Turbulent Natural Convection of Heat with Localized Heating and Cooling on Adjacent Vertical Walls in an Enclosure

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Abstract: ***In this work, we considered natural convection in a three dimensional rectangular enclosure in the form of a room with the heater placed on the floor and two windows each on the vertical adjacent walls .A fluid motion of the Boussinesq fluid in the three dimensional cavity has been considered .To enable the analysis of the flow and heat transfer rates, a set of non-dimensional equations governing a Newtonian fluid were used. The governing equations with the boundary conditions were described using three-point central and forward difference approximations for non-uniform mesh. The resulting finite difference equations were then solved using numerical methods .The solutions were presented at Reynolds number, Re=5,500 and Prandtl number, Pr=0.71. The results were presented in graphical form and discussed. The results show that the region in the middle of the room is warm and near the window is relatively cold.***

Keywords: Convection, turbulence, laminar, energy, buoyancy

1. Introduction

Heat is a form of energy. We can look at heat as a means of energy transfer and consider the process of thermal conductivity, convection and radiation. Thermal conduction can be viewed as exchange of kinetic energy between the molecules (particles of matter). Molecules with less energy gain energy as they collide with more energetic molecules. The rate of conduction depends on the properties of the materials.

A fluid is a substance that flows, that is, its constituent particles may continuously change their position relative to each other. In convection, we consider the velocity of the fluid with respect to solid surface and thereby combining the energy equations or the first law of thermodynamics with the momentum and continuity relation of the fluid mechanics. We can define convection as transfer of energy by movement of a substance. When the movement results from a difference in density, as with air around a fire, it is referred to as natural convection. When the substance is forced to move by a fan/pump as in some hot air and hot water heating systems, the process is referred to as forced convection. Convection currents assist in the boiling of water. In a tea kettle on a hotplate; the lower parts are warmed first. The warmed water has lower density and rises to the top; while the denser cool water at the surface goes to the bottom to take its place. We can note that the difference in temperature brings about density difference and the situation brings about buoyant forces. The forces will make the denser parts of the fluid to move downwards. The same process occurs when a radiator raises the temperature of a room. An automobile engine is maintained at a safe operating temperature by a combination of conduction and forced convection.

When we consider natural convection in an enclosure, the buoyant forces make the fluid flow. The term natural convection is used to denote closed cavity flow while free convection is an induced flow around a body of infinite medium. Convection phenomena due to buoyancy forces drives flow inside a closed cavity subjected to differential heating is encountered in many practical engineering problems which include thermo insulation of building, heat transfer through double window, etc. In convection, we consider two types of flow:- laminar or turbulent flow. Laminar flow is where we have steady flow while in turbulent flow we have unsteady flow. Turbulent flow is irregular flow in which the various quantities show a random variation with time and space coordinates.

The study of free convection in the recent past has mainly focused on two geometries: - Single isothermal constant flux vertical plate in isothermal standing surrounding and the enclosed rectangular cavity with heated and cooled walls.

The former is of parabolic character, that is, determines how unknown varies in both space and time and can be computed using numerical methods while the latter is of elliptic character, that is, characterizes the state boundary value problem which has to be converted into parabolic form before being solved. Due to the elliptic nature of internal buoyant flows, natural convection flows require a complete solution of the Navier-Srokes equation mainly by a digital computer. However, most of the fluid dynamics problems involving natural convection could solve by experimental, theoretical and numerical methods. Various techniques are available for numerical solutions governing differential equations fluid flow: Finite difference method, finite element /finite volume method and Boundary element method.

A number of researchers have carried out investigations on this area. [1] dedicated part of his research paper on natural convection in an enclosure. Results showed that turbulent convection depended on high Ralyleigh number. [2] studied Laminar and Turbulent flow, experimentally. [3] studied in detail turbulent flow in a 3-dimensional enclosure. Results indicated that the rate of heat transfer is higher for a larger window. [4] gave paper on the use of Mesh Generation Function. Heating was on the floor and the two opposite walls cooled. [5] presented paper on buoyancy driven convection. Results were obtained when window and heater were on the same wall [6] studied convection in a room. Results showed that flow and cooling rate depended on the angle at which cold down downdraught from the window and rising warm air from the heater mounted below it flows into enclosure. [7] studied turbulent heat flow convection in an enclosure. [8] Studied fundamentals of fluids. [9] presented work on three dimensional in a cavity with localized heating. [10] gave a study focused mainly of two geometries – single isothermal constant flux vertical plate in isothermal standing. [11] –studied natural convection. Results indicated different boundary conditions.

