Utility of Intuitionistic Fuzzy Optimization Technique in Solving an Inventory Model

Soumen Banerjee

Department of Mathematics, Raja Peary Mohan College, Uttarpara, Hooghly, West Bengal (A Govt. Aided College)

Abstract: ***A probabilistic inventory model under uniform demand is discussed here, considering deterministic constraint. The model is analyzed using fuzzy geometric programming technique and intuitionistic fuzzy optimization techniques and it is observed that more optimized value of the objective function is obtained in case of intuitionistic fuzzy optimization technique.***

Keywords: Multi Objective Stochastic Inventory model, Fuzzy Geometric Programming, Intuitionistic Fuzzy Optimization

1. Introduction

Yang and Cao (2005) analyzed geometric programming with max-product fuzzy relation equation constraints. They (2006, 2007) also considered two different problems based on geometric programming. Wu. (2008). Described optimizing the geometric programming problem with single-term exponents subject to max-min fuzzy relational equation constraints. Ouyang and Chang (2002) attempted to apply the fuzzy set concepts to deal with the ambiguous lost sales rate. Several fuzzy models for single-period inventory problem were discussed by Lushu, Kabadi and Nair (2002). Mahapatra, G.S. and Roy, T.K. (2006) used General Fuzy Programming technique on a reliability optimization model. Cao (1993) and his recent book (2002) discussed fuzzy geometric programming with zero degree of difficulty. Das et. al. (2000) developed a multi-item inventory model with quantity dependent inventory costs and demand dependent unit cost under imprecise objective function and constraint and solved by GP technique. Mondal et. al. (2005) developed a multi-objective inventory model and solved it by GP method. A multi-objective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004). Islam and Roy (2006) solved another fuzzy economic production quantity model under space constraint by GP method. Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov[1999]. Intuitionistic fuzzy soft sets are considered by Maji Biswas and Roy(2001). Nikolova, Nikolov, Cornelis and Deschrijver(2002) presented a Survey of the research on intuitionistic fuzzy sets. Rough intuitionistic fuzzy sets are analyzed by Rizvi, Naqvi and Nadeem(2002). Angelov (1997) implemented the Optimization in an intuitionistic fuzzy environment. Pramanik and Roy (2005) solved a vector optimization problem using an Intuitionistic Fuzzy goal programming. A transportation model is solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming.

In this paper, a stochastic inventory model under uniform demand is discussed here. . It is illustrated numerically that the proposed intuitionistic Fuzzy Optimization is more effective than the Fuzzy Geometric Programmig Technique..

2. Mathematical Model

2.1 Backorder Case: Stock out Cost per Unit

Here the policy is to order a lot size Q when the inventory level drops to a reorder point r ant it is supposed that the inventory position of an item is monitored after every transaction. The demand in any given interval of time is a random variable and the expected value of demand in a unit of time, say a year, is D. We let x denote the demand during the lead-time and f (x) denote its probability distribution.

With backorders, there is no loss of sales, since the customer awaits the arrival of the order if stock is not available. The expected safety stock is defined as

$$S = \int_0^\infty (r-x)\,f(x)dx = r\int_0^\infty f(x)dx - \int_0^\infty xf(x)dx = r-\bar{x}$$

The number of backorders per lead-time is zero if x – r < 0 and x – r if x – r > 0. The expected number of backorders per lead-time is

$$E\,(x>r) = \int_r^\infty (x-r)\,f(x)dx$$

Here, annual safety stock cost = holding cost + stock out cost

$$\text{i.e. } TC = SH + \frac{AD}{Q}\int_r^\infty (x-r)\,f(x)dx$$

$$= H(r-\bar{x}) + \frac{AD}{Q}\int_r^\infty (x-r)\,f(x)dx$$

The following mathematical notations are used:
For the i^{th} item:-
r_i = reorder point in units,
S_i = safety stock in units,
H_i = holding cost per unit of inventory per year,
A_i = backordering cost per unit,
x = lead time demand in units (a random variable),

$\bar{x}$ = average lead time demand in units,
$x - r_i$ = size of stock out in units
p_i = purchasing price of each product
TC = expected annual cost of safety stock,
F= total floor space area
B=total budget

2.2 Multi Objective Stochastic Inventory Model with Deterministic Constraint

$$MinTC_i(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n) =$$
$$S_i H_i + \frac{K_i D_i}{Q_i} \int_{r_i}^{\infty} (x - r_i) f_i(x) dx$$

subject to the constraints

$$\sum_{i=1}^{n} p_i Q_i \le B$$
$$Q_i, r_i > 0 \forall i = 1,2,......., n. \quad (2.1)$$

3. Mathematical Analysis

3.1 Fuzzy Geometric Programming Problem

Multi-objective geometric programming (MOGP) is a special type of a class of MONLP problems. Biswal (1992) and Verma (1990) developed a fuzzy geometric programming technique to solve a MOGP problem. Here, we have discussed a fuzzy geometric programming technique based on max-min and max-convex combination operators to solve a MOGP.

To solve the MOGP we use the Zimmerman's technique. The procedure consists of the following steps.

