The Network Transportation Problem with Volume Discount on Shipping Cost

Issaka Haruna [1], Mubarack Ahmed[2], Akweittey Emmanuel[3]

[1]Tutor, Aduman Senior High School, Kumasi. Past Mphil. Student, Department of mathematics, Kwame Nkrumah University Of Science and Technology, Kumasi, Ashanti, Ghana.

[2] Lecturer Information Technology Department, Garden City University College , Kumasi, Ashanti, Ghana

[1]Tutor, Aduman Senior High School, Kumasi. Past MSc. Student,Department of mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ashanti, Ghana.

Lecturer, Department of Mathematics, Presbyterian University College, Kumasi, Ashanti, Ghana

Abstract: ***It is assumed that the cost of goods per unit shipped from a given source to a given destination is fixed regardless of the amount shipped. But in actuality the cost is not always fixed. Volume discount however is sometimes allowed for large shipments. This project therefore seeks to develop a mathematical model using optimization techniques to bridge the gap between demand and supply by discounting so as to minimize total transportation cost. The problem that will be addressed in this study centers on the transportation problems experienced by freight companies. Volumes of goods to be shipped incur costs hence acquiring volume discounts could effectively lead to reduced shipping costs. However, there are transportation problems that hinder the materialization of improved total output through reduced costs of shipping. We shall provide algorithms and different solution procedures to the different cases that might arise. The main aim of this study is to design mathematical program that would improve the total output of freight companies especially since they deal with shipping of goods by volume. Whether maximum profit will be realized with discounts on large volumes or not means to determine the best transportation route that would lead to low transportation cost and the effective transportation of these goods. A test of this algorithm on the GALCO company limited(Kumasi, Ghana), recorded the following results as a feasible solution; x=(x11 =15, x12 =0, x13=0, x21=5, x22=10, x23=8, x31=0, x32=0, x33=10). This will result in a total transportation cost of 1500x15 + 5000x5 + 10000x10 + 8000x8 + 10000x10= GHC 311,500. This value as will be shown is far below the original value. These results are subject to readjustment and reassignment. Sometimes there may be different ways to model a particular problem but choosing the best approach minimizes the complicity of the problem and time to solve. Since any programming problem with constraint matrix structure the same as the transportation problem, can be regarded as a transportation type problem regardless of its physical meaning and because of its simple structure, modeling such problems as transportation problem requires much less effort to solve than modeling it differently.***

Keywords: destination, minimize, maximize, source, transportation, respondent

1. Introduction

When considering transportation a lot of factors must be considered. These factors include port selection, inland movement, port to port carrier selection and so on. In addition to these factors distribution-related issues must also be given attention to. Even freight companies that deal in large volume movements are bound to encounter serious transportation problems. The understanding of these transportation problems especially that which affects shipping cost is critical. Volume discounts are meant to target shipping cost and in minimizing these cost, volume discount must be acquired.

A very important area of application is the management and efficient use of scarce resources to increase productivity. The application areas include operational problems (distributions of goods, production scheduling, production and machine sequencing), planning problems (capital budget facility allocation) and design problems (telecom and transport network design).

Shetty (1959) formulated an algorithm to solve transportation problems taking nonlinear costs. The case when a convex production cost is included at each supply center besides the linear transportation cost was considered.

The transportation problem, network integer programming problem, deals with the distribution of any commodity from any group of 'sources' to any group of destinations or 'sinks' in the most cost effective way with a given 'supply' and 'demand' constraints . Depending on the nature of the constraint function, the transportation problem can be grouped into linear and nonlinear transportation problem.

In the linear transportation problem (ordinary transportation problem) the cost per unit commodity shipped from a given source to a given destination is constant, regardless of the amount shipped. It is always supposed that the mileage (distance) from every source to every destination is fixed. To solve such transportation problem we have the streamlined simplex algorithm which is very efficient. The problem with the production capacity of each source fixed with constant unit transportation cost was originally formulated by Hitchcock (1941) and was subsequently dealt with independently by Koopmans during World War II.

However two cases arise that makes the transportation problem fail to be linear. First, the cost per unit commodity transported may not be fixed for volume discounts sometimes are available for large volume shipments. This would make the cost function piecewise linear. In this instance the problem may be reformulated into a piecewise linear or concave programming problem with linear constraints.

Second, in special conditions such as transporting emergency materials when natural calamity occurs or transporting military during war times, where carrying network may be destroyed, directions from some sources to some destinations are no longer regular. As a result, the choice of different routs could lead to nonlinear (quadratic, convex …) objective function.

In both of the above cases solving the transportation problem is not as simple as that of the linear case. In this work, solution procedures to the generalized transportation problem taking nonlinear cost function are considered. We then formulate the nonlinear transportation problem as; For a given set of n sources of commodity with known supply capacity and a set of m destinations with known demands; The objective function of transportation cost, nonlinear, and differentiable for a unit of product from each source to each destination. We are required to find the amount of product to be supplied from each source (may be the market) to meet the demand of each destination in such a way as to minimize the total transportation cost. The purpose of this paper is to find out whether giving discounts on transportation charges could minimize total transportation cost thereby increasing total revenue of both producers and retailers; and some of the problems aforementioned. We then apply existing general nonlinear programming algorithms to it making a suitable modification in order to use the special structure of the problem.