2. Mathematical Formulation

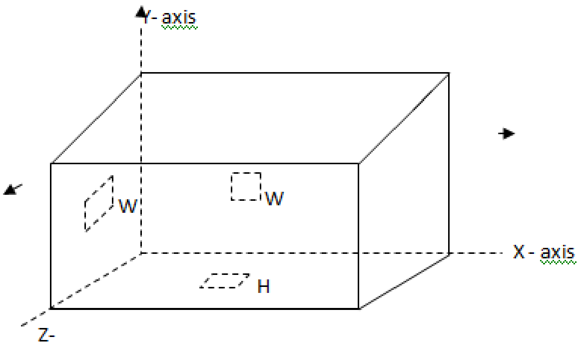

Figure 1: Geometry of the problem

Here we shall consider natural convection in an enclosure which comes as a result of heating and cooling. The heater is placed on the floor while two windows are placed on vertical adjacent walls. The window sizes and positions have been fixed and the heater is kept fixed on the floor. The movement of the fluid (air) in the various regions of the enclosure brought about by the temperature difference is our main concern.

3. The Governing Equations

3.1 The Continuity Equation

The law of conservation of mass states that the rate of increase of mass within a controlled volume is equal to the net rate of influx through the controlled surface .The continuity equation can be written as [2]

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u_j)}{\partial x_j}=0 \quad (1)$$

For steady state, equation (1) above can be written as

$$\frac{\partial(\rho u_j)}{\partial x_j}=0 \quad (2)$$

3.2 The Momentum Equation

The equation is derived from Newton's second law of motion, which states that the sum of the body forces and surface forces acting on a system is equal to rate of change of linear momentum. Thus, the momentum equation can be expressed as

$$\frac{\partial}{\partial x_i}(\rho u_i)+\frac{\partial}{\partial x_j}(\rho u_j u_i)=-\frac{\partial \rho}{\partial x_i}+\rho g_1+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\mu_s\delta_{ij}\frac{\partial u_k}{\partial x_k}\right] \quad (3)$$

3.3 Energy Equation

This is derived from the first law of thermodynamics which states that the rate of energy increase in a system is equal to the heat added to the system and the work done on the system .Assuming no external heat sources, the energy equation is often written as [2].

$$\frac{\partial}{\partial t}(\rho c_p T)+\frac{\partial}{\partial x_j}(\rho c_p u_j T)=\frac{\partial}{\partial x_j}\left(\lambda\frac{\partial T}{\partial x_j}\right)+\beta T\left(\frac{\partial p}{\partial t}+\frac{\partial u_j p}{\partial x_j}\right)+\Phi \quad (4)$$

Where ф represents the dissipation function

4. Nature of turbulence

Reynolds Decomposition

Reynolds decomposition of the Governing equations involve the separation of variables φ in the case of velocity component $u_{i,}$ pressure p, density ρ and temperature T into mean (time- average) value $\overline{\varphi}$ and the fluctuating part φ' ; that is

$$\varphi(x_i,t)=\overline{\varphi}(x_i)+\varphi'(t) \quad (5)$$

The mean value is given by:

$$\overline{\varphi}=\frac{1}{\Delta t}\int_0^{t_0+\Delta t}\varphi(x_i,t)dt$$

For practical purposes, Δt must be finite .Thus equation (5) becomes $\overline{\varphi}(x_i,t)=\varphi(x_j)$ (6)

5. Final Set of Equations

For simplicity reasons, the over-bar denoting time-mean values of the prime denoting the fluctuation quantities can be omitted. Upper and Lower case of the letters will be used to refer to the mean-time values of the variable and the fluctuating quantities for the case of velocity components and fluid properties, pressure P and temperature T. The final form of equations for turbulent natural convectional flow becomes:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho U_j+\overline{\rho u_j}\right)=0 \quad (7)$$

$$\frac{\partial}{\partial t}\left(\rho U_i+\overline{\rho u_i}\right)+\frac{\partial}{\partial x_i}\left(\rho U_i U_j+U_i\overline{\rho u_j}\right)=-\frac{\partial p}{\partial x_i}+\rho g_i+\frac{\partial}{\partial x_i}\left(\tau_{ij}-U_i\overline{\rho u_j}-\overline{\rho u_i u_j}-\overline{\rho u_i u_j}\right) \quad (8)$$

$$\frac{\partial}{\partial t}\left(c_p\rho T+c_p\overline{\rho T}\right)+\frac{\partial}{\partial x_j}\left(c_p\rho U_j T\right)=\frac{\partial p}{\partial t}+U_j\frac{\partial p}{\partial x_j}+$$

$$\tau_y=\mu\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_j}\right)+\mu_s\delta_{ij}\frac{\partial U_k}{\partial x_k}$$

$$u_j \frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial T}{\partial x_j} - c_p \overline{u_i t} - c_p \overline{u_i t} \right) + \Phi \quad (9)$$

