

Thermal Analysis of Moving Porous Fins with MoS_2 Nanoparticles Using HPM

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Abstract

This study investigates the thermal performance of a trapezoidal moving porous fin immersed in MoS_2 based nanofluids using the Homotopy Perturbation Method (HPM). By transforming the governing equations into dimensionless forms and analyzing them parametrically, the research demonstrates the effect of dimensionless physical quantities such as Peclet number, Sherwood number, and radiative parameters on the temperature distribution. Results validate the superiority of nanofluids in enhancing thermal dissipation over conventional fluids, offering a valuable framework for applications in automotive and aerospace industries.

Keywords : Moving porous fin, trapezoidal geometry, thermal analysis, nanofluids, Homotopy Perturbation Method.

1. Introduction

Study of heat transfer through objects in a medium have always been of great interest to many researchers due to its wide range of applications. They play a major role in real world scenarios like in radiators, heat exchangers, microwave oven, air conditioner etc., Understanding transfer of heat is also crucial in industrial, mechanical and engineering fields. Such systems require an increased heat transfer rate during operation. This can be achieved through extended surfaces called fins which are widely used from small devices like laptops to complex devices like automobile devices to enhance heat transfer. A generalized heat transfer correlation for louver fin geometry studied by Chang and Wang [1]. The temperature distribution along a horizontal fin has been analyzed using a straight forward model for radiative heat transfer and an estimated correlation for natural convection is given by D.W. Mueller Jr. et al. [2]. [3]-[9] Show some more studies observed in this direction.

Examining fins in motion is becoming more popular of late than looking at fins attached to a dependable surface, considering their frequent use in hydrogen fuel cells, power plants, motorcycle engines, automobile radiators, aerospace engineering and similar applications. While Mustafa Turkyilmazoglu [10] worked on heat transmission using exponential fins in motion and under the influence of heat generation, Partner L. Ndlovu performed thermal analysis of radial moving fins [11]. Fehmi Gamaoun et al. [12] investigated on convection, fluctuating thermal conductivity and uneven thermal distribution of moving dovetail fin with radiation. In addition [13, 14], have made further studies.

Further, the use of porous fins have received an extensive amount of attention in recent years as it provides optimized thermal performance by increasing the effective surface area for both convective as well as radiative heat transfer, as compared to a solid fin of comparable weight. S. Kiwan and M. A. Al-Nimr [15] have studied on thermal performance of porous fins and compared the results with that of convectional solid fins. The performance and optimum dimensions of porous fins with different models are discussed by Balaram Kundu and Dipankar Bhanja [16]. An analytical model for the determination of fin efficiency, temperature distribution and optimum design parameters of a porous moving fin is determined by Dipankar Bhanja [17]. Darcy model is employed to formulate the heat transfer equation is given by Seyfolah Saedodin et al. [18]. The influence of convection on the porous fin was discussed under different tip conditions by Kiwan [19]. Shwetha et al. [20] investigated the transient heat transfer

in a radial moving porous fin of a longitudinal trapezoidal structure. D.D. Ganji et al. [21] studied temperature distribution in a porous fin with temperature dependent internal heat generation which were calculated by three highly accurate and simple analytical methods such as DTM, CM and LS methods. Countless experimental and numerical studies have been carried out to provide a better investigation about the heat transfer mechanism through the porous medium [22, 23]. In order to maximize the fin's heat transmission efficiency, many more design parameters have been taken into consideration in recent investigations. Various fin shapes have been employed in numerous studies to investigate the porosity, mobility and heat transfer rate [24, 25, 26].

Conventional fluids, such as water, engine oil and ethylene glycol are normally used as heat transfer fluids. These fluids enable to attain substantial heat transfer rates in thermally designed devices, since they have comparatively low coefficient of thermal conductivity. By mixing nanoparticles with coolants, heat transfer rate can be enhanced. In recent years, researches gained interest on improving the heat transfer rates by using nanofluids. Yimin Xuan et al. [27] worked on to produce a nanofluids, a dispersion composed of base fluid and nanophase particles. Dhinesh Kumar Devendiran et al. [28] investigated on preparation of nanofluids, the chemical and physical characteristics of nanofluids, thermophysical properties and applications of nanofluids. Omid Mahian et al. [29] have demonstrated the use of nanofluids in solar thermal engineering systems to overcome the use of fossil fuels and to initiate the employment of alternative energy source like solar energy. Characteristics of nanofluid viscosity such as particle size, shape, temperature and even its volume fraction effects are explored by R. Saidur et al. [30]. Lazarus Godson et al. [31] were studied on enhancement of thermal conductivity, Brownian motion, thermophoresis and diffusiophoresis i.e., unique features of nanofluids.