2. Methodology

A typical application of the transportation problem is to determine an optimal plan for shipping goods from various sources to various destinations given supply and demand constraints in order to minimize total shipping cost. We sometimes assume that the cost of goods per unit shipped from a given source to a given destination is fixed regardless of the amount shipped. When volume discounts are offered, the objective function or the constraint functions assume a nonlinear form. We therefore use the nonlinear method of solution to solve such a problem. In this section, we consider a transportation problem with nonlinear cost function. We find different solution procedures depending on the nature of the objective function.

For the purposes of this paper, we shall formulate the Karish Kun Takah (KKT) condition as below.
Given a differentiable function

$C: \mathbb{R}^{nm} \longrightarrow \mathbb{R}$

Consider the nonlinear transportation problem (NTP) below.
Min $C(x)$
Subject to
$Ax = b,\ x \geq 0$

Where

$$x = \begin{pmatrix} x_{11} \\ \cdot \\ \cdot \\ x_{ij} \\ \cdot \\ \cdot \\ x_{nm} \end{pmatrix} \quad b = \begin{pmatrix} s_1 \\ \cdot \\ s_n \\ \cdot \\ d_1 \\ \cdot \\ d_m \end{pmatrix} \quad A = \begin{pmatrix} 1\ 1\ 1\ 1 \ldots\ldots\ldots\ldots \\ \ldots 1\ 1\ 1 \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots 1\ 1\ 1 \ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots 1\ 1\ 1\ 1 \ldots \\ 1\ 1\ 1 \ldots\ldots\ldots\ldots 1\ 1 \end{pmatrix}$$

The optimal feasible point x should satisfy the KKT conditions

$$\frac{\partial z}{\partial x_{ij}} = \frac{\partial c(x)}{\partial x_{ij}} - (u_i, v_j) = 0, \text{ and}$$

$x_{ij} \geq 0,$

Where $(u,v) = (u_1, u_2 \ldots u_n, v_1\ v_2 \ldots v_m)$.
From the above conditions and $\lambda_k \geq 0$, we get,

$$\frac{\partial z}{\partial x_{ij}} = \frac{\partial c(x)}{\partial x_{ij}} - (\overline{u_i + v_j}) \geq 0$$

$$x_{ij}\frac{\partial z}{\partial x_{ij}} = \frac{\partial c(x)}{\partial x_{ij}} - (\overline{u_i + v_j}) \geq 0$$

$x_{ij} \geq 0.$

It follows that if

$$\frac{\partial c(x_{ij})}{\partial x_{ij}} - (u_i + v_j) \geq 0$$

For all x_{ij} non-basic, stop and x is KKT point.

2.1 General Solution Procedure for the Nonlinear Transportation Problem

Initialization

Find an initial basic feasible solution x

Iteration
Step 1 If x is KKT point, stop. Otherwise go to the next step.
Step 2 Find a new feasible solution that improves the cost function and return to step 1.
Cgonsider the transportation problem below

Minimize

$$\sum_{i=1}^{3} \cdot \sum_{j=1}^{3} C_{ij} x_{ij} = d_j$$

Subject to

$$x_{11} + x_{12} + x_{13} = 150$$
$$x_{21} + x_{22} + x_{23} = 175$$
$$x_{31} + x_{32} + x_{33} = 275$$
$$x_{11} + x_{21} + x_{31} = 200$$
$$x_{12} + x_{22} + x_{32} = 100$$
$$x_{13} + x_{23} + x_{33} = 600$$

$$C_{12}(x_{12}) = 7x_{12} - 0.01{x^2}_{12}$$
$$C_{13}(x_{13}) = 10x_{13} - 0.02{x^2}_{13}$$
$$C_{21}(x_{21}) = 7x_{21} - 0.03{x^2}_{21}$$
$$C_{31}(x_{31}) = 4x_{31} - 0.02{x^2}_{31}$$
$$C_{32}(x_{32}) = 5x_{32} - 0.02{x^2}_{32}$$
$$C_{33}(x_{33}) = 12x_{33} - 0.015{x^2}_{33}$$

$$C_{22}(x_{22}) = 11x_{22} - 0.01x$$

We then use the West Corner rule to obtain the initial basic feasible solution.

$x = (x_{B11}, x_{12}, x_{13}, x_{B21}, x_{B22}, x_{B23}, x_{31}, x_{32}, x_B)$
= (15, 0, 0, 5, 10, 8, 0, 0, 10).

