Design and Development of Model Predictive Controller for Binary Distillation Column

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Abstract: Model Predictive Control (MPC) is an advanced process control technique used in process related industries that can predict the future behavior of the process state/output over the finite time horizon by predicting the change in the dependent variables of the modeled system that will be caused by changes in the independent variables. It can compute the future input signals at each step by minimizing a cost function under inequality constraints on the manipulated and/or controlled variables. The goal of this paper is to develop a Model Predictive Controller for constrained and unconstrained input/output, on SISO system as well as MIMO system (a non-linear binary distillation column). The objective is to maintain the specification of the product concentration outputs xB and xD due to disturbance F (feed flow-process disturbance) and xF (feed concentration) with the inputs R and S (manipulated variables). In this paper, performance indices like settling time, overshoot, ISE, IAE and ITAE errors of MPC controller are compared with conventional multi loop PI controller for both SISO and MIMO systems.

Keywords: Model Predictive Controller, distillation column, control horizon, model horizon

1. Introduction

Distillation is the separation method in the petroleum and chemical industries for purification of final products. They are used to enhance mass transfer or for transferring heat energy. The control structure for the distillation column is based on L-V (Liquid-Vapour) structure or the energy balance method. In this control configuration the vapour flow rate V and the liquid flow rate L are the control inputs. The main objective is to maintain the specification of the product concentration outputs xB and xD (controlled variables) due to disturbance F (feed flow) and xF (feed concentration). The MPC (Model Predictive Control) has been selected for controlling the distillation column. MPC is based on open loop control and close loop control method. It generates an online feedback control by using the open-loop optimization[7]. The basic concept involved in MPC design is to predict the future plant response by the help of a process model and always trying to minimize a finite horizon objective function which consists of a sum of future predicted errors and control moves.

This paper presents specific details about the simulated case study of MPC, over the Wood and Berry distillation column model. The Wood-Berry model is a 2×2 transfer function model of a pilot plant distillation column that separates methanol and water. Then it is extended to Distillation Column model of Benzene Toluene mixture, then Oggunnaite and Pay Distillation model [8]. The system outputs are the distillate and bottoms compositions, xD and xB [wt %] respectively, which are controlled by the reflux and steam flow rates, R and S [lb/min]. The unmeasured feed flow rate, F, acts as a process disturbance [1].

A SISO MPC controller for a simple first order transfer function performance is comparing with PI controller. Same way, a Binary Distillation Column, a non-linear 2 input 2 output system (MIMO system) is designed with MPC controller. Then comparing all the performance results like settling time, overshoot, ISE, IAE, ITAE errors: with conventional multiloop PI controller. The MPC simulation was performed using MATLAB and the Model Predictive Control Toolbox. In order to reduce the simulation time needed to explore a variety of design options and design parameters, unconstrained MPC was employed. However, the strategy presented in this paper is identical for both constrained and unconstrained MPC.

The goal of this paper is to understand the model predictive controller for constrained and unconstrained input/output, on SISO system as well as MIMO i.e, maintaining the specification of the product concentration outputs xB and xD (controlled variables) due to disturbance F (feed flow) and xF (feed concentration) with the inputs R and S (manipulated variables).

2. Distillation Column

2.1 Process Description

Distillation is a process that separates two or more components into an overhead distillate and bottoms. The bottoms product is most probably liquid, while the distillate may be liquid or a vapor or both. [1]. A typical distillation column contains a vertical column where trays are used to enhance the component separations. A reboiler is used to provide heat for the necessary vaporization from the bottom of the column and a condenser, to cool and condense the vapor from the top of the distillation column, then a reflux drum to hold the condensed vapour so that liquid reflux can be recycled back from the top of the column.
The distillation column contains one feed stream and two product streams. The feed contains a mole percent of the component called $z_F$. The product stream at the top has a composition referred as $x_D$. The product stream leaving the bottom contains a composition of $x_B$ of the light component. The column is having two sections. The top section is rectifying section and the bottom section is stripping section.

The important aspects of the steady-state operation, dynamics and control of continuous distillation columns are summarized in [10]. The treatment is mainly limited to two-product distillation columns separating relatively ideal binary mixtures.