(10)

$$\Phi = \tau_y \frac{\partial U_i}{\partial x_j} + \mu \overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}} \quad (11)$$

6. Non-Dimensionalization

There are more parameters in the governing equations than the equations themselves. The main rationale in non-dimensionalization is to reduce the number of parameters particularly when dealing with incompressible fluids. The non-dimensionalization is performed based on the following set of general scaling variables.

$$U = U'_j U_* \; X = X'_i L_r \;\; P = P' P_R \;\; \Theta = \frac{T - T_*}{\Delta T_*} \;\; K = K' U_*$$

$$\varepsilon = \varepsilon' \frac{U_*^3}{L_R} \;\; t = t' \frac{L_R}{U_*} \;\; \mu = \mu' \mu_R \;\; \mu_s = \mu'_s \mu_R \;\; V = V' \mu_R$$

$$\rho = \rho' \rho_R \;\; c_p = c_p c_{pR} \;\; \lambda = \lambda' \lambda_R$$

Since a number of non-dimensional schemes are possible, equations (7) to (9) are written respectively in the general form as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho U_j + \overline{\rho u_j} \right) = 0 \quad (12)$$

$$\frac{\partial}{\partial t}\left(\rho U_i + \overline{\rho u_i} \right) + \frac{\partial}{\partial x_j}\left(\rho U_i U_j + U_i \overline{\rho u_j} \right) = -M_1 \frac{\partial P}{\partial x_i} + M_2 \rho g_i + \frac{\partial}{\partial x_j}\left(M_3 \tau_{ij} - u_j \overline{\rho u_j} - \overline{\rho u_i u_j} - \rho u_i u_j \right) \quad (13)$$

$$\frac{\partial}{\partial t}\left(c_p \rho \theta + c_P \overline{\rho \Theta} \right) + \frac{\partial}{\partial x_j}\left(c_p \rho u_j \Theta \right) = T_1 \left[\frac{\partial P}{\partial t} + u_i \frac{\partial P}{\partial x_j} + u_j \frac{\partial P}{\partial x_j} \right] + \frac{\partial}{\partial x_j}\left(T_2 \lambda \frac{\partial \Theta}{\partial x_j} - c_P \overline{\rho u_i \theta} - c_P \overline{\rho u_i \theta} \right) + T_3 \phi \quad (14)$$

The coefficients are as shown below;

$$M_1 = \frac{P_R}{\rho_R U_*^2} \;\; M_2 = \frac{g L_R}{U_*^2} \;\; M_3 = \frac{\mu_R}{\rho_R U_* L_R}$$

$$T_1 = \frac{\Pr}{c_{PR} \rho_R \Delta T_*} \;\; T_2 = \frac{\lambda_R}{c_{PR} \rho_R U_* L_R} \;\; T_3 = \frac{\mu_R U_*}{c_{PR} \rho_R \Delta T_* L_R}$$

7. Methods of Solution

The non-linear governing equations are hereby expressed in finite difference form. The resulting equations are then solved using central and forward difference approximations. Equations (11) and (12) become equations (13) and (14) respectively.

$$\frac{U_{i,j+1} - U_{i,j-1}}{2k} = \frac{1}{R_e}\left(\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{h^2} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-}}{k^2} \right.$$

$$\left. - \nu \left(\frac{U_{i+1,j} - U_{i+1,j}}{2h} + \frac{U_{i,j+1} - U_{i,j-1}}{2k} \right) - \frac{E_u}{\rho_R} \frac{\partial \rho}{\partial y} - \frac{\Theta g_1}{(F_r)^2} + H.O.T \quad (15).$$

U(x,y,t)=sinπx +cosπy ,u(x,0,t) =0, u(x,1,t) =0 ,u(0,y,t)=0 , u(1,y,t) =0

$$\frac{\theta_{i,j+1} - \theta_{i,j-1}}{2k} = \frac{1}{P_r R_e}\left(\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{h^2} + \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{k^2} \right) - 2\nu \left(\frac{\theta_{i+1,j} - \theta_{i-1,j}}{2h} + \frac{\theta_{i,j+1} - \theta_{i,j-1}}{2k} \right) \quad (16)$$

