourth Zagreb co-polynomial of strong double graph of C_4

$\overline{M_4}(SD[G],x) = \sum_{uv \notin E(G)} x^{d_u(d_u+d_v)}$

$= |E_{5,5}| \sum_{uv \in E(G)} x^{(2d_u+1)+[(2d_u+1)+(2d_v+1)]}$

$= (2n+4)\,x^{32}$.

Theorem 6.10. Fifth Zagreb co-polynomial of strong double graph of C_4 is $(2n+4)\,x^{32}$.

Proof. Fifth Zagreb co-polynomial of strong double graph of C_4

$\overline{M_5}(SD[G],x) = \sum_{uv \notin E(G)} x^{d_u(d_u+d_v)}$

$= |E_{5,5}| \sum_{uv \in E(G)} x^{(2d_v+1)+[(2d_u+1)+(2d_v+1)]}$

$= (2n+4)\,x^{32}$.

Zagreb indices and co-indices

1) First Zagreb index of double graph of P_4

$M_1 D[G] = \frac{\partial M_1(D[G],x)}{\partial x}\Big|_{x=1}$

(n-2)3+(n-3)4

=7n-18.

2) Fourth Zagreb co-index of double graph of C_4

$\overline{M_4} D[G] = \frac{\partial \overline{M_4}(D[G],x)}{\partial x}\Big|_{x=1}$

$=4(n^2+2n)162$

$=648(n^2+2n)$.

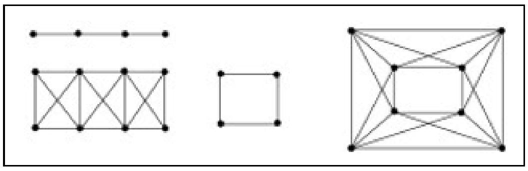

Figure 1: The double graphs of P_4 and C_4.

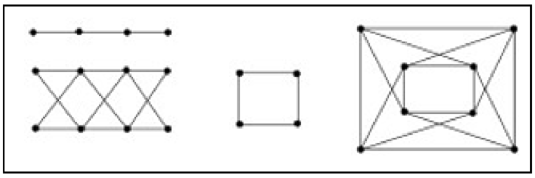

Figure 2: The strong double graphs of P_4 and C_4.

Table 1: Zagreb indices, co-indices of path, cycle, double, strong double graphs of P_4 and C_4.

Zagreb indices → Graph↓	M_1	M_2	M_3	M_4	M_5	$\overline{M_1}$	$\overline{M_2}$	$\overline{M_3}$	$\overline{M_4}$	$\overline{M_5}$
P_4	7n-18	6n-16	n-2	11n-30	14n-36	5n-12	3n-7	2n-5	8n-18	5n-12
D[G]P_4	112n-256	384n-256	16n	320n-512	704n-2048	56n+32	-(12n+16)	17n-4	232n-288	156n-288
SD[G]P_4	48n-156	193n-618	4n-10	372n-1236	306n-1236	48n-164	193n-682	4n-10	306n-1364	372n-1364
C_4	4n	4n	0	8n	8n	2n	N	0	2n	2n
D[G]C_4	128n	512n	0	1024n	1024n	96n	288n	0	576n	576n
SD[G]C_4	22(2n+4)	121(2n+4)	0	242(2n+4)	242(2n+4)	-8(2n+4)	16(2n+4)	0	32(2n+4)	32(2n+4)

4. Conclusion

Zagreb polynomials, co-polynomials and corresponding Zagreb indices of path, cycle, double, strong double graphs for path graph P_4 and cycle graph C_4 are studied.

References

[1] M. R. R. Kanna, S. Roopa and L. Parashivamurthy, Topological indices of Vitamin D_3, International Journal of Engineering and Technology, 7(4) (2018) 6276-6284.

[2] V. R. Kulli, On Banhatti-Sombor indices, SSRG International Journal of Applied Chemistry, 8(1) (2001) 21-25.

[3] M. Imran, S. Akhter, Degree based topological indices of double graphs and strong double graphs, Discrete Mathematics and Applications, 9(5) (2017) 1750066-15.

[4] E. Munarini, C. Perelli, A. Scaliola and N. Z. Salvi, Double graphs, Discrete Mathematics, 302(2002) 242-254.

[5] V. M. Diudea, J. Chem. Inf. Comput. Sci., 37(1997) 292-299.

[6] M. Togan, A. Yurttas, A. S. Cervic and N. Congul, Zagreb indices and multiplicative Zagreb indices of double graphs of subdivision graphs, TWMS Journal of Applied Engineering Mathematics, 9(2) (2019) 404-412.

[7] M. Rafiullah, H. Muhammad, A. Siddiqui, M. K. Siddiqui and M. Dhlamini, On degree-based topological indices for strong double graphs, Hindawi, Journal of Chemistry, Volume 2021, Article Id-4852459, 12 pages.

[8] M. S. Sardar, M. A. Ali, F. Asfraf and M. Cancan, On topological indices of double and strong double graphs of silicon carbide Si2C3-I[p, q], Eurosian Chem. Commun. , 5(2023) 37-49.
[9] K. Kiruthika, Zagreb indices and Zagreb co-indices of some graph operations, International Journal of Advanced Research in Engineering and Technology, 7(3) (2016) 25-41.
[10] I. Gutmn, B. Furtula, Z. K. Vukicevic and G. Popivoda, On Zagreb indices and co-indices, MATCH Mathematical and Computer Chemistry, 74(2015) 5-16.
[11] P. Sarkar, A. Pal, General fifth M-polynomials of benzene ring implanted in the P-type surface in 2D network, Biointerface Research in Applied Chemistry, 10(6) (2020) 6881-6892.
[12] L. Yan, W. Gao, Eccentric related indices of an infinite class of nanostar dendrimers (II), Journal of Chemical and Pharmaceutical Research, 8(5) (2016) 359-362.
[13] N. K. Raut, The Zagreb group indices and polynomials, International Journal of Modern Engineering Research, 6(10) (2016) 84-87.
[14] M. R. Farahani, Zagreb indices and Zagreb polynomials of polycyclic aromatic hydrocarbons, Journal of Chemical Acta, 2(2013) 70-72.
[15] K. Mirajkar, S. Konnur, Distance-based topological indices of double graphs and strong double graphs, Ratio Mathematics, Volume 48, 2023, 1-20.
[16] T. A. Chisti, H. A. Ganie and S. Pirzada, Properties of strong double graphs, Journal of Discrete Mathematical Science and Cryptography, 4(17) (2014) 311-319.
[17] A. U. Rehaman, W, Khalid, Zagreb polynomials and redefined Zagreb indices of line graph of HAC5C6C7[p, q] nanotube, Open Journal of Chemistry, 1(1) (2018) 26-35.
[18] B. Basavanagoud, P. Jakkannavar, On the Zagreb polynomials of transformation graphs, International Journal of Scientific Research in Mathematical and Statistical Sciences, 5(6) (2018) 328-335.
[19] M. R. Farahani, Zagreb indices and Zagreb polynomials of polycyclic aromatic hydrocarbons PAHs, Journal of Chemical Acta, 2(2013) 70-72.
[20] Narsing Deo, Graph Theory, Prentice-Hall of India, New Delhi (2007).
[21] N. Trinajstić, Chemical Graph Theory, CRC Press, Boca Raton, FL., 1992.
[22] R. Todeschini, and V. Consonni, Handbook of Molecular Descriptors, Wiley-VCH:Weinheim, 2000.