### 2.2 Determine Process Variables

1. Assumed feed rate, composition, purity of distillate and bottoms, and the quality of the feed are known.
2. Perform overall material and component balances to determine the compositions of the distillate and bottoms.

\[
F z_F = (x_D D_i) + (x_B B_o) \quad (1)
\]
\[
F = (D_i) + (B_o) \quad (2)
\]

Where

- $F$ = Feed rate of input stream
- $z_F$ = Composition of light component in feed
- $x$ = Mole Fraction of light in distillate
- $D_i$ = Total distillate amount
- $B_o$ = Total bottom amount

Distillation is the most popular and least expensive means of separating mixtures of liquids. If components to be separated have a high relative volatility difference and are thermally stable, distillation is hard to beat. Distillation Column working is well described in [12]. Distillation has low energy efficiency and requires thermal stability of compounds at their boiling points. If the mixture is azeotropic, then more advanced types of separation must be considered. Distillation column shell should be thicker to withstand pressure of the column. Other than petroleum industries, chemical, pharmaceutical and, food industries widely using this technique of distillation column in their separation units.

### 3. Model Predictive Controller

#### 3.1 Concept

Model Predictive Control (MPC) is an advanced method of process control that has been in use in the process industries in chemical plants and oil refineries since the 1980s. MPC also known as Receding horizon control or Moving horizon control uses an explicit dynamic plant model to predict the effect of future reactions of the manipulated variables on the output and the control signal obtained by minimizing the cost function. This predication takes into account, constraints on both the inputs and outputs of the process. An optimal input sequence is calculated. The measurements are then sent back to the controller, and a new optimizing problem is solved.

![Structure of MPC controller](image)

### 3.2 Overview

MPC is a feedback implementation of optimal control using:

- Finite prediction horizon
- On-line computation

When used for linear dynamic systems, the MPC controller uses a process model and a constrained, on-line optimization to determine the optimal future control move sequence. The first control move is implemented and the calculations are then repeated at the next control calculation interval. Three variants of MPC algorithm (linear MPC, multiple MPC and Non linear MPC) for power plant boiler and compared in the terms of control performance versus complexity trade-off in [2].

The models used in MPC are generally intended to represent the behavior of complex dynamical systems. MPC models predict the change in the dependent variables of the modeled system that will be caused by changes in the independent variables. The performance of the controller depends on how well the dynamics of the system being captured by the input–output model that is used for the design of the controller. A detailed study on stability and optimality of constrained model predictive control is given in [5].

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**Figure 1:** Schematic Diagram of Distillation Column

**Figure 2:** Structure of MPC controller
3.3 Principle of MPC

MPC is based on iterative, finite horizon optimization of a plant model. At time \( t \), the state is sampled and a cost minimizing control strategy is computed for a relatively short time horizon in the future.

![Figure 3: Principle of MPC](image)

As told earlier, MPC is a multivariable control algorithm that uses an internal dynamic model of the process or system, past control moves and an optimization cost function \( J \) over the receding prediction horizon, to calculate the optimum control moves \[7\]. The optimization cost function is as follows:

\[
J = \sum_{i=1}^{N} \omega_i (r_i - x_i)^2 + \sum_{i=1}^{N} \omega_i u_i^4
\]

without violating constraints (low/high limits) where

\( x_i \) is i-th controlled variable
\( r_i \) is i-th reference variable
\( u_i \) is i-th manipulated variable
\( w_{x_i} \) is weighting coefficient for relative importance of \( x_i \)
\( w_{u_i} \) is weighting coefficient of relative big changes in \( u_i \)

This model can be made into a step-response model with help from the MATLAB command `tfd2step`. The final discrete step-response model will be on the form:

\[
y_{k+1} = \sum_{i=1}^{R-1} a_i \Delta u_{k-i+1} + a_R u_{k-R+1} + y_i
\]

Where \( y_{k+1} \) is the output at time \( k+1 \),
\( a \) is step-response coefficient
\( y_i \) is the outputs initial conditions
\( R \) is a constant.

The MPC controller trying to minimize a quadratic object functions.

\[
\min_{u_k} = \sum_{j=0}^{P} \left( y_{k+j} - y^* \right)^2 + \left( u_{k+j} - u^* \right)^2 + \omega_{x_k} (y_{k+j} - y^*) + \omega_{u_k} (u_{k+j} - u^*)
\]

Subject to the constraint \( \Delta U \geq c_{k+1} \) \[6\]

Where \( P \) is the prediction horizon
\( N \) is the control horizon,
\( y^* \) is the reference output
\( u^* \) is the reference input
\( Q \) is the weights on the outputs
\( R \) is the weights on the inputs

This function is solved using a quadratic solver to ensure fast optimizing \[11\].