Nanofluids play a crucial role in enhancing heat transfer efficiency in fins, as it is having higher thermal conductivity compared to conventional fluids. They reduce thermal resistance and increase dissipation rates of heat, which make cooling systems in their devices for more efficient. This results in compact designs for electronic and automotive devices and energy conversion applications. Remarkable studies are carried out for analyzing the thermal profile of fins in the presence of nanofluids. Tehmina Ambreen and Man-Hoe Kim examined the combined impact of using nanofluid and changing the form of the fin cross-section on the heat transfer properties of a micro pin-fin heat sink, using a discrete phase model (DPM) [32]. Jahanshahi et al. [33] conducted an experiment to induce free convection in an $H_2O - SiO_2$ nanofluid. The heat transmission within a longitudinal permeable fin that is fully saturated with nanofluid has been investigated by Baslem et al. [34]. Madhura et al. [35] investigated the influence of nanoparticles on natural convection flow with heat and mass transfer rates of nanofluids.

Thus above research publications revealed the importance of thermal analysis using fins with nanofluids. Also, according to literature survey and author's knowledge, it is observed that investigation of heat analysis using moving porous fins with trapezoidal geometry in the presence of nanofluids, particularly, MoS_2 nanoparticle with water as nanofluid is not yet considered for the investigation. To fill this research gap, the present work addressed the thermal performance of moving porous trapezoidal fin which is immersed in water based mononanofluid formed by spherical shaped MoS_2 solid nanoparticle uniformly dispersed in water. More specifically the current analysis shows the importance of trapezoidal profile fin and advantage of using MoS_2 nanoparticles, indeed: *i*) The fins with trapezoidal profile over other profile namely triangular, parabolic and rectangular, the trapezoidal profile fins are widely used in many applications like aerospace, automotive and marine applications. The minor modifications in trapezoidal geometry leads to other shapes, like triangular, parabolic and rectangular. *ii*) The nanoparticle MoS_2 widely used in biomedicine to treat cancer cells. Besides, it is used in spanning catalysis, optoelectronics, as lubricants in aerospace and automobile industries. Moreover, in this research, authors are also focusing to present the importance of HPM to solve nonlinear ODE with adiabatic boundary conditions. HPM technique provides series form of convergent solutions, which are flexible for converting to computations terms and in some special cases, it is possible to get the solutions with high accuracy using only one iteration.

The present analysis is structured as follows: Section 1 provides introduction, literature review and core object of the present work. Section 2 describes the mathematical formulation of the problem. In section 3, governing equations are solved by using homotopy perturbation method. Further, in section 4, significance of physical parameters on thermal performance, advantage of using nanofluid model on temperature distribution and validation of results with previously published work are presented through graphical representations and physical interpretations are illustrated. The concluding remarks are presented in section 5.

2. Mathematical Formulation

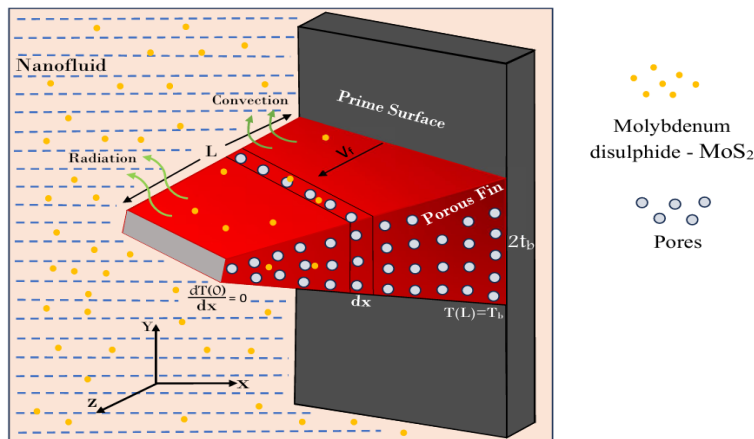


Figure 1: Moving radiative-convective fin with trapezoidal cross section.

