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K-Banhatti Indices, Polynomials, K-Banhatti Sombor Indices, Multiplicative Gourava Indices of OTIS Swapped, Bi-Swapped and K-Swapped Networks

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Abstract: ***Let G be connected graph with vertex set V(G) and edge set E(G).The first K-Banhatti index is defined as $B_1(G) = \sum_{ue}(d_G(u) + d_G(e))$, where $d_G(e)=d_G(u)+d_G(v)-2$ and e = uv, u~e for the vertex u and an edge e are adjacent in the graph G [1]. In this paper some K-Banhatti indices, polynomials, K-Banhatti Sombor indices, multiplicative Gourava indices and sum degree-based indices are studied for OTIS swapped, Bi-swapped and K-swapped networks.***

Keywords: K-Banhatti indices, polynomials, K-Banhatti Sombor indices, multiplicative Gourava indices, OTIS swapped network and sum degree-based indices

1. Introduction

Let G be a simple, finite, connected graph with vertex set V(G) and edge set E(G). The degree of a vertex u ∈ V(G) is denoted by $d_G(u)$ and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv [2]. A topological index is a numerical parameter mathematically derived from the graph structure.

The degree and valence of a compound in chemistry are closely related with each other [3]. The fifth GA index which is sum degree-based index was introduced in [4]. K-Banhatti indices for the molecular graph were studied in many papers like [5-8]. A new interconnection network is proposed based on a tree height h, T_h and a hypercube of dimension d, Q_d topologies, referred to as the chained-cubic tree, CCT(h,d) were investigated in [9].Optical transpose interconnection systems present new optoelectronic computer architecture that takes benefits from both optical and electronic technologies [10-11]. OTIS networks have a base graph G on n vertices and consists of n disjoint copies of G [12]. OTIS networks are implemented using free-space optoelectronic technologies [13]. A method where edges in a graph are swapped to improve properties like connectivity, efficiency or shortest paths. Vertex swapping changing node connections, while preserving degree sequences, often used in randomization techniques. Multiplicative Gourava indices of some nanotubes were studied in [14] and inverse multiplicative fifth sum connectivity index and multiplicative fifth sum connectivity index in [15].

In an OTIS network, a given network like mesh is duplicated and nodes in one copy are swapped in a structured manner with nodes in the other. Let R_k be a k-regular graph on n vertices and OR_k be OTIS swapped network with the basis R_k,then OR_k network has two types of edges based on the degree of end vertices [16-20]:$E_{(k,k+1)}$ and $E_{(k+1,k+1)}$.

Bi-swapping extends the standard OTIS transpose swap by adding a second. In Bi-swapped OTIS network each node swaps twice, meaning it may have two interlayer degrees instead of one. Bi-swapped network is a k+1 regular graph that is: the set of neighbors is the same for every vertex. Thus, the sum of degrees of vertices adjacent to every vertex u is $(k+1)^2$.The $Bsw(R_k)$ is a k+1 regular graph of order $2n^2$ and size $(k+1)n^2$ [21-23]. In a K-swapping OTIS network, instead of swapping nodes directly, nodes are swapped in a controlled manner based on K. The value of K determines swapping patterns, affecting edge connections. In a K-swapped OTIS network, the swapping rule is altered by an integer parameter, modifying the connection between layers. Let G be K-swapped network of t-regular graph, then K-swapped network has degree, d = t+K-1 and size $\frac{dKn^2}{2}$ [24].

Some K-Banhatti indices such as $B_1(G)$, $B_2(G)$, $HB_1(G)$, $HB_2(G)$, SB(G), PB(G), ${}^{m}B_1(G)$, ${}^{m}B_2(G)$ and $H_b(G)$ of chloroquine and hydroxychloroquine were computed by V.R.Kulli [25] for Covid-19, these topological indices are defined by equations (1-9). The first and the second K-Banhatti polynomials for graph a G are defined by equations (10-11) [26], where $d_G(e)$ is degree of an edge e in G, defined as $d_G(e)=d_G(u)+d_G(v)-2$. The Banhatti Sombor index and reduced Banhatti Sombor index are defined and studied in [27], we introduce K-Banhatti Sombor index and reduced K-Banhatti Sombor index as represented by equations (12-13). Multiplicative reduced Sombor index was introduced by Amin et al.in [28]. Multiplicative Gourava indices are defined by equations (14-17). The fifth M_1 and M_2 multiplicative Zagreb indices were studied in [29]. Fifth multiplicative sum connectivity index and inverse fifth multiplicative sum connectivity index are defined by equations (18-19) and multiplicative reduced Sombor index by equation (20). The Sanskruti index S(G) of a graph G was studied by Hosamani [30] which is sum degree-based index, represented by equation (21).

