International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Extension of the Linear Exponential Distribution and its Applications

M. A. El-Damcese[1], Gh. Marei[2]

[1]Mathematical Department, Faculty of Science, Tanta University, Tanta, Egypt

[2] Mathematical Department, Faculty of Science, Zagazig University, Zagazig , Egypt

Abstract: ***In this paper we introduce a new lifetime distribution by the Poisson distribution with drifted supporting set {1,2,3,...} is used for extending the families of the linear exponential distribution. The properties of this distribution are discussed in this paper. The maximum likelihood estimates of the unknown parameters are obtained.***

Keywords: Linear failure rate distribution, Poisson distribution, reliability analysis, maximum likelihood method.

1. Introduction

Probability distributions are often used in survival analysis for modeling data, because they offer insight into the nature of various parameters and functions, particularly the failure rate (or hazard) function. Throughout the last decades, a considerable amount of research was devoted to the creation of lifetime models with more than the classical increasing and decreasing hazard rates; apparently, the motivation for this trend was to provide with more freedom of choice in the description of complex practical situations [1–6].

The linear exponential (LE) distribution, having exponential and Rayleigh distributions as sub-models, is a very well known distribution for modeling lifetime data and for modeling phenomenon with linearly increasing failure rates. However, the LE distribution does not provide a reasonable parametric fit for modeling phenomenon with decreasing, non linear increasing, or non-monotone failure rates such as the bathtub shape, which are common in firmware reliability modeling, biological studies, see Lai et al. [7]and Zhang et al. [8].

In [9], a method is given, which uses the Poisson distribution is used to add a new parameter to the exponential and Weibull distributions.

In this paper, we introduce a new distribution with three parameters, referred to as the Poisson distribution is used to add a new parameter to the linear exponential distribution. Several properties are derived, such as density, failure rate, inverse failure rate, mean lifetime, moments, residual lifetime, Furthermore, estimation by maximum likelihood

2. The New Reliability Function

The linear failure rate distribution $LFRD(\alpha,\beta)$ with the parameters $\alpha \succ 0$ and $\beta \succ 0$, will be denoted by, has the following reliability function[9]

$$\overline{F}(t;\alpha,\beta) = e^{-(\alpha t+\frac{\beta}{2}t^2)}; \quad \text{for} \quad \alpha \succ 0, \beta \succ 0 \tag{1}$$

The probability density function (pdf) corresponding to $\overline{F}(t;\alpha,\beta)$ is given by

$$f(t;\alpha,\beta) = (\alpha+\beta t)\, e^{-(\alpha t+\frac{\beta}{2}t^2)}; \quad \text{for} \quad \alpha \succ 0, \beta \succ 0 \tag{2}$$

Let $\overline{F}$ be a one-parameter survival function and N be a Poisson random variable with
parameter λ, having the drifted supporting set {1,2,3,...}, or equivalently let M be a
Poisson variable with parameter λ and N = M + 1. For all t∈**R**, define a new reliability function by [10].

$$\overline{G}(t) = \sum_{n=1}^{\infty} \overline{F}^{\,n}(t) P[N=n] = \sum_{n=1}^{\infty} \overline{F}^{\,n}(t) \frac{e^{-\lambda}\lambda^{n-1}}{(n-1)!} = \overline{F}(t) e^{-\lambda[1-\overline{F}(t)]} \tag{3}$$

or equivalently reliability function and cumulative density function (cdf) by

$$\begin{cases} \overline{G}(t) = \overline{F}(t) e^{-\lambda F(t)}, t \in R, \lambda \ge 0 \\ G(t) = 1 - [1 - F(t)] e^{-\lambda F(t)}, t \in R, \lambda \ge 0 \end{cases} \tag{4}$$

For the new reliability function, it is true that $\overline{G}(t)$ is decreasing in λ, as λ becomes nearer 0, $\overline{G}(t)$ takes after $\overline{F}(t)$, otherwise we have $\overline{G}(t) \le \overline{F}(t), t \in R$