This mathematical frame work describes one-dimensional temperature distribution in a longitudinal porous moving fin with a trapezoidal profile. A convective-radiative trapezoidal porous fin with fin base thickness $2t_b$, length L , width W and moving with constant velocity V_f is considered, as shown in figure 1. Heat is transferred from the body into the fin at a temperature T_b and is dissipated through the fin sides by convection and radiation at temperatures T_b and T_s respectively. The fin tip is assumed to be adiabatic. The coefficient of thermal conductivity, coefficient of convective heat transfer and surface emissivity were assumed to be temperature-dependent and can be taken as,

$$k = k_a^*[1 + \lambda(T - T_a)], \quad (1)$$

$$h_{cov} = h_D \left(\frac{T - T_a}{T_b - T_a} \right)^m, \quad (2)$$

$$\epsilon = \epsilon_s[1 + \beta(T - T_s)], \quad (3)$$

$$t(x) = t_b + \delta((x/L) - 1) \quad (4)$$

where, k_a^* is the coefficient of thermal conductivity at T_a , λ is the thermal conductivity coefficient, which is determined by the fin material, h_D is the coefficient of convective heat transfer with respect to the temperature difference $T_b - T_a$ and ϵ_s is the coefficient of emissivity at temperature T_s , β is the surface emissivity coefficient, $t(x)$ is the thickness of fin, the thickness of the fin tip is defined by the geometric parameter δ .

By energy balance equation,

$$\sum_{in} E_{n1} - \sum_{out} E_{n1} = \frac{dE_{n1}}{dt} \quad (5)$$

Here $\sum_{in} E_{n1}$ and $\sum_{out} E_{n1}$ is the rates of energy input and output, $\frac{dE_{n1}}{dt}$ denotes the energy variation in the control volume.

Substituting the energy terms, the steady-state energy balance equation reduces to,

$$q_{con,x} + \dot{M}_f h - \left(q_{con,x} + \frac{dq_{con,x}}{dx} dx \right) - \left(\dot{M}_f h - \frac{d(\dot{M}_f h)}{dx} dx \right) - 2q_{cov} - 2q_{rad} - 2M_f C_p (T - T_a) = 0 \quad (6)$$

$$- \frac{dq_{con,x}}{dx} dx + \frac{d(\dot{M}_f h)}{dx} dx - 2q_{cov} - 2q_{rad} - 2M_f C_p (T - T_a) = 0 \quad (7)$$

The energy equation governed by law of conservation of energy is obtained to be,

$$\frac{d}{dx}[k_a^*(T)t(x)]\frac{dT}{dx} + \frac{d}{dx}[\rho V_f t(x) C_p T] - h_{cov}(T)(T - T_a) - \epsilon(T)\sigma(T^4 - T_s^4) - \frac{\rho g K^* \beta (T - T_a)}{\nu} C_p (T - T_a)^2 = 0 \tag{8}$$

The boundary conditions considered are :

$$\frac{dT(0)}{dx} = 0 \quad \text{at } x = 0, \tag{9}$$

$$T(L) = T_b \quad \text{at } x = L. \tag{10}$$

Using dimensionless parameters as in table 1, thermal and physical properties and characteristics of nano fluid as in table 2 and table 3. The equation (8) reduces to following form,

Table 1 : The Dimensionless parameters

Non-dimensional numbers	Formulae
Dimensionless temperature(θ)	$\theta = \frac{T}{T_b}$
Dimensionless ambient temperature(θ_a)	$\theta_a = \frac{T_a}{T_b}$
Dimensionless surface temperature(θ_s)	$\theta_s = \frac{T_s}{T_b}$
Dimensionless axial co-ordinate(X)	$X = \frac{x}{L}$
Thermal conductivity parameter (A_1)	$A_1 = \lambda T_b$
Emissivity parameter (B_1)	$B_1 = \beta_f T_b$
Shape of fin cross section(C_1)	$C_1 = \frac{\delta}{t_b}$
Convective parameter(Nc)	$Nc = \frac{h_b L^2 T_b^m}{k_f t_b (T_b - T_a)^m}$
Radiative parameter(N_r)	$Nr = \frac{\sigma \epsilon_s L^2 T_b^3}{k_f t_b}$
Peclet(Pe)	$Pe = \frac{LV_f(\rho C_p)_f}{k_f}$
Shrewood number(Sh)	$Sh = \frac{g K^* (\rho \beta)_f (\rho C_p)_f T_b L^2}{\mu_f k_f t_b}$