The prediction horizon (P) tells the controller how many sample steps ahead should be used when minimizing the object function. The control horizon (M) tells the controller how many control steps should be used when minimizing. The larger M is compared to P, the bigger the chance is that the controller will find an input sequence to minimize the function. But that may lead to an aggressive use of input and an unstable system. The values of these parameters will be a trade-off between good performance and time limits.

A high weight on one of the outputs will force the controller to keep that value closer to the reference than the value of the other outputs. A high weight on one of the inputs will reduce the input activity. If the weight on the input is increased indefinitely, the activity will be reduced to zero, and there will no longer be any feedback action. The most common way of tuning the weights is to use them to scale all the inputs and outputs of the object function \[2\]. A MPC-controller can handle constraints both for the inputs and outputs. The constraints will be formulated like:

\[
y_{k+1} \leq y_{k+1} \leq y_{k+1} \leq y_{k+1}
\]

These limitations can lead to an infeasible solution set for the controller. Some advantages of MPC include straightforward formulation, based on well understood concepts and explicit handling of constraints. Its development time much shorter than for competing advanced control methods. This is by optimizing a finite time-horizon. Its ability to anticipate future events and can take control action accordingly is an important factor.

4. Simulation Results and Discussion

4.1 SISO System

A simple SISO system is taken here for the study – FOPDT system.

\[
\frac{Y(s)}{X(s)} = \frac{1}{10s + 1} e^{-8s}
\]

PI Controller is designed with values of \( K_p=14.4249 \), \( K_i=8.8268 \). For the same system tuning of MPC Controller for SISO system is based upon \[9\]. The tuning strategy achieves set point tracking with minimal overshoot. Tuned values of MPC are: Control Horizon (M) = 5, Prediction Horizon (P) = N = 53, Control Interval = 0.3
MPC can be simulated with MATLAB software and Model Predictive Control toolbox. And the output response of the PI Controller and the MPC controller are given below:

**Figure 4:** Comparison of PI and MPC controller SISO

**Table 1:** Performance of PI controller and MPC controller on SISO System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PI Controller</th>
<th>MPC Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling time</td>
<td>9.27</td>
<td>3.7</td>
</tr>
<tr>
<td>Overshoot</td>
<td>1.637</td>
<td>1.04</td>
</tr>
<tr>
<td>ISE</td>
<td>2.1</td>
<td>1.846</td>
</tr>
<tr>
<td>IAE</td>
<td>1.963e-14</td>
<td>1.007e-15</td>
</tr>
<tr>
<td>ITAE</td>
<td>2.622</td>
<td>0.6918</td>
</tr>
</tbody>
</table>

4.2. MPC - MATLAB coding for SISO System

Matlab coding is efficient and user friendly.

4.2.1 For Unconstrained Case

**Figure 5:** Input and output graphs of Unconstrained MPC of SISO

4.2.2 For Constrained Case

**Figure 6:** Input and output graphs of Constrained MPC for SISO

4.3. MIMO System

The Wood and Berry model is taken as primary, which is a 2×2 transfer function model of a pilot plant distillation column that separates methanol and water. The system outputs are the distillate and bottoms compositions, xD and xB, which are controlled by the reflux and steam flow rates, R and S.

**Equation 10:**

\[
\begin{bmatrix}
12.0e^{-s} \\
10.9s + 1
\end{bmatrix}
\begin{bmatrix}
2.88e^{-s} \\
14.4s + 1
\end{bmatrix}
\begin{bmatrix}
d(s)
\end{bmatrix}
\]

4.4 Multi Loop PI Controller Tuning

There are different methods for the tuning of multi loop PI controllers. Among these Decoupling techniques on BLT tuning method of Luyben[6] is the easy way to tune multi loop PI controllers. The control of a binary distillation column is generally a difficult problem, due to the nonlinearity and time-delay properties in the process. Since binary distillation column is a MIMO system, multiloop PI controllers are the choice. A simple tuning method for multiloop PI controllers is described in [3]. The method is suited for PI algorithms with no proportional and derivative kick. Here we had tuned PI controller for Wood and Berry Distillation Column:

Kc1 = 0.375, $\zeta_{11} = 8.29$
Kc2 = -0.075, $\zeta_{12} = 23.6$

4.5 MPC Tuning Parameters

The closed-loop MPC simulation was performed using MATLAB and the Model Predictive Control Toolbox [4].
The manipulated variables (MV’s) were R and S, and the controlled variables (CV’s) were xD and xB. Model predictive control for linear constrained systems has been proven as a useful control solution for many practical applications. As in SISO system, the tuning of MPC Controller for MIMO system is based upon [9]. Tuned values of MPC are: Control Horizon (M) = 4, Prediction Horizon (P) = N = 96, Control Interval = 0.485