The cdf of a new three parametric exponential family as

$$G(t;\alpha,\beta,\lambda) = 1 - e^{-(\alpha t+\frac{\beta}{2}t^2)} e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]}, t \ge 0, \lambda \ge 0 \tag{5}$$

and the reliability function is

$$\overline{G}(t;\alpha,\beta,\lambda) = e^{-(\alpha t+\frac{\beta}{2}t^2)} e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]}, t \ge 0, \lambda \ge 0 \tag{6}$$

Let the distribution function $\overline{F}(t;\alpha,\beta)$ in (1) have a density function $f(t;\alpha,\beta)$ and denote its failure rate function by $f(t;\alpha,\beta)/\overline{F}(t;\alpha,\beta)$, and similarly of $G(t;\alpha,\beta,\lambda)$ by failure rate function $z(t;\alpha,\beta,\lambda)$. The pdf corresponding to $G(t;\alpha,\beta,\lambda)$ is given by

$$g(t;\alpha,\beta,\lambda)=\{1+\lambda e^{-(\alpha t+\frac{\beta}{2}t^2)}\}(\alpha+\beta t)\,e^{-(\alpha t+\frac{\beta}{2}t^2)}e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]},t\geq 0,\lambda\geq 0 \quad (7)$$

and the failure rate function is

$$z(t;\alpha,\beta,\lambda)=[1+\lambda\, e^{-(\alpha t+\frac{\beta}{2}t^2)}](\alpha+\beta t),t\geq 0,\lambda\geq 0 \quad (8)$$

For some values of λ, α and β, the density functions $g(t;\alpha,\beta,\lambda)$ are drawn in Figure 1 and their reliability functions $\overline{G}(t;\alpha,\beta,\lambda)$ are drawn in Figure 2.

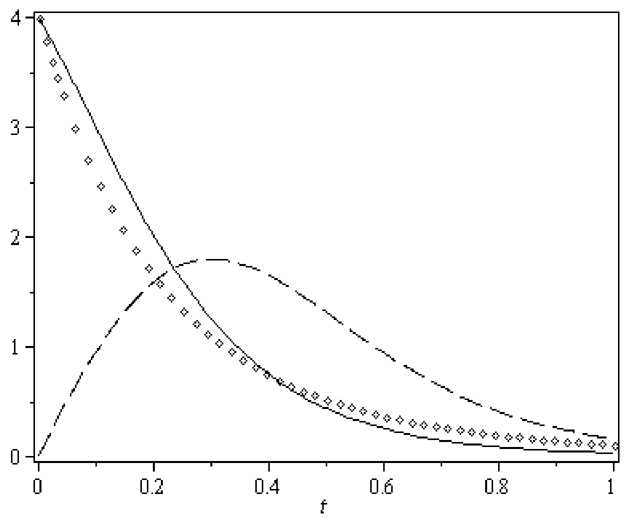

Figure 1: Probability density functions $g(t;\alpha,\beta,\lambda)$ for α=1, β=2.5, λ=3 (solid line), α=0, β=2.5, λ=3 (dashed line), and α=1, β= 0, λ=3 (dotted line).

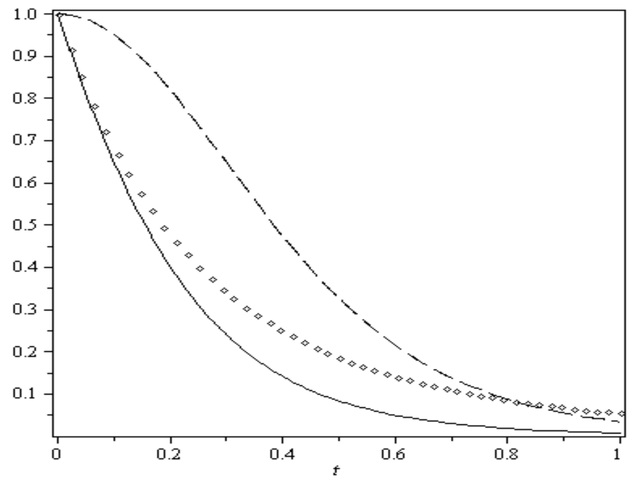

Figure 2: Reliability functions $\overline{G}(t;\alpha,\beta,\lambda)$ for α=1, β=2.5, λ=3 (solid line), α=0, β=2.5, λ=3 (dashed line), and α=1, β= 0, λ=3 (dotted line).