Hamilton-Crosser model is employed to characterize the behaviour of spherical shaped nano particle. The applied model is illustrated in the below tables:

Table 2 : The Thermal and Physical properties of nanofluid

Properties	Nanofluid
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$
Heat capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$
Thermal expansion Coefficient	$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f + 2\phi(k_s - k_f)}{k_s + 2k_f - \phi(k_s - k_f)}$
Effective dynamic viscosity	$\mu_{nf} = \frac{\mu_f}{(1 + \phi)^{2.5}}$

Table 3 : The Thermal and Physical characteristics of water and Nano particle

Property	Water/Base fluid	MoS ₂
$\rho(kg/m^2)$	997.1	5060
$C_p(J/kg/K)$	4179	397.21
$k(W/mK)$	0.613	904.4
$\beta \times 10^{-5}(1/K)$	21	2.8424

Using the non dimensional numbers (8) reduces to,

$$\frac{d}{dX} \left[[1 + A_1(\theta - \theta_a)] \frac{t(X)}{t_b} \frac{d\theta}{dX} \right] - Nc(\theta - \theta_a)^{m+1}(F_1) - Nr[1 + B_1(\theta - \theta_s)](\theta^4 - \theta_s^4)(F_1) - Pe \frac{t(X)}{t_b} \frac{d\theta}{dX} (F_1 F_2) - [Sh(\theta - \theta_a)^2 (F_1 F_2 F_3 F_4)] = 0. \tag{11}$$

where F_1, F_2, F_3 and F_4 are defined as,

$$F_1 = \left(\frac{k_f}{k_{nf}} \right), F_2 = \left(\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \right), F_3 = \left(\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} \right), F_4 = \left(\frac{\mu_f}{\mu_{nf}} \right). \tag{12}$$

The associated reduced boundary conditions are,

$$\theta'(0) = 0, \theta(1) = 1. \tag{13}$$

The rate of heat transfer from the fin to its surrounding environment is defined as:

$$Q_f = \int_{X=0}^{X=1} [Nc(\theta - \theta_a)^{m+1} + Nr[1 + B_1(\theta - \theta_s)](\theta^4 - \theta_s^4)] dX \tag{14}$$

Fin efficiency is defined as the ratio of the rate of heat transfer from the fin in real-life conditions to the rate if the temperature of the whole fin is assumed to be equal to its base temperature:

$$\eta = \frac{\int_{X=0}^{X=1} [Nc(\theta - \theta_a)^{m+1} + Nr[1 + B_1(\theta - \theta_s)](\theta^4 - \theta_s^4)] dX}{Nc(1 - \theta_a)^{m+1} + Nr[1 + B_1(1 - \theta_s)](1 - \theta_s^4)} \tag{15}$$

3. Homotopy Perturbation Method

The Homotopy Perturbation Method is a semi-analytical technique used to solve non-linear differential equation introduced by Ji-Huan He in 1999 [36, 37, 38].

We consider the following non linear ODE

$$\xi(u) - \phi(r) = 0, r \in \Omega \tag{16}$$

with boundary condition

$$\zeta \left(u, \frac{\partial u}{\partial X} \right) = 0, r \in \Gamma \tag{17}$$

where $\xi(u)$ and ζ are the general differential and boundary operators, respectively. $\phi(r)$ represents a specified analytical function, Γ is the boundary of the domain Ω .

where $\xi(u)$ is defined as follows:

$$\xi(u) = L(u) + N(u) \tag{18}$$

where L stands for the linear and N stands for the non-linear part, (16) rewritten as :

$$L(u) + N(u) - \phi(r) = 0 \tag{19}$$

To implement the HPM, a homotopy $\theta(r, q) : \Omega \times [0, 1] \rightarrow R$ is defined to satisfy:

$$H(\theta, q) = (1 - q)[L(\theta) - L(u_0)] + q[L(\theta) + N(\theta) - \phi(r)] = 0 \tag{20}$$

By simplifying Eq.(19) and Eq.(20) can be obtained as:

$$H(\theta, q) = L(\theta) - L(u_0) + qL(u_0) + q[N(\theta) - \phi(r)] = 0 \tag{21}$$