The partial derivatives at x are given as follows:

$$\frac{\partial f(\bar{x})}{\partial x_{11}} = 3, \quad \frac{\partial f(\bar{x})}{\partial x_{21}} = 4, \quad \frac{\partial f(\bar{x})}{\partial x_{22}} = 9,$$

$$\frac{\partial f(\bar{x})}{\partial x_{23}} = 10.25, \qquad \frac{\partial f(\bar{x})}{\partial x_{33}} = 3.$$

Now we find,

$$\frac{\partial z}{\partial x_{Bij}} = \frac{\partial f(\bar{x})}{\partial x_{Bij}} - (u_i + v_j) = 0$$

ie

$$\frac{\partial f(x)}{\partial x_{Bij}} = (u_i + v_j).$$

Hence

$$u_1 + v_1 = 3, \quad u_2 + v_1 = 4, \quad u_2 + v_2 = 9,$$

$$u_2 + v_3 = 10.25, \qquad u_3 + v_3 = 3.75$$

setting $u_1 = 0$, we obtain

$$u_0 = 0, \quad u_2 = 1, \quad u_3 = -5.5$$

$$v_1 = 3, \quad v_2 = 8 \quad v_3 = 9.$$

Hence the reduced costs for the non basic variables are

$$\frac{\partial z}{\partial x_{12}} = \frac{\partial f(\bar{x})}{\partial x_{12}} - (u_1 + v_2) = 5$$

$$\frac{\partial z}{\partial x_{13}} = \frac{\partial f(\bar{x})}{\partial x_{13}} - (u_1 + v_3) = 0.75$$

$$\frac{\partial z}{\partial x_{31}} = \frac{\partial f(\bar{x})}{\partial x_{31}} - (u_3 + v_1) = 6.5$$

$$\frac{\partial z}{\partial x_{32}} = \frac{\partial f(\bar{x})}{\partial x_{32}} - (u_3 + v_2) = 1.25$$

Since all are non-negative, x is **kkt** point and optimal solution to the problem.

3. Conclusions and Discussions

After the research, it was observed that the company could save between 25% and 37% on transport. It had no planed means and route of transport for its products. An initial cost of about GHS 600,000 which was spent on transport per year had drastically reduced to GHS 311,500, hence saving the company huge sums of money. The percentage saved is calculated as: percentage saved =(amount saved/old cost) x100 = ((600,000 – 311500)/600,000) x 100= 48.08%.

Sometimes, there may be different ways to model a particular problem, but choosing the best approach minimizes the complexity of the problem and time to solve.

Since any programming problem with constraint matrix structure the same as the transportation problem, it can be regarded as a transportation type problem regardless of its physical meaning and because of its simple structure, modeling such problems as transportation problem requires much less effort to solve than modeling it differently.

In this paper the nonlinear transportation problem is considered as a nonlinear programming problem and algorithms to solve this particular problem are given. The first algorithm is similar to that of the transportation simplex algorithm except for the nonlinearity assumption.

The second algorithm is dependent on the simplex algorithm of Zangwill that we modified to use the special property of the coefficient matrix of the transportation problem so that we may take shortcuts to make problem solving simple. For the purposes of even distribution of each product, reasonable reallocation could be made to unassigned destinations.

We then conclude that given discounts on cost of transportation could lead to increased productivity of producers. This as a result of the fact that wholesalers and retailers, will have to pay less on transport for buying in large quantities; subsequently, consumers will buy at a lower cost comparatively.

Reference

[1] Abad, P. L. (1988). Determining optimal selling price and lot-size when the supplier offers all-unit Quantity Discounts, *Decision Science*, page 19, 622-634.
[2] Adby P.R and Dempster M.A.H. (1976). Introduction to Optimization Methods.
[3] Arcelus, F.J.; J. Bhadury and Srinivasan G (1996). Buyer-seller quantity discount strategies under profit maximizing objectives: *A game-theoretical analysis*, page 49-72.
[4] Banerjee, A. (1986). Quantity discount-pricing model to increase profits, *Management Science*. 32, 1513-1517.
[5] Bazaraa M.S. (1993). Nonlinear programming: *theory and algorithms*, *John Wiley and Sons.*
[6] Caputo A.C. (2006). A genetic approach for freight transportation planning, *Industrial Management and Data Systems*, Vol. 106 No. 5.
[7] Frank Sharp J. (May, 1970). A Decomposition Algorithm for multifacility production transportation problem with nonlinear production costs, *Econometrics*, Vol. 38 No.3
[8] Frederick S., Hiller and Gerald J. Lieberman (1990). Introduction to mathematical programming, *McGraw-Hill, Incopration*, *Second Edition*.
[9] Hitchcock F.L. (1941). The Distribution of a Product from Several Sources to Numerous Localities, *journal of Mathematics and Physics* Vol.20, page 224-230.
[10] Reep J. and Leaengood S. (June 2002). Transportation problem: A Special Case of Linear Programming Problems, *Operations Research Society of America*.
[11] Sa G. (1969). Branch- and- Bound and Approximate Solutions to the Capacitated Plant Location Problem, *Operations Research*, Vol. 17, page 1016-111.
[12] Shetty C.M. (1959). A Solution to the Transportation Problem with Nonlinear Costs," *Operation Research*. Vol. 7. No.5. page 571-580.
[13] Simons (March, 1962). Nonlinear Programming for Operation research, *Society for Industrial and Applied Mathematics*, Vol.10, No.1.
[14] Soland R.M. (1974). Optimal Facility Location with Concave Costs" *Ops. Res.* Vol. 22, No.2. page 373-382.
[15] http://en.wikipedia.org/wiki/Transportation-problem