Temperature and boundary conditions;

Θ(x,y,0) =sinπx +cosπy ,Θ(x,0,t) =50^0c ,Θ(x,1,t)=20^0c ,Θ(0,y,t) =10^0c ,Θ(1,y,t)=20^0c ,Θ(0.5,0.5.t) =10^0c Θ(0.5,0.5,t)=10^0c

I also used the following constants;

Re=5,500, Pr=0.71, Eu =2.71828, Fr=1 and g=10

8. Results and Discussions

8.1 Flow Fields

The results of this study are to understand the structure of the velocity profile and temperature distribution due to the temperature difference on the two vertical walls and the floor. The remaining walls of the enclosure were assumed to be fully insulated. The solutions presented are for Reynolds number Re =5, 500 and Prandtl number, Pr =0.71. In the flow fields we have the vector plots at selected places. In the x-y plane, the vector plots are in the plane z =0.5, z =0.1 and z =0.9, see Fig 1(a), Fig. 2(a) and Fig. 3(a) respectively. The structure of the flow where the plane is at z =0.5 shows two circular motions in different directions –one in clockwise while the other in anti-clockwise direction. At high Reynolds number, a strong convective motion develops and heat is transferred from the bottom to the regions. The warm fluid gains energy; becomes less dense gains in velocity resulting in an upward movement while on the sides the cold air descends. This is due to the buoyancy effect. The velocity of the descending fluid is strongest near the window. On the side, where there is no window, there is a mixing up of warm and cold fluid resulting into low movement of the fluid particles. Due to symmetry at the planes z =0.1 and z =0.9, the flow patterns remain the same.