Here $q=0$ Eq.(20) become linear equation and $q=1$ Eq.(21) become non-linear equation :

$$H(\theta, 0) = [L(\theta) + L(u_0)] = 0 \tag{22}$$

$$H(\theta, 1) = [L(\theta) + N(\theta) - \phi(r)] = 0 \tag{23}$$

The transformation of q from 0 to 1 introduces non-linearity $L(\dot{\nu}) - L(\dot{\lambda}_0) = 0$ into original linear equation $\xi(\dot{\nu}) - \phi(\dot{\lambda}_0) = 0$. The solution to Eq.(20) and Eq.(21) is assumed to be given as a power series in q :

$$\theta = \theta_0 + q\theta_1 + q^2\theta_2 + q^3\theta_3 + q^4\theta_4 + \dots \tag{24}$$

Substituting $q=1$

$$\theta = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \dots \tag{25}$$

3.1 The HPM Solution to the problem

$$\begin{aligned} H(\theta, q) = & L(\theta) - L(u_0) + qL(u_0) \\ & + q \left[\left(A_1(\theta - \theta_a) \left(\frac{t(X)}{t_b} - 1 \right) + A_1(\theta - \theta_a) + \left(\frac{t(X)}{t_b} - 1 \right) \right) \frac{d^2\theta}{dX^2} \right. \\ & + A_1 \frac{t(X)}{t_b} \left(\frac{d\theta}{dX} \right)^2 + \frac{1}{t_b} (1 + A_1(\theta - \theta_a)) \frac{dt(X)}{dX} \frac{d\theta}{dX} \\ & - Pe \frac{t(X)}{t_b} \frac{d\theta}{dX} (F_1 F_2) - Nc(\theta - \theta_a)^{m+1} (F_1) \\ & \left. - Nr [1 + B_1(\theta - \theta_s)] (\theta^4 - \theta_s^4) (F_1) - Sh(\theta - \theta_a)^2 (F_1 F_2 F_3 F_4) \right] = 0 \end{aligned} \tag{26}$$

Comparing the co-efficients of q powers we get,

$$q^0 : \frac{d^2\theta_0}{dX^2} - \frac{d^2u_0}{dX^2} = 0 \tag{27}$$

$$\begin{aligned} q^1 : & \frac{d^2\theta_1}{dX^2} + \frac{d^2u_0}{dX^2} + \left[A_1 C_1 (X - 1) (\theta_0 - \theta_a) \frac{d^2\theta_0}{dX^2} \right] + A_1 (\theta_0 - \theta_a) \frac{d^2\theta_0}{dX^2} \\ & + C_1 (X - 1) \frac{d^2\theta_0}{dX^2} + A_1 [C_1 (X - 1) + 1] \left(\frac{d\theta_0}{dX} \right)^2 + C_1 \left[A_1 (\theta_0 - \theta_a) \right. \\ & \left. + 1 \right] \frac{d\theta_0}{dX} - Pe [C_1 (X - 1)] \frac{d\theta_0}{dX} (F_1 F_2) - Nc (\theta_0 - \theta_a)^{m+1} (F_1) \\ & - Nr B_1 (\theta_0^5 - \theta_0^4 \theta_s - \theta_s^4 \theta_0 + \theta_s^5) (F_1) - Nr (\theta_0^4 - \theta_s^4) (F_1) \\ & - Sh (\theta_a^2 - 2\theta_a \theta_0 + \theta_0^2) (F_1 F_2 F_3 F_4) = 0 \end{aligned} \tag{28}$$

$$\begin{aligned}
q^2 : \quad & \frac{d^2\theta_2}{dX^2} + \left[A_1 C_1 (X-1) \left(\frac{d^2\theta_1}{dX^2} \theta_0 + \frac{d^2\theta_0}{dX^2} \theta_1 - \frac{d^2\theta_1}{dX^2} \theta_a \right) \right] + A_1 \left(\frac{d^2\theta_1}{dX^2} \theta_0 \right. \\
& \left. + \frac{d^2\theta_0}{dX^2} \theta_1 - \frac{d^2\theta_1}{dX^2} \theta_a \right) C_1 (X-1) \frac{d^2\theta_1}{dX^2} + 2A_1 [C_1 (X-1) + 1] \frac{d\theta_0}{dX} \frac{d\theta_1}{dX} \\
& + A_1 C_1 \left(\frac{d\theta_1}{dX} \theta_0 + \frac{d\theta_0}{dX} \theta_1 - \frac{d\theta_1}{dX} \theta_a \right) + C_1 \frac{d\theta_2}{dX} - Pe [C_1 (X-1) + 1] \frac{d\theta_1}{dX} (F_1)(F_2) \\
& - Nc(m+1)\theta_1(\theta_0 - \theta_a)^m (F_1) - [Sh(2\theta_0\theta_1 - 2\theta_1\theta_a)(F_1 F_2 F_3 F_4)] \\
& - Nr(4\theta_0^3\theta_1 + B_1(5\theta_0^4\theta_1 - 4\theta_0^3\theta_1\theta_s - \theta_1\theta_s^4))(F_1) = 0
\end{aligned} \tag{29}$$