1) First K-Banhatti index: $B_1(G)=\sum_{ue}[d_G(u) + d_G(e)]$.
2) Second K-Banhatti index: $B_2(G)=\sum_{ue}[d_G(u) \times d_G(e)]$.

3) First hyper K-Banhatti index: $HB_1(G)= \sum_{ue}[d_G(u) + d_G(e)]^2$.
4) Second hyper K-Banhatti index: $HB_2(G)= \sum_{ue}[d_G(u) \times d_G(e)]^2$.
5) Sum connectivity K-Banhatti index: $SB(G)= \sum_{ue} \frac{1}{\sqrt{[d_G(u)+d_G(e)]}}$.
6) Product connectivity K-Banhatti index: $PB(G)= \sum_{ue} \frac{1}{\sqrt{[d_G(u)\times d_G(e)]}}$.
7) Modified first K-Banhatti index: ${}^m_1B(G) = \sum_{ue} \frac{1}{[d_G(u)+d_G(e)]}$.
8) Modified second K-Banhatti index: ${}^m_2B(G) = \sum_{ue} \frac{1}{[d_G(u)\times d_G(e)]}$.
9) K-Banhatti harmonic index: $H_b(G) = \sum_{ue} \frac{2}{[d_G(u)+d_G(e)]}$.
10) First K-Banhatti polynomial: $KB_1(G,x)= \sum_{ue} x^{[d_G(u)+d_G(e)]}$.
11) Second K-Banhatti polynomial: $KB_2(G,x)= \sum_{ue} x^{[d_G(u)\times d_G(e)]}$.
12) K-Banhatti Sombor index: $BSO(G)= \sum_{ue} \sqrt{d_G(u)^2 + d_G(e)^2}$.
13) Reduced K-Banhatti Sombor index: $RBSO(G)= \sum_{ue} \sqrt{(d_G(u) - 1)^2 + (d_G(e) - 1)^2}$.
14) Multiplicative first Gourava index: $\Pi GO_1(G)= \prod_{uv\epsilon E(G)}[(d_G(u) + d_G(v)) + (d_G(u) \times d_G(v))]$.
15) Multiplicative second Gourava index: $\Pi GO_2(G)= \prod_{uv\epsilon E(G)}[(d_G(u) + d_G(v)) \times (d_G(u)d_G(v))]$.
16) Multiplicative hyper first Gourava index: $H\Pi GO_1(G)= \prod_{uv\epsilon E(G)}[(d_G(u) + d_G(v)) + (d_G(u) \times d_G(v))]^2$.
17) Multiplicative hyper second Gourava index: $H\Pi GO_2(G)= \prod_{uv\epsilon E(G)}[(d_G(u) + d_G(v)) \times (d_G(u)d_G(v))]^2$.
18) Multiplicative fifth sum connectivity index: $S_5\Pi(G) =\prod_{uv\epsilon E(G)} \frac{1}{\sqrt{S_u+S_v}}$.
19) Multiplicative inverse fifth sum connectivity index: $IS_5\Pi(G) =\prod_{uv\epsilon E(G)} \sqrt{S_u + S_v}$.
20) Multiplicative reduced Sombor index: $MRSO(G) = \prod_{uv\epsilon E(G)} \sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2}$.
21) Sanskruti index: $S(G) = \sum_{uv\in E(G)}(\frac{S_uS_v}{S_u+S_v-2})^3$.

All the symbols and notations used in this paper are standard and taken mainly from books of graph theory [31-33]. In this paper some K-Banhatti indices, polynomials, K-Banhatti Sombor indices, multiplicative Gourava indices and sum degree-based indices are studied for OTIS swapped, Bi-swapped and K-swapped networks.