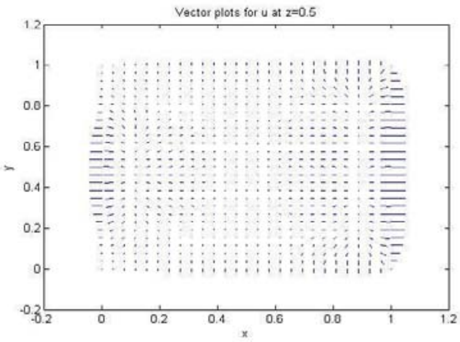

Figure 1(a): Velocity vector plots at the plane z =0.5

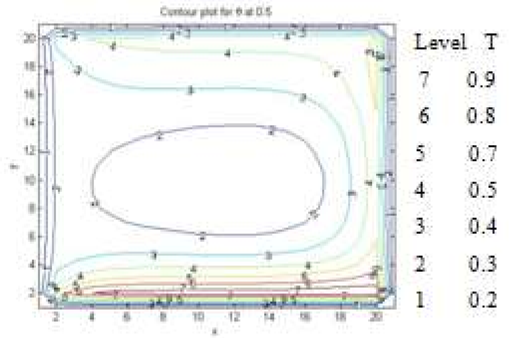

Figure 1(b): Isotherms at the plane z =0.5

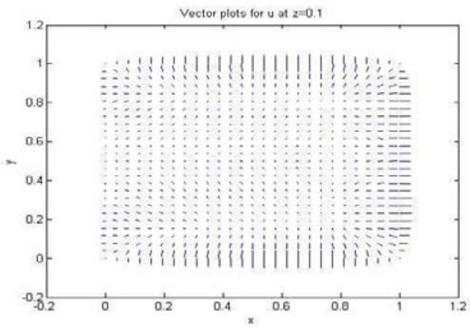

Figure 2 (a): Velocity vector plots at the plane z =0

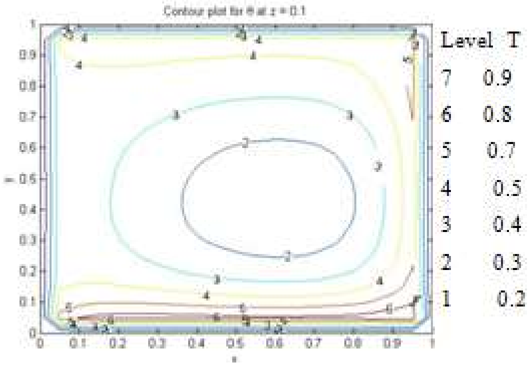

Figure 2 (b): Isotherms at the plane z =0.1

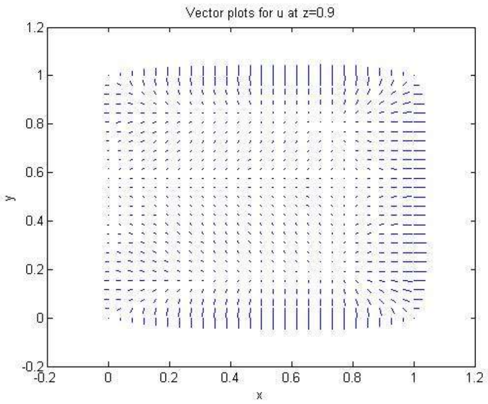

Figure 3(a): Velocity vector plots at the plane z =0.9

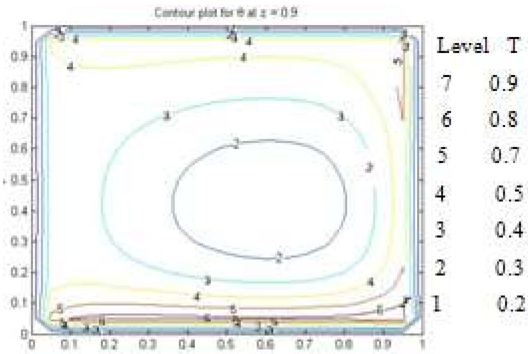

Figure 3(b): Isotherms at the plane z=0.9

8.2 Temperature Fields

The warm air above the heater rises to the ceiling and as it moves it disperses on both sides into circular motions, as shown, but more pronounced on the wall containing the window. Isotherms were obtained at z =0.5, z= 0.1 and z = 0.9 see Fig. 1(b), Fig 2(b), Fig 3(b) respectively. Due to symmetry, results at z =0.1 and z =0.9 appeared basically similar. The temperature decreases as you move up. At the centre of the room, there is relatively warm temperature. This comes as a result of the hot air rising up from the heater mixing up the cold fluid from the window. The turbulent natural convection does play an important role in the variation of temperature in an enclosure.

9. Conclusion

The objective of the study was to look at the temperature distribution brought about by the heat transfer in an enclosure. The heat transfer was by convection. The solutions were obtained for Reynolds number 5,500 and Prandtl number 0.71. The geometry considered is a 3-dimensional rectangular enclosure in form of a room with convectional heater positioned on the floor and the cold area (windows) on the two adjacent walls as shown in Fig 1.

The Boussinesq approximations were used, allowing the conservation equations to be simplified. The governing equations with boundary conditions were discretized using a three-point central and forward difference approximation. The results show that in an enclosure environment, the natural turbulence flow plays an important role in the temperature distribution. Regions of varying temperatures are created either across the room or in an upward direction. This helps in keeping of some items at the stated temperature.

10. Nomenclature

.ρ, Density
τ , Viscosity stress tensor.
φ General variable
$\bar{\varphi}$ Average value of .φ
φ' Fluctuating value of φ
Cp Specific heat capacity at constant pressure
Eu Euler number = $\frac{P_R}{\rho_R U_c^2}$
g Acceleration due to gravity in N/ m^2
h Mesh interval in case of uniform mesh with the same interval in each direction
i,j,k Integer variables
Lm Characteristic length scale in m.
P Thermodynamic pressure in N/m2
Pr Prandtl number= $\frac{\mu_R C_{PR}}{K_R}$
Fr Froude number= $\frac{u}{\sqrt{gL_R}}$
Ra Rayleigh Number= $\frac{\rho_R^2 C_{PR} g\beta\Delta T_* L_R^3}{\mu_R K_R}$
Re Reynolds Number= $\frac{\rho_R U_1 L_R}{\mu_R}$
t Time in sec.

u u' ,U Instantaneous velocity component in the x - direction, fluctuation velocity and mean velocity in the x- direction in m.
U* Characteristic velocity in m/s
v v' ,V Instantaneous velocity component in y-direction, fluctuation velocity and mean velocity in y-direction in m/s.
x,y,z Co-ordinate direction in i, j, and k direction .
β, Coefficient of volumetric expansion in m3
θ Non-dimensional or fluctuating temperature
λ, Thermal conductivity
.μ, First coefficient of viscosity
i ,j ,k Denotes the i^{th}, j^{th}, k^{th} mesh points in the x ,y , and z directions respectively .
H.O.T Higher order terms

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