respectively. Recently, it is observed, see [11], that the reversed hazard function plays an important role in the reliability analysis. The reversed hazard function of the $G(t;\alpha,\beta,\lambda)$ is

$$r(t;\alpha,\beta,\lambda)=\frac{g(t;\alpha,\beta,\lambda)}{G(t;\alpha,\beta,\lambda)}$$

$$=\frac{\{1+\lambda e^{-(\alpha t+\frac{\beta}{2}t^2)}\}(\alpha+\beta t)\,e^{-(\alpha t+\frac{\beta}{2}t^2)}e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]}}{1-e^{-(\alpha t+\frac{\beta}{2}t^2)}e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]}} \quad (9)$$

The mean residual lifetime for a new three parametric exponential family distribution is given by [12]

$$m(a;\alpha,\beta,\lambda)=E(T-a\,|\,T\geq a)=\int_a^\infty\frac{g(t;\alpha,\beta,\lambda)}{1-G(a;\alpha,\beta,\lambda)}\ln(\frac{g(t;\alpha,\beta,\lambda)}{1-G(a;\alpha,\beta,\lambda)})dt$$

$$=\ln(1-G(a;\alpha,\beta,\lambda))-\frac{1}{1-G(a;\alpha,\beta,\lambda)}\int_a^\infty g(t;\alpha,\beta,\lambda)\ln(g(t;\alpha,\beta,\lambda))dt \quad (10)$$

3. Statistical Properties

In this section we provide some of basic statistical properties of the $g(t;\alpha,\beta,\lambda)$.

3.1. Quintile, median and mode

The quintile x_q of the $g(t;\alpha,\beta,\lambda)$ is the real solution of the following equation: $G(t_q;\alpha,\beta,\lambda)$ -q=0 thus

$$\lambda\, e^{-(\alpha t_q+\frac{\beta}{2}t_q{}^2)} - (\alpha t_q + \beta t^2{}_q/2) - \lambda - \ln(1-q) = 0 \qquad (11)$$

Using (11), the median of the $g(t;\alpha,\beta,\lambda)$ can be obtained, by setting q =1/2, then for α =1, β =2.5, λ =3 , we find $t_{0.5} \approx 0.0463196$

The mode of $g(t;\alpha,\beta,\lambda)$ can be obtained as a solution of the following non-linear equation with respect to t.

$$[(\beta-\lambda(\alpha+\beta t)^2 e^{-(\alpha t+\frac{\beta}{2}t^2)} - (\alpha+\beta t)^2) + \lambda e^{-(\alpha t+\frac{\beta}{2}t^2)}(\beta-\lambda(\alpha+\beta t)^2 e^{-(\alpha t+\frac{\beta}{2}t^2)} - 2(\alpha+\beta t)^2)] \qquad (12)$$
$$e^{-\lambda[1-e^{-(\alpha t+\frac{\beta}{2}t^2)}]} e^{-(\alpha t+\frac{\beta}{2}t^2)} = 0$$

It is not possible to obtain the explicit solution in the general case. It has to be obtained numerically. For different special cases, the explicit forms may be obtained.