2. Materials and Methods

The complete graph denoted by k_n with n vertices and OR_k be the swapped network. In a basic OTIS network, the swap operation follows the rule: Each node (i,j) in layer 1 is connected to node (j, i) in layer 2. If the base network has degree d, then in an OTIS network: the total degree is d+1. The graph OR_k contains n^2 vertices and $\frac{n^2(k+1)-n}{2}$ edges. Bi-swapped network is a k+1 regular graph that is the set of neighbors is the same for every vertex. Bi-swapped (R_k) is a k+1 regular graph of order $2n^2$ and size $(k+1)n^2$. In Bi-swapped network each node swaps twice, the total degree is d+2. Let G be K-swapped network of t-regular graph, then K-swapped network has degree, d = t+ K -1 and size $\frac{dKn^2}{2}$. The OTIS swapped network is shown in figure (1) and Bi-swapped network in figure (2).

3. Results and Discussion

OTIS swapped network OR_k

Let R_k be k-regular graph of n vertices and OR_k be OTIS swapped network with basis network R_k.

Theorem 1.1. First K-Banhatti index of OTIS swapped network is

$$(3k - 1)nk + (3k + 1)[\frac{n^2(k+1)-n(1+2k)}{2}] .$$

Proof. From table (1) and equation (1), we have

$B_1(OR_k)= \sum_{ue}[d_G(u) + d_G(e)]$
$= (k+2k-1)|E_1|+[(k+1)+2k]|E_2|$
$= (3k-1)nk+(3k+1)\left[\frac{n^2(k+1)-n(1+2k)}{2}\right]$.

Theorem 1.2. First hyper K-Banhatti index of OTIS swapped network is

$(3k-1)^2nk+(3k+1)^2[\frac{n^2(k+1)-n(1+2k)}{2}]$.

Proof. From table (1) and equation (3), we have

$HB_1(OR_k)= \sum_{ue}[d_G(u) + d_G(e)]^2$
$= [(k)+(2k-1)]^2|E_1|+[(k+1) + (2k)]^2|E_2|$
$= (3k-1)^2nk+(3k+1)^2[\frac{n^2(k+1)-n(1+2k)}{2}]$.

Theorem 1.3. Sum connectivity K-Banhatti index of OTIS swapped network is $\frac{nk}{\sqrt{3k-1}}+\frac{n^2(k+1)-n(1+2k)}{2\sqrt{3k+1}}$.

Proof. From table (1) and equation (5), we have

$SB(OR_k)= \sum_{ue} \frac{1}{\sqrt{[d_G(u)+d_G(e)]}}$
$= \frac{1}{\sqrt{k+(2k-1)}}|E_1|+\frac{1}{\sqrt{(k+1)+2k}}|E_2|$
$= \frac{nk}{\sqrt{3k-1}}+\frac{n^2(k+1)-n(1+2k)}{2\sqrt{3k+1}}$.

Theorem 1.4. Modified first K-Banhatti index of OTIS swapped network is $\frac{nk}{3k-1} +\frac{n^2(k+1)-n(1+2k)}{2(3k+1)}$.

Proof. From table (1) and equation (7), we have

${}^m_1B(OR_k) = \sum_{ue} \frac{1}{[d_G(u)+d_G(e)]}$
$= \frac{1}{k+(2k-1)}|E_1|+\frac{1}{(k+1)+2k}|E_2|$
$= \frac{nk}{3k-1} +\frac{n^2(k+1)-n(1+2k)}{2(3k+1)}$.

Theorem 1.5. First K-Banhatti polynomial of OTIS swapped network is $nkx^{3k-1}+\frac{n^2(k+1)-n(1+2k)}{2}x^{3k+1}$.

Proof. From table (1) and equation (10), we have

$KB_1(OR_k, x) = \sum_{ue} x^{[d_G(u)+d_G(e)]}$
$= |E_1|x^{k+(2k-1)}+|E_2|x^{(k+1)+2k}$

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$= nkx^{3k-1} + \frac{n^2(k+1)-n(1+2k)}{2}x^{3k+1}$.

Theorem 1.6. K-Banhatti Sombor index of OTIS swapped network is $nk\sqrt{k^2+(2k-1)^2} + \frac{n^2(k+1)-n(1+2k)}{2}\sqrt{(k+1)^2+4k^2}$.