To get the solution that is physically meaningful, it was assumed that $\theta(0) = K$ where K is an arbitrary constant. The boundary conditions are calculated from equation as

$$\theta = \theta_0(0) + q\theta_1(0) + q^2\theta_2(0) + q^3\theta_3(0) + q^4\theta_4(0) + \dots = K \tag{30}$$

$$\frac{d\theta(0)}{dX} = \frac{d}{dX} [\theta_0(0) + q\theta_1(0) + q^2\theta_2(0) + q^3\theta_3(0) + q^4\theta_4(0) + \dots] = 0 \tag{31}$$

Arranging coefficients of equal powers of q , one obtains the boundary conditions as:

$$\theta_0(0) = K, \theta_1(0) = \theta_2(0) = \theta_3(0) = \theta_4(0) = \dots = 0, \tag{32}$$

$$\frac{d\theta_0(0)}{dX} = \frac{d\theta_1(0)}{dX} = \frac{d\theta_2(0)}{dX} = \frac{d\theta_3(0)}{dX} = \frac{d\theta_4(0)}{dX} = \dots = 0 \tag{33}$$

Solving equations from Eqs.(22)-(25) with the boundary conditions (28) and (29), we obtain $\theta_0, \theta_1, \theta_2, \theta_3, \dots$

$$\theta_0 = u_0 = K$$

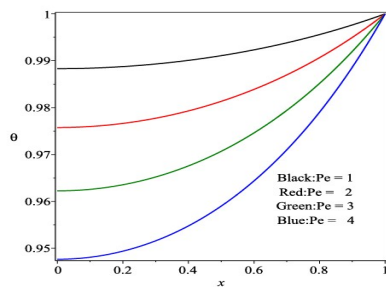
$$\begin{aligned}
\theta_1 = \frac{X^2}{2} \left[B_1 K^5 Nr F_1 - Nc F_1 (K - \theta_a)^m \theta_a + Nr F_1 \theta_s^4 (-1 + B_1 \theta_s) + K^4 (Nr F_1 \right. \\
\left. - B_1 Nr F_1 \theta_s) + K (Nc F_1 (K - \theta_a)^m - B_1 Nr F_1 \theta_s^4) + Sh F_1 F_2 F_3 F_4 (K^2 + \theta_a^2 - 2K\theta_a) \right] \\
\theta_2 = \left[\left(\frac{A_1 C_1 X^2}{2} - \frac{A_1 C_1 X^3}{6} \right) (K - \theta_a) - A_1 (K - \theta_a) \frac{X^2}{2} - C_1 \frac{X^3}{6} + C_1 \frac{X^2}{2} \right. \\
\left. - A_1 C_1 (K - \theta_a) \frac{X^3}{6} - C_1 \frac{X^3}{6} - Pe F_1 F_2 C_1 \frac{X^4}{12} + Pe F_1 F_2 C_1 \frac{X^3}{6} - Pe F_1 F_2 \frac{X^3}{6} \right. \\
\left. + Nc F_1 (m+1) (K - \theta_a)^m \frac{X^4}{24} + 2Sh F_1 F_2 F_3 F_4 (K - \theta_a) \frac{X^4}{24} + \frac{X^4}{24} Nr F_1 (4K^3 \right. \\
\left. + 5B_1 K^4 - 4B_1 K^3 \theta_s - B_1 \theta_s^4) \right] (B_1 K^5 Nr F_1 - Nc F_1 (K - \theta_a)^m \theta_a \\
+ Nr F_1 \theta_s^4 (-1 + B_1 \theta_s) + K^4 (Nr F_1 - B_1 Nr F_1 \theta_s) + K (Nc F_1 (K - \theta_a)^m \\
- B_1 Nr F_1 \theta_s^4) + Sh F_1 F_2 F_3 F_4 (K^2 + \theta_a^2 - 2K\theta_a))
\end{aligned}$$

The boundary condition $\theta(1) = 1$ allows us to solve for K , providing a specific value that makes the approximate solution compatible with the given constraints.