3.2 Moments

If T has the $g(t;\alpha,\beta,\lambda)$ distribution, then m^{th} moment of T, is given as follows:

$$E[T^m] = m\int_0^\infty t^{m-1}\overline{G}(t;\alpha,\beta,\lambda)dt$$
$$= \mathrm{m}e^{-\lambda}\int_0^\infty t^{m-1}e^{-\alpha t-\beta t^2/2}\sum_{n=0}^\infty \frac{(\lambda e^{-\alpha t-\beta t^2/2})^n}{n!}dt$$
$$= \mathrm{m}e^{-\lambda}\sum_{n=0}^\infty\sum_{k=0}^\infty \frac{\lambda^n(-\alpha(n+1))^k}{n!k!}\int_0^\infty t^{m-1}e^{-\beta(n+1)t^2/2}dt \qquad (13)$$
$$= \frac{\mathrm{m}e^{-\lambda}}{2}\left(\frac{2}{\beta}\right)^{m/2}\Gamma\left(\frac{m}{2}\right)\sum_{n=0}^\infty\sum_{k=0}^\infty \frac{\lambda^n(-\alpha)^k(n+1)^{k-m/2}}{n!k!}$$

for m = 1,2, . . ., where Γ(.) is the gamma function.

The moments of the new extended exponential distribution are given by

For β=0 , α > 0:

$$E[T^m]_{\beta=0} = m\int_0^\infty t^{m-1}\overline{G}(t;\alpha,\lambda)dt$$
$$= \mathrm{m}e^{-\lambda}\int_0^\infty t^{m-1}e^{-\alpha t}\sum_{n=0}^\infty\frac{(\lambda e^{-\alpha t})^n}{n!}dt \qquad (14)$$
$$= \frac{\Gamma(m+1)e^{-\lambda}}{\alpha^m}\sum_{n=0}^\infty\frac{\lambda^n}{n!(n+1)^m}$$

So that, the mean and variance of $g(t;\alpha,\beta,\lambda)$ respectively, are

$$E[T] = \frac{1-e^{\lambda}}{\alpha\lambda} \text{ and } Var[T] \le \frac{\pi^2}{3\alpha^2} - \left(\frac{1-e^{\lambda}}{\alpha\lambda}\right)^2$$

For α =0 , β >0:

The moments of the new extended Rayleigh distribution are given by

$$E[T^m]_{\alpha=0} = m\int_0^\infty t^{m-1}\overline{G}(t;\beta,\lambda)dt$$

$$= \text{m}e^{-\lambda}\int_0^\infty t^{m-1}e^{-\beta t^2/2}\sum_{n=0}^\infty \frac{(\lambda e^{-\beta t^2/2})^n}{n!}dt \quad (15)$$

$$= \frac{me^{-\lambda}\Gamma(\frac{m}{2})}{2(\beta/2)^{m/2}}\sum_{n=0}^\infty \frac{\lambda^n}{n!(n+1)^{m/2}}$$

3.3. Order Statistics

It will also be useful to derive the pdf of the rth order statistic $T_{(r)}$ of a random sample $T_1,.., T_n$ drawn from the distribution was proposed by equation (5) with parameters α, β and λ. From [13], the pdf of $T_{(r)}$ is given by

$$g_{r:n}(t) = \frac{[G(t)]^{r-1}[\overline{G}(t)]^{n-r}g(t)}{B(r,\text{n - r}+1)} \quad (16)$$

where B(., .) is the beta function.

Using $h(t)=\alpha+\beta t$, $H(t)=\alpha t+\beta t^2/2$, and substituting (5), (6) and (7) into (16), we get

$$g_{r:n}(t) = n[1+\lambda e^{-H(t)}]h(t)\binom{n-1}{\text{r - 1}}\sum_{i=0}^{r-1}(-1)^i\binom{r-1}{\text{i}}e^{-(n+i+1-r)[\lambda+H(t)-\lambda e^{-H(t)}]} \quad (17)$$

4. Parameter Estimations

In this section, we derive the maximum likelihood estimates of the unknown parameters *α, β* and *λ* of $g(t;\alpha,\beta,\lambda)$ based on a complete sample. Let us assume that we have a simple random sample $T_1, T_2,\ldots,T_n$ from $g(t;\alpha,\beta,\lambda)$ The likelihood function of this sample is

$$L=\prod_{i=1}^n g(t_i;\alpha,\beta,\lambda) \quad (18)$$

Substituting from (7) into (18), we get

$$L=\prod_{i=1}^n\{1+\lambda e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}\}(\alpha+\beta t_i)\,e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}e^{-\lambda[1-e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}]} \quad (19)$$