Proof. From table (1) and equation (12), we have
$BSO(OR_k) = \sum_{ue}\sqrt{d_G(u)^2+d_G(e)^2}$
$= \sqrt{k^2+(2k-1)^2}|E_1| + \sqrt{(k+1)^2+4k^2}|E_2|$
$= nk\sqrt{k^2+(2k-1)^2} + \frac{n^2(k+1)-n(1+2k)}{2}\sqrt{(k+1)^2+4k^2}$.

Theorem 1.7. Multiplicative first Gourava index of OTIS swapped network is
$[(2k+1)+k(k+1)]^{nk} \times [2(k+1)+(k+1)^2]^{\frac{n^2(k+1)-n(1+2k)}{2}}$.

Proof. From table (1) and equation (14), we have
$\Pi GO_1(OR_k) = \prod_{uv\epsilon E(G)}[(d_G(u)+d_G(v))+(d_G(u)\times d_G(v))]$
$= [k+(k+1)+k(k+1)]^{|E_1|} \times [2(k+1)+(k+1)^2]^{|E_2|}$
$= [(2k+1)+k(k+1)]^{nk} \times [2(k+1)+(k+1)^2]^{\frac{n^2(k+1)-n(1+2k)}{2}}$.

Theorem 1.8. Sanskruti index of OTIS swapped network is $(\frac{2k^2}{2k-1})^3 nk$.

Proof. From figure (1) and equation (21), we have
$S(OR_k) = \sum_{uv\epsilon E(G)}(\frac{S_uS_v}{S_u+S_v-2})^3$, where S_u denote the degree sum of all vertices of G that are adjacent to u,
$= (\frac{2k\times 2k}{2k+2k-2})^3|E|$
$= (\frac{2k^2}{2k-1})^3 nk$.

Bi-swapped network

Let R_k be the k-regular graph of order n and $Bsw(R_k)$ be the Bi-swapped network with the basis R_k. It is observed from figure (2) that $Bsw(R_k)$ has order $2n^2$ and size $(k+1)n^2$.

Theorem 2.1. First K-Banhatti index of Bi-swapped network is $(k+1)(3k+1)n^2$.

Proof. From figure (2) and equation (1), we have
$B_1(Bsw(R_k)) = \sum_{ue}[d_G(u)+d_G(e)]$
$= (k+1+2k)|E|$
$= (k+1)(3k+1)n^2$.

Theorem 2.2. First hyper K-Banhatti index of Bi-swapped network is $(k+1)(3k+1)^2n^2$.

Proof. From figure (2) and equation (3), we have
$HB_1(Bsw(R_k)) = \sum_{ue}[d_G(u)+d_G(e)]^2$
$= [(k+1)+(2k)]^2|E|$
$= (k+1)(3k+1)^2n^2$.

Theorem 2.3. Sum connectivity K-Banhatti index of Bi-swapped network is $\frac{(k+1)n^2}{\sqrt{3k+1}}$.

Proof. From figure (2) and equation (5), we have
$SB(Bsw(R_k)) = \sum_{ue}\frac{1}{\sqrt{[d_G(u)+d_G(e)]}}$
$= \frac{1}{\sqrt{(k+1)+(2k)}}|E|$
$= \frac{(k+1)n^2}{\sqrt{3k+1}}$.

Theorem 2.4. Modified first K-Banhatti index of Bi-swapped network is $\frac{(k+1)n^2}{(3k+1)}$.

Proof. From figure (2) and equation (7), we have
${}^{m}_{1}B(Bsw(R_k)) = \sum_{ue}\frac{1}{[d_G(u)+d_G(e)]}$
$= \frac{1}{(k+1)+(2k)}|E|$
$= \frac{(k+1)n^2}{(3k+1)}$.

Theorem 2.5. First K-Banhatti polynomial of Bi-swapped network $(k+1)n^2x^{3k+1}$.

Proof. From figure (2) and equation (10), we have
$KB_1(Bsw(R_k),x) = \sum_{ue} x^{[d_G(u)+d_G(e)]}$
$= |E|x^{(k+1)+2k}$
$= (k+1)n^2x^{3k+1}$.

Theorem 2.6. K-Banhatti Sombor index of Bi-swapped network is $\sqrt{(k+1)^2+4k^2} \times (k+1)n^2$.