Step 1. Solve the MOGP as a single GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions. Repeat the process k times for k different objectives. Let x^1, x^2, ………, x^k be the ideal solutions for the respective objective functions, where

$$x^r = (x_1^r, x_2^r, ………………….., x_n^r)$$

Step 2. From the ideal solutions of Step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each solution, the pay-off matrix of size (k x k) can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ .... \\ x^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & .... & f_k(x) \end{array} \\ \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & .... & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & .... & f_k(x^2) \\ .... & .... & .... & .... \\ f_1(x^k) & f_2(x^k) & .... & f_k^*(x^k) \end{bmatrix} \end{array}$$

Step 3. From the Step 2, find the desired goal L_r and worst tolerable value U_r of $f_r(x)$, r = 1, 2, …….., k as follows:
$L_r \le f_r \le U_r$, r = 1, 2, …….., k
Where, $U_r = \max \{f_r(x^1), f_r(x^2),…….,f_r(x^k)\}$
$L_r = \min \{f_r(x^1), f_r(x^2),…….,f_r(x^k)\}$

Step 4. Define a fuzzy linear or non-linear membership function $\mu_r [f_r(x)]$ for the r-th objective function $f_r(x)$, r = 1, 2, …….., k
$\mu_r [f_r(x)] = 0$ or $\to 0$ if $f_r(x) \ge U_r$
$= d_r(x)$ if $L_r \le f_r(x) \le U_r$ (r = 1, 2, …… , k)
$= 1$ or $\to 1$ if $f_r(x) \le L_r$
Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Step 5. At this stage, either a max-min operator or a max-convex combination operator can be used to formulate the corresponding single objective optimization problem.

A. Through a Max-Min operator

According to Zimmerman (1978) the problem can be solved as:

$$\mu_D(x^*) = Max(Min(\mu_1(f_1(x)), \mu_2(f_2(x)),........, \mu_k(f_k(x))))$$

subject to
$g_j(x) \le b_j$, j=1, 2, ….., m, x > 0
which is equivalent to the following problem as:
Max α …(3.1)
Subject to
$\alpha \le \mu_r [f_r(x)]$, for r = 1, 2, …….., k
$g_j(x) \le b_j$, j=1, 2, ….., m, x > 0

The parameter α is called an aspiration level and represents the compromise among the objective functions. After reducing the problem into a standard form of a PGP problem, it can be solved through a GP technique.

3.2 Formulation of Intuitionistic Fuzzy Optimization [IFO]

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints, we can write:

$$\max \mu_i(\bar{X}), \bar{X} \varepsilon R, i = 1,2,......, K + n$$
$$\min \upsilon_i(\bar{X}), \bar{X} \varepsilon R, i = 1,2,......, K + n$$

Subject to

$$\upsilon_i(\bar{X}) \ge 0,$$
$$\mu_i(\bar{X}) \ge \upsilon_i(\bar{X})$$
$$\mu_i(\bar{X}) + \upsilon_i(\bar{X}) < 1$$
$$\bar{X} \ge 0$$

Where $\mu_i(\bar{X})$ denotes the degree of membership function of $(\bar{X})$ to the i^{th} IF sets and $\nu_i(\bar{X})$ denotes the degree of non-membership (rejection) of $(\bar{X})$ from the i^{th} IF sets.

4. A Stochastic Model: Demand follows Uniform distribution

We assume that demand for the period for the i^{th} item is a random variable which follows uniform distribution and if the decision maker feels that demand values for item i below a_i or above b_i are highly unlikely and values between a_i and b_i are equally likely, then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \begin{cases} \frac{1}{b_i - a_i} & \text{if } a_i \le x \le b_i \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2, \ldots, n.$$

$$TC_i(Q_i, r_i) = H_i(r_i - \mu_i) + \frac{K_i D_i (b_i - r_i)^2}{2Q_i(b_i - a_i)}$$

Where, $\mu_i = \frac{a_i + b_i}{2}$.

5. Numerical Examples

To solve the model (2.1) we consider the following data:
$H_1 = 9$; $D_1=2300$; $a_1=15$; $b_1=40$; $K_1 = 10$; $H_2 = 10$; $D_2=2000$; $a_2=30$; $b_2=50$; $K_2 = 9$; B=12000.
[All the cost related terms are measured in '$']

Using section 3, section 4 and by the help of the above mentioned data the solution of Table - 1 is obtained.

Table 1: Solution of the model (2.1) with Uniform Demand by FGPT and IFO

METHOD	TC_1^*($)	TC_2^*($)	Q_1	Q_2	r_1	r_2	ASPIRATION LEVEL
IFO	32.54	1710.78	411	358	29	22	μ_1=0.892 μ_2=0.774
FGPT	36.76	1722.84	429	377	32	31	μ_1=0.881 μ_2=0.790

From Table - 1 we conclude that, Intuitionistic Fuzzy Optimization Technique obtained more minimized values of TC_1 and TC_2, in comparison to Fuzzy Geometric Programming Technique [FGPT].

6. Conclusion

A Multi-objective stochastic inventory model with Uniform demand is discussed here. The stockout cost is more minimized in case of Intuitionistic Fuzzy Optimization technique than fuzzy geometric programming technique. So, utility of Intuitionistic Fuzzy Optimization technique is established here. Like uniform distribution, exponential, normal distribution can also be considered as lead-time demand.

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