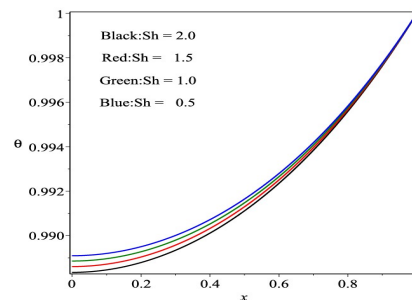
4. Results and Discussion

The solution obtained by HPM has been displayed through graphs and discussed parametrically. The nonlinear parameters Pe , Sh , Nr and Nc that impact the temperature performance have been analyzed with the help of plotted graphs, keeping the other constant values considered for the analysis: $A_1 = 0.5$, $B_1 = 1$, $C_1 = 0.2$, $\theta_a = 1$, $\theta_s = 1$.

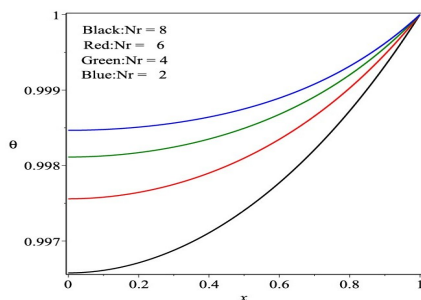
In figure 2, sub figure (2a) provides the impact of speed of moving fin on thermal distribution of the system. There is a gradual decrease in temperature profile as the pecllet number values increases because



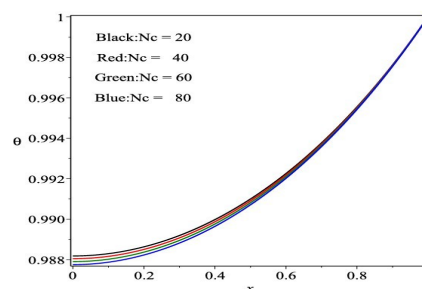
(2a) Change of dimensionless temperature distribution with rising Pe values for trapezoidal profile.



(2b) Change of dimensionless temperature distribution with rising Sh values for trapezoidal profile.



(2c) Change of dimensionless temperature distribution with rising Nr values for trapezoidal profile.



(2d) Change of dimensionless temperature distribution with rising Nc values for trapezoidal profile.

Figure 2: Variation of temperature profile with non dimensional parameters

of increasing advective effect on the surface of fin. As a result exposure time of the fin to the surrounding fluid reduces during the heat loss process through convection.

Sub figure (2b) illustrates the relationship between the temperature and porosity parameter, revealing a decrease in temperature as the value of porosity increases. This phenomenon occurs because larger porosity enhances heat transfer due to changes in the permeability parameter, driven by the combination of Darcy and Rayleigh numbers. As the Rayleigh number amplifies, fin permeability intensifies, boosting fluid flow through pores and increasing Buoyancy force and convection. This results in improved thermal transfer from the fin.

The impact of the radiative parameter on the temperature profile is depicted in sub figure (2c). From the graph, it is evident that increasing the radiative parameter results in a decrease in temperature. This is due to a pronounced radiation effect that results in heat dissipation from the fin's surface, leading to a cooling effect on the fin.

Sub figure (2d) illustrates the impact of dimensionless conductive parameter on the temperature distribution along a porous trapezoidal fin's surface. As the convective parameter increases, the temperature profile decreases due to enhanced heat transfer to the surroundings from the fin surface. Correspondingly, temperature declines and leads to increase in heat transmission rate of the system.

In figure 3, sub figures (3a), (3b), (3c) and (3d) show the comparison of dimensionless parameters such as Pe, Sh, Nr and Nc in the presence and absence of nanofluid. From the graphs, it is noticed that in the presence of nanofluid, the temperature distribution is more when compared with the absence of nanofluid. This is because, nanofluid play an important role for enhancing the efficiency of the heat transfer in the fins, as it is having higher thermal conductivity compared to convectational fluids. They reduce thermal resistance and increase the dissipation rate of heat.

5. Validation

Figure 4 presents a comparison of the current work with and without the porosity term, omitting the nanofluid model. It is noted that Arman et al. [39] and the current work are in good agreement.