Proof. From figure (2) and equation (12), we have
$BSO(Bsw(R_k)) = \sum_{ue}\sqrt{d_G(u)^2+d_G(e)^2}$
$= \sqrt{(k+1)^2+4k^2} \times |E|$
$= \sqrt{(k+1)^2+4k^2} \times (k+1)n^2$.

Theorem 2.7. Multiplicative first Gourava index of Bi-swapped network is $[2(k+1)+(k+1)^2]^{(k+1)n^2}$.

Proof. From figure (2) and equation (14), we have
$\Pi GO_1(Bsw(R_k)) = \prod_{uv\epsilon E(G)}[(d_G(u)+d_G(v))+(d_G(u)\times d_G(v))]$
$= [2(k+1)+(k+1)^2]^{|E|}$
$= [2(k+1)+(k+1)^2]^{(k+1)n^2}$.

Theorem 2.8. Sanskruti index of Bi-swapped network is $\left(\frac{(k+1)^4}{2(k+1)^2-2}\right)^3(k+1)n^2$.

Proof. From figure (2) and equation (21), we have
$S(Bsw(R_k)) = \sum_{uv\epsilon E(G)}(\frac{S_uS_v}{S_u+S_v-2})^3$, where S_u denote the degree sum of all vertices of G that are adjacent to u,
$= \left(\frac{(k+1)^2\times(k+1)^2}{2(k+1)^2-2}\right)^3|E| = \left(\frac{(k+1)^4}{2(k+1)^2-2}\right)^3(k+1)n^2$.

K-swapped network

Theorem 3.1. First K-Banhatti index of K-swapped network is $(3t+3K-5)\left(\frac{t+K-1}{2}\right)Kn^2$.

Proof. From K-swapped network, equation (1), we have degree d=t+K-1, $|E| = \left(\frac{t+K-1}{2}\right)Kn^2$ and $d_G(e) = 2t+2K-4$, we have
$B_1(G) = \sum_{ue}[d_G(u)+d_G(e)]$

= [(t+ K -1)+(2t+2K -4)]|E|
= (3t+3K -5)$\left(\frac{t+K-1}{2}\right)$ Kn^2.

Theorem 3.2. First hyper K-Banhatti index of K-swapped network is (3t+3K -5)^2 $\left(\frac{t+K-1}{2}\right)$ Kn^2.

Proof. From K-swapped network, equation (3), we have degree d=t+K -1, |E|=$\left(\frac{t+K-1}{2}\right)$ Kn^2 and $d_G(e) = 2t + 2K - 4$,we have
$HB_1(G)= \sum_{ue}[d_G(u) + d_G(e)]^2$
= $[(t+ K -1)+(2t+2K -4)]^2|E|$
= (3t+3K -5)^2 $\left(\frac{t+K-1}{2}\right)$ Kn^2.

Theorem 3.3. Sum connectivity K-Banhatti index of K-swapped network is $\frac{1}{\sqrt{(3t+3K-5)}}\left(\frac{t+K-1}{2}\right)$ Kn^2.

Proof. From K- swapped network and equation (5), we have
$SB(G)= \sum_{ue}\frac{1}{\sqrt{[d_G(u)+d_G(e)]}}$
$= \frac{1}{\sqrt{(t+K-1)+(2t+2K-4)}}|E|$
$= \frac{1}{\sqrt{(3t+3K-5)}}\left(\frac{t+K-1}{2}\right) Kn^2$.

Theorem 3.4. Modified first K-Banhatti index of K-swapped network is $\frac{1}{(3t+3K-5)}\left(\frac{t+K-1}{2}\right)$ Kn^2.

Proof. From K- swapped network and equation (7), we have
${}^m_1B(G) = \sum_{ue}\frac{1}{[d_G(u)+d_G(e)]}$
$= \frac{1}{[(t+K-1)+(2t+2K-4)]}|E|$
$= \frac{1}{(3t+3K-5)}\left(\frac{t+K-1}{2}\right) Kn^2$.

Theorem 3.5. First K-Banhatti polynomial of K-swapped network is $(\frac{t+K-1}{2})Kn^2\ x^{3t+3K-5}$.