It can be written as;

$$L=(\alpha^n+\beta^n T_1)e^{-(\alpha T_1+\beta T_2)}e^{-\lambda[n-Y]}(1+\lambda^n e^{-(\alpha T_1+\beta T_2)}) \quad (20)$$

Where $T_j=\frac{1}{j}\sum_{i=1}^n t_i^j$, $j=1,2$ and $Y=\sum_{i=1}^n e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}$

The log-likelihood function becomes

$$\text{lnL} = \ln(\alpha^n+\beta^n T_1)-(\alpha T_1+\beta T_2)-\lambda(n-Y)+\ln(1+\lambda^n e^{-(\alpha T_1+\beta T_2)}) \quad (21)$$

Setting the first partial derivatives of lnL with respect α, β and λ to zero, the likelihood equations are

$$0= -\frac{\lambda^n T_1 e^{-(\alpha T_1+\beta T_2)}}{1+\lambda^n e^{-(\alpha T_1+\beta T_2)}}+\frac{n\alpha^{n-1}}{(\alpha^n+\beta^n T_1)}-T_1-\lambda\sum_{i=1}^n t_i e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)} \quad (22)$$

$$0=\frac{n\beta^{n-1}T_1}{(\alpha^n+\beta^n T_1)}-\frac{\lambda}{2}\sum_{i=1}^n t_i^2 e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}-\frac{T_2\lambda^n e^{-(\alpha T_1+\beta T_2)}}{1+\lambda^n e^{-(\alpha T_1+\beta T_2)}}-T_2 \quad (23)$$

$$0= -n+\sum_{i=1}^n e^{-(\alpha t_i+\frac{\beta}{2}t_i^2)}+\frac{n\lambda^{(n-1)}e^{-(\alpha T_1+\beta T_2)}}{1+\lambda^n e^{-(\alpha T_1+\beta T_2)}} \quad (24)$$

The maximum likelihood estimates can be obtained by solving the non-linear equations numerically for α, β and λ. This can be done using Mathematica and Maple, among other packages. The relatively large number of parameters can cause problems especially when the sample size is not large. A good set of initial values is essential.

Asymptotic Confidence Bounds: Since the MLEs of the unknown parameters α, β, λ cannot be obtained in closed forms, then it is not easy to derive the exact distributions of the MLE of these parameters. Thus, we derive the approximate confidence intervals of the parameters based on the asymptotic distributions of the MLE of the parameters. It is known that the asymptotic distribution of the MLE $(\hat{\alpha}, \hat{\beta}, \hat{\lambda})$ is given by see [14], $((\hat{\alpha}, \hat{\beta}, \hat{\lambda}) - (\alpha, \beta, \lambda)) \to N(0, I_0^{-1})$, where I_0^{-1} is the variance covariance matrix of the unknown parameters (α, β, λ), where I_0^{-1} is the inverse of the observed information matrix

$$I_0^{-1} = \begin{pmatrix} \frac{\partial^2 \ln L}{\partial^2 \alpha^2} & \frac{\partial^2 \ln L}{\partial \alpha \partial \beta} & \frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & \frac{\partial^2 \ln L}{\partial^2 \beta^2} & \frac{\partial^2 \ln L}{\partial \beta \partial \lambda} \\ \frac{\partial^2 \ln L}{\partial \lambda \partial \alpha} & \frac{\partial^2 \ln L}{\partial \lambda \partial \beta} & \frac{\partial^2 \ln L}{\partial^2 \lambda^2} \end{pmatrix}^{-1} \tag{25}$$

thus

$$I_0^{-1} = \begin{pmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha}) & Cov(\hat{\alpha}, \hat{\lambda}) \\ Cov(\hat{\beta}, \hat{\alpha}) & Var(\hat{\beta}) & Cov(\hat{\beta}, \hat{\lambda}) \\ Cov(\hat{\lambda}, \hat{\alpha}) & Cov(\hat{\lambda}, \hat{\beta}) & Var(\hat{\lambda}) \end{pmatrix} \tag{26}$$