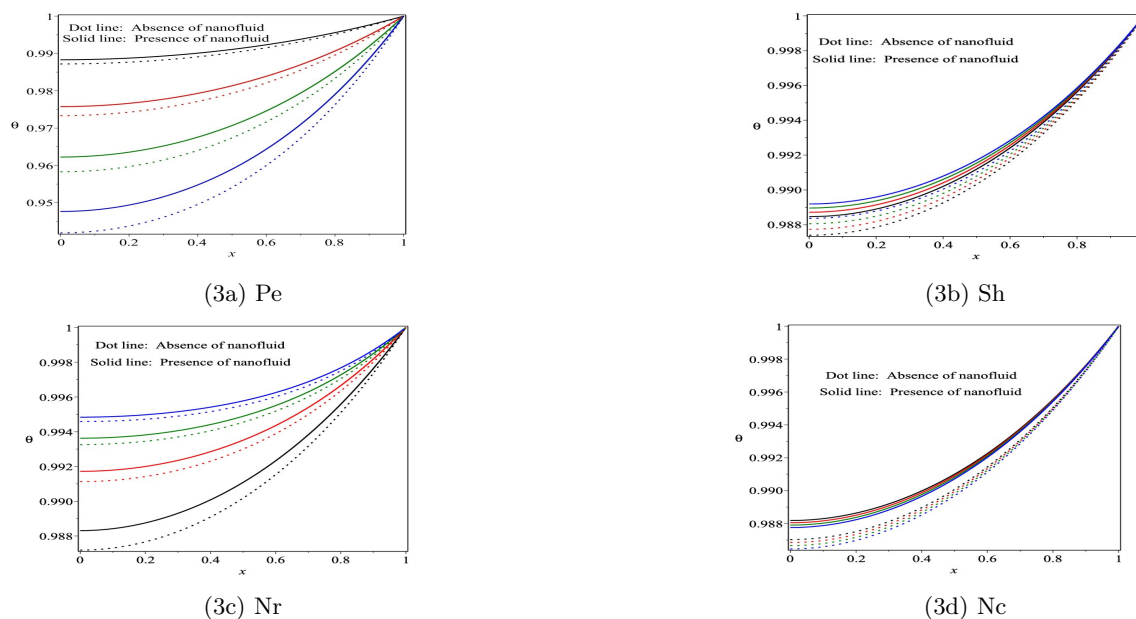


Figure 3: Comparison of Dimensionless parameters with and without Nanofluid.

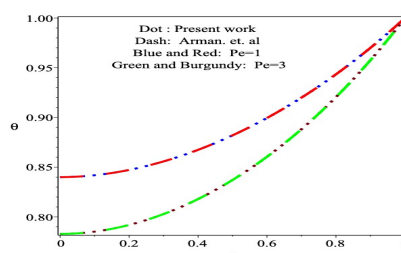


Figure 4: Validation of Peclet number.

6. Conclusion

The thermal behavior of a moving longitudinal fin of trapezoidal profile immersed in MoS_2 -water mononanofluid has been studied. The HPM technique is employed to solve the second order nonlinear ordinary differential equation obtained from solving the governing equation. The effect of dimensionless parameters in the reduced equation has been analyzed graphically. The below are some of the most significant findings of the report:

- The Homotopy Perturbation Method (HPM) is a powerful tool for solving complicated nonlinear equations, particularly used in analyzing the thermal efficiency of radiative-convective moving fins.
- With rise in pecelet number Pe , temperature distribution decreases, due to enhanced convective transport.
- An ascending value of porosity parameter Sh and radiative parameter Nr , decreases the temperature distribution.
- For higher convective parameter Nc reduces the dimensionless temperature of the fin base.
- The comparison of dimensionless parameters like Pe , Sh , Nr and Nc in the presence and absence of nanofluid is depicted in figure 6.6. The graphs show that the temperature distribution increases in the presence of nanofluid than it is in the absence of nanofluid.
- Figure 6.7 compare the present work by neglecting the nanofluid model and in the absence of porosity term. From the graph, it is observed that the present work is in good agreement with Arman et al. [39].
- Molybdenum disulphide (MoS_2) significantly enhance the thermal performance of trapezoidal fin. From the graph it is noticed that in the presence of Molybdenum disulphide (MoS_2) the temperature distribution enhances.

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