Proof. From K- swapped network and equation (10), we have
$KB_1(G,x) = \sum_{ue} x^{[d_G(u)+d_G(e)]}$
$= |E|x^{(t+K+1)+(2t+2K-4)}$
$= (\frac{t+K-1}{2})Kn^2x^{3t+3K-5}$.

Theorem 3.6. K-Banhatti Sombor index of K-swapped network is $[(t+K-1)^2+(2t+2K-4)^2]^{\frac{1}{2}}\left(\frac{t+K-1}{2}\right)Kn^2$.

Proof. From table (1) and equation (12), we have
$BSO(G)= \sum_{ue}\sqrt{d_G(u)^2 + d_G(e)^2}$
$= [(t+K-1)^2+(2t+2K-4)^2]^{\frac{1}{2}}|E|$
$= [(t+K-1)^2+(2t+2K-4)^2]^{\frac{1}{2}}\left(\frac{t+K-1}{2}\right)Kn^2$.

Theorem 3.7. Multiplicative first Gourava index of K-swapped network is $[2(t+K-1)+(t+K-1)^2]^{(\frac{t+K-1}{2})Kn^2}$.

Proof. From K-swapped network and equation (14), we have
$\Pi GO_1(G)= \prod_{uv\epsilon E(G)}[(d_G(u)+d_G(v))+(d_G(u)\times d_G(v))]$
$= [2(t+K-1)+(t+K-1)^2]^{|E|}$
$= [2(t+K-1)+(t+K-1)^2]^{(\frac{t+K-1}{2})Kn^2}$.

Theorem 3.8. Sanskruti index of K-swapped network is $(\frac{(t+K-1)^2}{2t+2K-4})^3\left(\frac{t+K-1}{2}\right)Kn^2$.

Proof. From K-swapped network and equation (21), we have
$S(G)= \sum_{uv\epsilon E(G)}(\frac{S_uS_v}{S_u+S_v-2})^3$,where S_u denote the degree sum of all vertices of G that are adjacent to u,
$= (\frac{(t+K-1)(t+K-1)}{2t+2k-4})^3|E|$
$= (\frac{(t+K-1)^2}{2t+2K-4})^3\left(\frac{t+K-1}{2}\right)Kn^2$.

The computed values of some topological indices of OTIS swapped, Bi-swapped and K-swapped network: $B_2(G)$, $HB_2(G)$,PB(G), ${}^m_2B(G)$, $H_b(G)$, $KB_2(G,x)$,RBSO(G), $\Pi GO_2(G)$, $H\Pi GO_1(G)$, $H\Pi GO_2(G)$, $S_5\Pi(G)$, $IS_5\Pi(G)$ and MRSO(G) are given in table (2).

Table 1: Edge partition of OTIS swapped network OR_k.

$(d_G(u), d_G(v))$	(k, k+1)	(k+1, k+1)
Number of edges	nk	$\frac{n^2(k+1)-n(1+2k)}{2}$

Table 2: Topological indices of OTIS swapped, Bi-swapped and K-swapped networks.

Toplogical indices/polynomial	OTIS swapped network	Bi-swapped network	K-swapped network
$B_2(G)$	$(2k-1)nk^2+2k(k+1)[\frac{n^2(k+1)-n(1+2k)}{2}]$	$2k(k+1)^2n^2$	(t+ K -1)(2t+2K -4)$\left(\frac{t+K-1}{2}\right)Kn^2$
$HB_2(G)$	$(2k-1)^2nk^3+2(k(k+1))^2\times[n^2(k+1)-n(1+2k)]$	$4(k+1)^3n^2k^2$	$[(t+K-1)\times(2t+2K-4)]^2\left(\frac{t+K-1}{2}\right)Kn^2$
PB(G)	$\frac{nk}{\sqrt{k(2k-1)}}+\frac{n^2(k+1)-n(1+2k)}{2\sqrt{(k+1)2k}}$	$\frac{(k+1)n^2}{\sqrt{(k+1)2k}}$	$\frac{1}{\sqrt{(t+K-1)\times(2t+2K-4)}}\left(\frac{t+K-1}{2}\right)Kn^2$
${}^m_2B(G)$	$\frac{n}{(2k-1)}+[\frac{n^2(k+1)-n(1+2k)}{4k(k+1)}]$	$\frac{n^2}{2k}$	$\frac{Kn^2}{2(2t+2K-4)}$
$H_b(G)$	$\frac{2nk}{3k-1}+[\frac{n^2(k+1)-n(1+2k)}{3k+1}]$	$\frac{2(k+1)n^2}{3k+1}$	$\left(\frac{t+K-1}{3t+3K-5}\right)Kn^2$
$KB_2(G,x)$	$nkx^{k(2k-1)}+\frac{n^2(k+1)-n(1+2k)}{2}x^{2k(k+1)}$	$(k+1)n^2x^{2k(k+1)}$	$(\frac{t+K-1}{2})Kn^2\ x^{(t+K-1)(2t+2K-4)}$

