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Edge Augmentation in Graphs: Graph Transformation of G to G^* Under Specified Condition and Resultant Properties

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Abstract: ***In this Paper, we explore a specific class of Transformation involving the augmentation of a graph G by adding edges to form a new graph G^* under certain conditions.***

Keywords: Graph Transformation, Identity Transformation, Linear Transformation

1. Introduction

A transformation is a mathematical rule used to change one geometric figure into another, often described by a specific formula. In the context of linear algebra, linear transformations are functions that map vectors from one space to another, subject to two key conditions: additivity and scalar multiplication. These transformations are fundamental in the study of vector spaces. One significant transformation, known as the Kelmans transformation, was introduced by Alexander Kelmans . This transformation has since played a crucial role in the development of graph theory. In the paper On Graph Transformation, we define transformation of graph. In this paper we define some results of Graph Transformation and also define another transformation of Graphs and some results.

Definition 1. Let $G = (p,q)$ be a graph. A graph G^* is said to be a transformation, then there exists a mapping $T: G \to G^*$ which satisfies following conditions;

1) $|V(G)| = |V(G^*)|$
2) u^* and $v^*$are adjacent in G^* if either u and v are adjacent in G or $d(u) + d(v) \leq p$ in G.

Example.

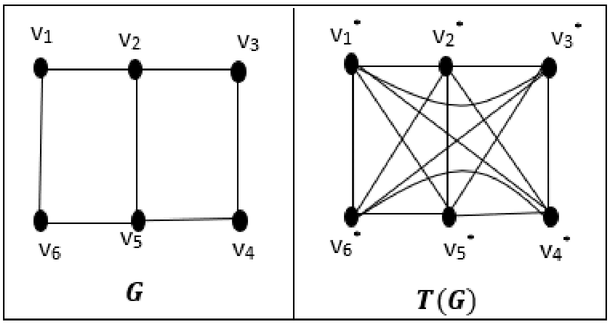

Figure 1

Definition 2. If $|E(G^*)| = |E(G)| + 1$, then the transformation is said to be linear.

Definition 3. If $T(G) = G$, then T is said to be the identity transformation.

Theorem 1. Let G be a (p,q) graph. If $\Delta(G) > \frac{p}{2}$, $\delta = \frac{p}{2}$, exactly two non-adjacent vertices have degree $\frac{p}{2}$, then the transformation is linear.

Proof. Let u and v be the two non-adjacent vertices of G having degree $\frac{p}{2}$ and all other vertices having degree greater than $\frac{p}{2}$. Sum of degree of all non-adjacent vertices is greater than p except u and v $[d(u) + d(v) = p]$. Therefore, we can add only one edge (u^*, v^*) in $T(G)$, which implies $T(G)$ is linear.

Some Observations.

In this section, we give some immediate consequences as observations without proof.

1) Let G be a (p,q) graph. If $q > \binom{p-1}{2} + 2$, then $T(G) = G$.
2) Let $A(G)$ be the $p \times p$ adjacency matrix of a graph G. If each row sum of $A(G)$ is greater than $\frac{p}{2}$, then $T(G) = G$.
3) If $A(G)$ has row sum greater than or equal to $\frac{p+1}{2}$, where p is odd, then $T(G) = G$.
4) If $A(G)$ has row sum less than or equal to $\frac{p}{2}$, then $T(G) \cong K_p$.
5) If $A(G)$ has row sum less than or equal to $\frac{p-1}{2}$, where p is odd, then $T(G) \cong K_p$.
6) If $A(G)$ has exactly 2 rows i and j have the sum less than or equal to $\frac{p}{2}$, and all other rows have sum greater than $\frac{p}{2}$, and $a_{ij} = 0$, then the transformation $T(G)$ is linear.

2. Another Transformation

In this section, we define another transformation as follows

Definition 4: Let $G = (p,q)$ be a graph. A graph G^* is said to be a transformation, then there exists a mapping $T^*: G \to G^*$ which satisfies following conditions;

1. $|V(G)| = |V(G^*)|$

2. u^* and $v^*$are adjacent in G^* if either u and v are adjacent in G or $d(u) + d(v) > p$

Theorem 2. Let C_p be a cycle with p vertices. $T^*(C_p) \cong C_p$ for all p . That is $T^*(C_p)$ is an identity transformation

Proof: Since every vertex of a cycle has degree 2 and if u and v are two non-adjacent vertices in G, $d(u) + d(v) \leq p, if$ p ≥ 4. So $E(G) = E(G^*)$. Therefore $T^*(C_p) \cong C_p$ for all p

Theorem 3: Let G be a (p, q) graph, p is even and if G is a $\frac{p+2}{2}$ regular graph, then $T^*(G) \cong K_p$.

Proof: Let $u, v \in V(G), u \,\&v$ are non-adjacent in G. Since G is $\frac{p+2}{2}$ regular $d(u) + d(v) = p + 2 > p$. In T^* , $u^*$and v^* are adjacent. Since uand v are arbitrary, sum of degrees of any two non-adjacent vertices in G is greater than p. Therefore $T^*(G) \cong K_p$

Theorem 4: Let G be a (p, q) graph. If $\delta(G) = \frac{p}{2} + 1$, where p is even, then $T^*(G) \cong K_p$.

Proof. Let v be any arbitrary vertex with $d(v) = \delta(G)$.∶ Let $u \in V(G), u \,\&v$ are non − adjacent in G. Clearly $d(u) \geq \frac{p}{2} + 1$. Therefore $d(u) + d(v) > p + 2$, . which is greater than p. In T^* , $u^*$and v^* are adjacent. Since uand v are arbitrary, sum of degrees of any two non-adjacent vertices in G is greater than p. Therefore $T^*(G) \cong K_p$

Theorem 5: Let G be a (p, q) graph. If $\delta(G) = \frac{p+1}{2}$, where p is odd, then $T^*(G) \cong K_p$.

Proof. Let v be any arbitrary vertex with $d(v) = \delta(G)$.∶ Let $u \in V(G), u \,\&v$ are non − adjacent in G. Clearly $d(u) \geq \frac{p+1}{2}$.Therefore $d(u) + d(v) > p + 1$, . which is greater than p. In T^* , $u^*$and v^* are adjacent. Since uand v are arbitrary, sum of degrees of any two non-adjacent vertices in G is greater than p. Therefore $T^*(G) \cong K_p$

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