The derivatives in I_0 are given as follows

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = \frac{n(n-1)\alpha^{n-2}}{(\alpha^n + \beta^n T_1)} - \frac{n^2 \alpha^{2n-2}}{(\alpha^n + \beta^n T_1)^2} + \frac{\lambda^n T_1^2 e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} - \frac{\lambda^{2n} T_1^2 e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} + \lambda \sum_{i=1}^{n} t_i^2 e^{-(\alpha t_i + \beta t_i^2/2)} \tag{27}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} = -\frac{T_1 n^2 \alpha^{n-1} \beta^{n-1}}{(\alpha^n + \beta^n T_1)^2} + \frac{\lambda^n T_1 T_2 e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} - \frac{\lambda^{2n} T_1 T_2 e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} + \frac{\lambda}{2} \sum_{i=1}^{n} t_i^3 e^{-(\alpha t_i + \beta t_i^2/2)} \tag{28}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = -\frac{n \lambda^{n-1} T_1 e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} + \frac{n \lambda^{2n-1} T_1 e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} - \sum_{i=1}^{n} t_i e^{-(\alpha t_i + \beta t_i^2/2)} \tag{29}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \frac{n(n-1) T_1 \beta^{n-2}}{(\alpha^n + \beta^n T_1)} - \frac{n^2 \beta^{2n-2} T_1^2}{(\alpha^n + \beta^n T_1)^2} + \frac{\lambda^n T_2^2 e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} - \frac{\lambda^{2n} T_2^2 e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} + \frac{\lambda}{4} \sum_{i=1}^{n} t_i^4 e^{-(\alpha t_i + \beta t_i^2/2)} \tag{30}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = -\frac{n \lambda^{n-1} T_2 e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} + \frac{n \lambda^{2n-1} T_2 e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} - \frac{1}{2} \sum_{i=1}^{n} t_i^2 e^{-(\alpha t_i + \beta t_i^2/2)} \tag{31}$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{n(n-1)\lambda^{n-2} e^{-(\alpha T_1 + \beta T_2)}}{1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)}} - \frac{n^2 \lambda^{2n-2} e^{-2(\alpha T_1 + \beta T_2)}}{(1 + \lambda^n e^{-(\alpha T_1 + \beta T_2)})^2} \tag{32}$$

The above approach is used to derive the $100(1-\alpha)\%$ confidence intervals of the parameters α, β, λ as in the following forms

$$\hat{\alpha} \pm Z_{\alpha/2}\sqrt{Var(\hat{\alpha})}, \quad \hat{\beta} \pm Z_{\alpha/2}\sqrt{Var(\hat{\beta})}, \quad \hat{\lambda} \pm Z_{\alpha/2}\sqrt{Var(\hat{\lambda})} \tag{33}$$

Here, $Z_{\alpha/2}$ is the upper $(\alpha/2)$ th percentile of the standard normal distribution. It should be mentioned here as it was pointed by a referee that if we do not make the assumption that the true parameter vector (α, β, λ) is an interior point of the parameter space then the asymptotic normality results will not hold. If any of the true parameter value is 0, then the asymptotic distribution of the maximum likelihood estimators is a mixture distribution, see for example [15] in this connection. In that case obtaining the asymptotic confidence intervals becomes quite difficult and it is not pursued here.

5. Application

In this section we use the real data were $(t_1, t_2, \ldots, t_{10})$ = (31, 43, 56, 65, 73, 82, 96, 101, 111, 135), we assume that these data follow the distribution $G(t; \alpha, \beta, \lambda)$. First we compute the maximum likelihood estimator(s) for the parameters (α, β, λ), finally, we compute the asymptotic confidence intervals of the parameters (α, β, λ).