$RBSO(G)$	$nk\sqrt{(k-1)^2+(2k-2)^2} + \frac{n^2(k+1)-n(1+2k)}{2}\sqrt{k^2+(2k-1)^2}$	$\sqrt{k^2+(2k-1)^2} \times (k+1)n^2$	$\sqrt{(t+K-2)^2+(2t+2K-5)^2} \times \left(\frac{t+K-1}{2}\right)Kn^2$
$\Pi GO_2(G)$	$[(2k+1)\times k(k+1)]^{nk} \times [2(k+1)^3]^{\frac{n^2(k+1)-n(1+2k)}{2}}$	$[2(k+1)^3]^{(k+1)n^2}$	$[2(t+K-1)^3]^{(\frac{t+K-1}{2})Kn^2}$
$H\Pi GO_1(G)$	$[(2k+1)+k(k+1)]^{2nk} \times [2(k+1)+(k+1)^2]^{n^2(k+1)-n(1+2k)}$	$[2(k+1)+(k+1)^2]^{2(k+1)n^2}$	$[2(t+K-1)+(t+K-1)^2]^{(t+K-1)Kn^2}$
$H\Pi GO_2(G)$	$[(2k+1)(k^2+k)]^{2nk} \times [2(k+1)^3]^{n^2(k+1)-n(1+2k)}$	$[2(k+1)^3]^{2(k+1)n^2}$	$[2(t+K-1)^3]^{(t+K-1)Kn^2}$
$S_5\Pi(G)$	$\frac{n}{2}\sqrt{k}$	$\frac{(k+1)n^2}{\sqrt{2(k+1)^2}}$	$(\frac{1}{\sqrt{2(t+K-1)}})^{(\frac{t+K-1}{2})Kn^2}$
$IS_5\Pi(G)$	$2nk^{\frac{3}{2}}$	$\sqrt{2(k+1)^2} \times (k+1)n^2$	$(\sqrt{2(t+K-1)})^{(\frac{t+K-1}{2})Kn^2}$
$MRSO(G)$	$(\sqrt{(k-1)^2+k^2})^{nk} \times (\sqrt{2k^2})^{\frac{n^2(k+1)-n(1+2k}{2}}$	$(\sqrt{2k^2})^{(k+1)n^2}$	$\sqrt{2(t+K-2)^2}^{(\frac{t+K-1}{2})Kn^2}$

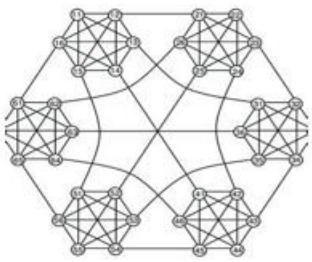

Figure 1: OTIS swapped network OR_k

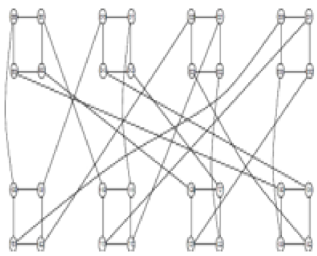

Figure 2: Bi-swapped network Bsw(C_4).

4. Conclusion

K-Banhatti indices are edge degree-based topological indices, in Bi-swapped network the vertex degree is k+1 and in K-swapped network degree, d=t+K-1. The K-Banhatti harmonic index is twice of the modified first K-Banhatti index for these networks. K-Banhatti indices, polynomials, K-Banhatti Sombor indices, multiplicative Gourava indices and sum degree-based indices are obtained for OTIS swapped, Bi-swapped and K-swapped networks.

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