**Table 2**: Performance of PI controller and MPC controller on MIMO system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PI Controller</th>
<th>MPC Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOP PRODUCT (vapour) – xD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Settling Point</td>
<td>77.9</td>
<td>59.18</td>
</tr>
<tr>
<td>Maximum Peak Overshoot</td>
<td>1.365</td>
<td>1.015</td>
</tr>
<tr>
<td>ISE</td>
<td>2.255</td>
<td>1.879</td>
</tr>
<tr>
<td>IAE</td>
<td>5.656</td>
<td>3.046</td>
</tr>
<tr>
<td>ITAE</td>
<td>64.08</td>
<td>16.92</td>
</tr>
<tr>
<td>BOTTOM PRODUCT (liquid) - xB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Settling Point</td>
<td>102</td>
<td>75.57</td>
</tr>
<tr>
<td>Maximum Peak Overshoot</td>
<td>1.13 with -ve overshoot</td>
<td>1.15</td>
</tr>
<tr>
<td>ISE</td>
<td>26.13</td>
<td>5.802</td>
</tr>
<tr>
<td>IAE</td>
<td>21.9</td>
<td>8.936</td>
</tr>
<tr>
<td>ITAE</td>
<td>269.2</td>
<td>86.69</td>
</tr>
</tbody>
</table>

**Figure 7**: Top and Bottom product of Distillation Column

**Figure 8**: Controlling of top and bottom product of BDC (using Unconstrained MPC)

**Figure 9**: Controlling of top and bottom product of Wood and Berry Binary Distillation Column for Constrained case

4.6 MPC for Other Two Distillation Models

4.6.1 Ogunnakke and Pay BDC Model[8]

\[
\begin{bmatrix}
-21.6e-4 & -21.6e-4 \\
-21.6e-4 & -21.6e-4 \\
-21.6e-4 & -21.6e-4 \\
-21.6e-4 & -21.6e-4 \\
\end{bmatrix}
\begin{bmatrix}
0.6e-3 \\
0.6e-3 \\
0.6e-3 \\
0.6e-3 \\
\end{bmatrix}
\begin{bmatrix}
\frac{R}{(S)} \\
\frac{R}{(S)} \\
\frac{R}{(S)} \\
\frac{R}{(S)} \\
\end{bmatrix}
\]
Figure 10: Controlling of top and bottom product of Ogunnaikke Model BDC for unconstrained case

G.2 Benzene - Toluene Separation BDC Model

\[ x_D(z) = \begin{bmatrix} 0.040z^{-0.003} \\ 0.022z^{-0.013} \\ 0.009z^{-0.016} \\ 0.027z^{-0.017} \\ 0.020z^{-0.017} \\ 0.017z^{-0.017} \end{bmatrix}, \quad x_B(z) = \begin{bmatrix} 0.020z^{-0.003} \\ 0.017z^{-0.013} \end{bmatrix} \]

Figure 11: Controlling of top and bottom product of Benzene- Toluene Separation BDC for unconstrained case

5. Conclusion

Wood and Berry Binary distillation column is taken as the primary system model. This non-linear 2 input 2 output system, is controlled with multi loop PID and MPC controller. Performance indices like settling point, overshoot, and ISE, IAE, ITAE errors of MPC controller is compared with conventional multi loop PI controller. The result depicts MPC is far better than the conventional in all manners as it provides smooth reference tracking with reduced peak overshoot and better closed loop performances such as ISE, IAE, ITAE.

The closed-loop MPC simulation was performed using MATLAB and the Model Predictive Control Toolbox. Both SIMULINK model and MATLAB program coding for MPC are developed and analyzed the performance. The MATLAB program coding is done for both constrained and unconstrained MPC. This shows why MPC is much suitable for industrial applications. Other than Wood and Berry Distillation Column, 2 more distillation models are taken for the study. Tuning technique suggested in paper[9] has been used for both SISO system as well as in MIMO system. Prediction horizon is tuned whereas Control horizon value is taken within a range of 2-6.

The larger control horizon (N) is compared to Prediction horizon (P), the bigger the chance is that the controller will find an input sequence to minimize the function. But that may lead to an aggressive use of input and an unstable system. Along with the tuning parameters, control interval, weights, and gain also having a great impact on the performance of the system.

6. Future Scope

The future scope of this paper work is the application of Non-linear MPC on non-linear plants which is much advantageous for such plants. Also the use of Fuzzy Logic on the MPC to anticipate the future errors can also be done as extension.

References


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