The MLE of the parameters is $\begin{bmatrix}\hat{\alpha}\\ \hat{\beta}\\ \hat{\lambda}\end{bmatrix}=\begin{bmatrix}4.0700\\ 2.1200\\ 0.4025\end{bmatrix}$

and by substituting the MLE of unknown parameters in equation (25), we get estimation of the variance covariance matrix as

$$I_0^{-1}=\begin{bmatrix}4.307\times10^{-5} & -7.081\times10^{-5} & 9.075\times10^{-3}\\ -8.503\times10^{-4} & 8.725\times10^{-6} & -4.917\times10^{-4}\\ 2.614\times10^{-3} & -9.566\times10^{-4} & 0.0164\end{bmatrix}$$

The approximate 95% two sided confidence intervals of the parameters (α,β,λ)

Confidence intervals			
Significance level	Parameter Estimation	Upper	lower
a=0.05	α	4.0829	4.0571
a=0.05	β	2.2579	2.1142
a=0.05	λ	0.6535	0.1515

6. Conclusion

The new distribution with three parameters, referred to as the Poisson distribution is used to add a new parameter to the linear exponential distribution given in this study. We discussed some reliability and statistical properties of new distribution. In this paper we have considered the problem of estimation of parameters of new distribution. Procedure for the maximum likelihood estimation has been discussed.

References

[1] C. Kus, “A new lifetime distribution, (2007).*Computational Statistics and Data Analysis*, vol. 51, no. 9, pp. 4497–4509.
[2] A. W. Marshall and I. Olkin, (2007) *Life Distributions*, Springer, New York, NY, USA.
[3] W. Barreto-Souza, A. L. de Morais, and G. M. Cordeiro,(2011). The weibull-geometric distribution, *Journal of Statistical Computation and Simulation*, vol. 81, no. 5, pp. 645–657.
[4] W. Barreto-Souza and F. Cribari-Neto,(2009). A generalization of the exponential-Poisson distribution,” *Statistics & Probability Letters*, vol. 79, no. 24, pp. 2493–2500.
[5] M. Chahkandi and M. Ganjali,(2009). On some lifetime distributions with decreasing failure rate, *Computational Statistics and Data Analysis*, vol. 53, no. 12, pp. 4433–4440.
[6] V. Pappas, K. Adamidis and S. Loukas A Family of Lifetime Distributions, International Journal of Quality, Statistics, and Reliability, Volume 2012, Article ID 760687.
[7] C.D. Lai, M. Xie, D.N.P. Murthy,(2001). Bathtub shaped failure rate distributions, in: N. Balakrishnan, C.R. Rao (Eds.), Handbook in Reliability, vol. 20, pp. 69–104.
[8] T. Zhang, M. Xie, L.C. Tang, S.H. Ng, (2005). Reliability and modeling of systems integrated with firmware and hardware, Int. J. Reliab. Quality Safety Eng, vol. 12 (3), pp. 227–239.
[9] M.E. Ghitany,(2006).Reliability properties of extended linear failure-rate distributions, Probability in the Engineering and Information Sciences, vol. 21, pp. 441 - 450.
[10] S. Çelebioğlu, (2010). On an extension of the exponential and Weibull distributions, *SDU Journal of Science (E-Journal), vol. 5 (1), pp. 137-146*
[11] R.C. Gupta and R.D. Gupta,(2007). Proportional reversed hazard model and its applications, Journal of Statistical Planning and Inference, vol. 137, no. 11, pp. 3525 - 3536.
[12] Ebrahimi, N. (1996). How to measure uncertainty in the residual life time distribution, The Indian Journal of Statistics, 58 series A, pp. 48–56.
[13] Balakrishnan AN, Nagaraja HN. (1992). A first course in order statistics. New York: Wiley-Interscience.
[14] Lawless, J. F. (2003). "Statistical Models and Methods for Lifetime Data". John Wiley and Sons, New York.
[15] Self, S. G. and Liang, K. Y. (1987). "Asymptotic properties of maximum likelihood estimators and likelihood ratio tests under nonstandard conditions", Journal of the Americal Statistical Association, 82, pp. 605 - 610.